Renormalized CFT/ Effective AdS

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Work in progress
Outline

• Motivation and goal
• Prescription
• Examples
• Conclusion and open questions
AdS/CFT correspondence: Maldacena conjecture `97

N=4 conformal SYM
\( g_s = g_{YM}^2 \)

Type IIB string theory on AdS\(_5 \times S^5\)
\( R^4 = 4\pi g_s N l_s^4 \)

Phenomenological works motivated by the duality:
Randall-Sundrum models...
AdS/QCD
“bottom-up” AdS/CMT...
• Rules of thumb for model-building

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<td>Strong dynamics that breaks CFT also breaks gauge symmetry</td>
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Copied from Csaki, Hubisz and Meade 05’
But unlike the original N=4/Type IIB duality, the “phenomenological” duality

- Finite N
- Non-supersymmetric
The “phenomenological” duality

- Finite N
- Non-supersymmetric

<table>
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• Goal: assuming existence of an effective AdS field theory/CFT correspondence, understand how bulk interactions renormalize CFT 2-point correlator beyond the leading order in $N$.

More specifically, calculate anomalous dimensions of single- and double-trace operators arising from bulk interactions. E.g.: scalar single-trace operator:

$$\Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m^2 + \gamma}$$
Aside: definition of single- and double-trace operators

Given a matrix-valued field $\Phi(N \times N$ hermitian matrix),

Single-trace operators: $\text{Tr} \Phi^n$

Double-trace operators: $\left(\text{Tr} \Phi^n\right)^2$

Example:

Single-trace operators: $\text{Tr} F^2$  $\psi \bar{\psi}$

Double-trace operators: $\left(\text{Tr} F^2\right)^2$  $\left(\psi \bar{\psi}\right)^2$
More on the effective AdS/CFT correspondence

**Conjecture:** (Heemskerk, Penedones, Polchinski and Sully `09; Fitzpatrick, Katz, Poland, Simmons-Duffin `10)

Sufficient conditions for CFT to have a local bulk dual

- a mass gap: all single-trace operators of spin greater than 2 have parametrically large dimension.

More specifically, $m_{KK} \sim 1/R$; $R$: AdS radius

String states $m_s \sim 1/l_s \sim \lambda^{1/4}/R$;

- a small parameter such as $1/N$

$M_5 \sim N^{2/3}/R$
5D AdS EFT

$m_{\text{planck}}$

$m_s$

$1/R$

$0$
• Setup

AdS\textsubscript{d+1}: metric \[ ds^2 = \frac{z_0^2}{z^2} + \sum_{i=1}^{d} dx_i^2 \]

scalar field theory with contact interactions

CFT: at the bottom of the spectrum, only one single-trace scalar operator \( \mathcal{O}(x) \)

Double-trace operator from OPE \( O \times O \)

\[ \mathcal{O}_{n,l}(x) \equiv \mathcal{O}(\partial_\mu \partial^\mu)^n (\partial_{\nu_1} \partial_{\nu_2} \cdots \partial_{\nu_l}) \mathcal{O}(x) \]

n: twist l: spin
Boundary action

To make precise sense of the AdS/CFT, needs to introduce a radial regulator $\varepsilon$: $z_0 \geq \varepsilon > 0$

$$S_\varepsilon = S_{\varepsilon}^{local} + S_{\varepsilon}^{non-local}$$

$$S = S_\varepsilon + \int d^d x \int_{z_0 > \varepsilon} dz_0 \mathcal{L}_{bulk}$$
$S_{\text{local}}$

- Consist of all (high-dimensional) local counter-terms to restore conformal invariance.

- Requiring the total action $S$ is independent of $\varepsilon$, holographic RGE of the boundary local operators at classical level:

$$\partial_\varepsilon S_{\text{local}} = - \int_{z=\varepsilon} d^d x (\Pi \partial_z \phi - \sqrt{-g} \mathcal{L}) = - \int d^d x \mathcal{H}$$

Lewandowski, May and Sundrum `02...;

Heemskerk and Polchinski `10;

$S_{\epsilon}^{\text{non-local}}$

- Encodes the correlators of the dual field theory.
- The ansatz:

\[
\langle \exp \int_{\epsilon} \phi_0 \mathcal{O} \rangle_{\text{CFT}} = Z_\epsilon(\phi_0)
\]

Generating function of CFT

Boundary value of bulk field

Boundary partition function

\[
\langle \mathcal{O}(x) \mathcal{O}(0) \rangle = \epsilon^{2(d-\Delta)} \langle \phi_0(x) \phi_0(0) \rangle_{\text{nonlocal}}^{1PI}
\]

Normalization
\( S_{\epsilon}^{\text{non-local}} \)

- Example: free AdS scalar theory with Dirichlet b.c:
  (mixed momentum-position rep.)

\[
S_{\epsilon} = \frac{1}{2} \int d^d k d^d k' \delta(\vec{k} + \vec{k}') \epsilon^{-d+1} \phi_0(\epsilon, \vec{k}) \partial_{z_0} \phi(z_0, \vec{k}') |_{z_0 = \epsilon} \\
= \frac{1}{2} \int d^d k d^d k' \delta(\vec{k} + \vec{k}') \epsilon^{-d+1} \phi_0(\vec{k}) \partial_{z_0} K(z_0, \vec{k}') |_{z_0 = \epsilon} \phi_0(\vec{k}')
\]

\[
\langle \mathcal{O}(\vec{k}) \mathcal{O}(\vec{k}') \rangle
\]

\[
\phi(z_0, \vec{k}) = K(z_0, \vec{k}) \phi_0(\vec{k})
\]

\[
K(z_0, \vec{k}) = \left( \frac{z_0}{\epsilon} \right)^{d/2} \frac{K_{\nu}(kz_0)}{K_{\nu}(k\epsilon)} \quad \text{Bulk to boundary propagator}
\]
Prescription

- After turning on bulk interactions, CFT gets renormalized
  \[ \mathcal{O}_R(x) = Z \mathcal{O}(x) \]

- Calculate correction to the two-point \( \langle \mathcal{O}(x)\mathcal{O}(0) \rangle^{(1)} \)
  from AdS: correction to the 1PI boundary action

\[ \epsilon^{2(d-\Delta)} \langle \phi_0(x)\phi_0(0) \cdots \rangle^{1PI}_{\text{non-local}} \]

Insertions of bulk interaction
The renormalized two-point functions are **Finite** after taking the regulator away $\epsilon \to 0$

$$\langle \mathcal{O}_R(x) \mathcal{O}_R(0) \rangle = Z^2 \langle \mathcal{O}(x) \mathcal{O}(0) \rangle$$

$Z$ factor absorbs the $\epsilon$ dependence, more specifically, $\log \epsilon$ dependence

$$\gamma \equiv -\epsilon \frac{\partial \log Z}{\partial \epsilon}$$
A toy example: mass perturbation

\[ \delta m^2 \phi^2 \]

Exact solution:

\[ \Delta_{\text{exact}} = \frac{d}{2} + \sqrt{\left(\frac{d}{2}\right)^2 + m^2 + \delta m^2} \]

\[ = \Delta_0 + \frac{\delta m^2}{2\nu} + O((\delta m^2)^2) \]

\[ \nu = \sqrt{\frac{d^2}{4} + m^2} \]
A bit of information beforehand:
Main ingredient for calculating anomalous dimensions from contact interactions

\[ \langle \phi(x) \phi(0) \rangle \int \frac{dz_0 d^dz}{z_0^{d+1}} \frac{1}{2} \delta m^2 \phi(z_0, z)^2 \langle_{\text{nonlocal}} \]

\[ = \delta m^2 \int \frac{dz_0}{z_0} \frac{1}{\epsilon^d} \int \frac{dk}{(2\pi)^d} \left( \frac{K_{\nu}(kz_0)}{K_{\nu}(k\epsilon)} \right)^2 e^{-ik \cdot x} \]

\[ = \cdots - 2\delta m^2 \int \frac{dz_0}{z_0} \int \frac{dk}{(2\pi)^d} \epsilon^{2\nu-d} \left( \frac{k}{2} \right)^{2\nu} \frac{\Gamma(1-\nu)}{\Gamma(1+\nu)} e^{-ik \cdot x} + \cdots \]

\[ = \cdots + \epsilon^{2\nu-d} \log \epsilon \frac{2\delta m^2}{\pi^{d/2}} \frac{\Gamma(\Delta)}{\Gamma(\Delta - d/2)} |x|^{-2\Delta} + \cdots , \]
\[ \delta m^2 \phi^2 \]

A bit of information beforehand:
Main ingredient for calculating anomalous dimensions from contact interactions

\[ \langle \phi_0(x) \phi_0(0) \rangle \int \frac{dz_0 d^d z}{z_0^{d+1}} \frac{1}{2} \delta m^2 \phi(z_0, z)^2 \nonlocal^{1PI} \]

\[ = \ldots + \epsilon^{2\nu - d} \log \epsilon + \frac{2\delta m^2}{\pi^{d/2}} \frac{\Gamma(\Delta)}{\Gamma(\Delta - d/2)} |x|^{-2\Delta} + \ldots \]

\[ Z^2 - 1 = -\frac{\delta m^2}{\nu} \log \epsilon + \text{finite terms} \]

\[ \gamma = -\epsilon \frac{\partial \log Z}{\partial \epsilon} = \frac{\delta m^2}{2\nu} \]

Compared to the exact solution:

\[ \Delta_{\text{exact}} = \frac{d}{2} + \sqrt{\left(\frac{d}{2}\right)^2 + m^2 + \delta m^2} \]

\[ = \Delta_0 + \frac{\delta m^2}{2\nu} + O((\delta m^2)^2) \]
On the CFT side,

\[ \langle O(x)O(0) \rangle = \frac{1}{|x|^{2(\Delta + \gamma)}} = \frac{1}{|x|^2} (1 - 2\gamma \log(|x|\Lambda)) \]

\[ Z^2 - 1 = 2\gamma \log \Lambda \quad \gamma = \frac{\partial \log Z}{\partial \log \Lambda} \]

UV/IR duality

\[ \Lambda \leftrightarrow \frac{1}{\epsilon} \]

\[ \gamma \equiv -\epsilon \frac{\partial \log Z}{\partial \epsilon} \]
Example 2: mass perturbation again
-- toy model of integrating out heavy states

\[ \mathcal{V} = -\frac{1}{2} m_1^2 \phi^2 - \frac{1}{2} m_2^2 \chi^2 - \delta m^2 \phi \chi, \quad \delta m^2 \ll m_1^2 \neq m_2^2 \]

Again one could obtain the exact solution:

Diagonalize mass matrix

\[
\begin{pmatrix}
  m_1^2 & \delta m^2 \\
  \delta m^2 & m_2^2
\end{pmatrix}
\]

\[ \Delta = \frac{d^2}{2} + \sqrt{\frac{d^2}{4} + m^2} \]

\[ \gamma_1 = \frac{(\delta m^2)^2}{m_1^2 - m_2^2} \frac{1}{2 \nu_1} \]
Example 2: continued

\[
\begin{align*}
\langle \phi_0(x)\phi_0(0) \rangle^{(1)} &= \int \frac{dz_0}{z_0^{d+1}} \frac{dw_0}{w_0^{d+1}} \int \frac{d^d k}{(2\pi)^d} e^{-ik\cdot x} K(z_0, k) G(z_0, w_0; k) K(w_0, k) \\
&= 2(\delta m^2)^2 \int_{\epsilon} z_0^{d-1} \frac{1}{z_0^d} K_{\nu_1}(kz_0) K_{\nu_2}(kw_0) \int_{\epsilon} w_0^{d-1} \frac{I_{\nu_2}(kw_0)}{w_0} \\
\gamma_1 &= \frac{(\delta m^2)^2}{m_1^2 - m_2^2} \frac{1}{2\nu_1}
\end{align*}
\]
Example 2: Continue

\[ V = -\frac{1}{2}m_1^2\phi^2 - \frac{1}{2}m_2^2\chi^2 - \delta m^2\phi\chi, \quad \delta m^2 \ll m_1^2 \ll m_2^2 \]

\[ \gamma_1 = \frac{(\delta m^2)^2}{m_1^2 - m_2^2} \frac{1}{2\nu_1} \]

\[ \delta m'^2 = -\frac{(\delta m^2)^2}{m_2^2} \]

\[ \gamma_1 = \frac{\delta m'^2}{2\nu_1} \]
Contact interaction $\phi^4$

- Single-trace operator

\[
\gamma = \frac{\delta m^2}{2\nu}
\]

\[
\delta m^2 = \int_0^{\alpha/z_0} \frac{d^d p}{(2\pi)^d} G(z_0, z_0; p)
\]

Momentum loop integration is divergent

\[
\int_\epsilon \frac{dz_0}{z_0} \int \frac{d^d k}{(2\pi)^d} \left( \frac{K_\nu(kz_0)}{K_\nu(k\epsilon)} \right)^2 e^{-ik\cdot x} = \delta m^2
\]

\[
\frac{\mu}{2} \int \frac{d^d p}{(2\pi)^d} G(z_0, z_0; p)
\]

Position-dependent cutoff

\[
\delta m^2 = \int_0^{\alpha/z_0} \frac{d^d p}{(2\pi)^d} G(z_0, z_0; p)
\]
Power countings of the divergences:

Flat space: \( \int (d^{d+1}p) \frac{1}{p^2} \sim \Lambda^{d-1} \)

Warped: \( \int (d^d p) G(z_0, z_0; p) \sim \int (d^d p) K_\nu(pz_0) I_\nu(pz_0) \sim \int (d^d p) \frac{1}{p} \sim \Lambda^{d-1} \)

Could also regulate the momentum loop with Pauli-Villas

Final answer is scheme-dependent
Double-trace operators

\[ O_{n,l}(x) \equiv O(\bar{\partial}_\mu \partial^\mu)^n (\bar{\partial}_{\nu_1} \partial_{\nu_2} \cdots \bar{\partial}_{\nu_l}) O(x) \]

Why should we care about these composite CFT operators?

- They encode information about scattering in AdS

- Anomalous dimension of double-trace operators and OPE coefficients, the two sets of data, contain all the dynamical information of CFT at \( O(1/N^2) \)
More recently,

- Anomalous dimension of double-trace gives important information about bulk locality
  
  Heemskerk, Penedones, Polchinski and Sully `09; Faulkner, Liu and Rangamani `10

- RGE of boundary local operators $\leftrightarrow$ multitrace-trace flow (classical level)
  
  Heemskerk and Polchinski `10; Faulkner, Liu and Rangamani `10

- Possible phenomenological applications of double-trace deformation
• Turning on $\phi^4$

$$\mathcal{O}_n(x) \equiv \mathcal{O}(\partial_\mu \partial^\mu)^n \mathcal{O}(x)$$

Would be renormalized

$$\Delta_n = 2\Delta + 2n + \gamma(n)$$

Single-trace operator dim.
Assume $\phi_0^2$ sources the double-trace, calculate

$$\langle \phi_0^2(x) \phi_0^2(0) \rangle \int \frac{dz_0 d^d z}{z_0^{d+1}} \frac{\mu}{4!} \phi(z_0, z)^4 \rangle_{_{\text{non-local}}}^{1PI}$$

Trick: Instead of using the single-particle propagator, use the two-particle propagator

$$\phi^2(x) \quad \phi^2(0) = \sum_n \frac{1}{N^n}$$

Single-particle propagator: $G_\Delta(x, 0)$

Two-particles’ propagator: $G_{\Delta_n}(x, 0)$

Analog of partial-wave decomposition
\[ d = 2 \quad \gamma(n) = \frac{\mu}{8\pi} \frac{1}{2\Delta + 2n - 1}, \]

\[ d = 4 \quad \gamma(n) = \frac{\mu}{16\pi^2} \frac{(n + 1)(\Delta + n - 1)(2\Delta + n - 3)}{(2\Delta + 2n - 1)(2\Delta + 2n - 3)} \]

Agrees with Heemskerk, Penedones, Polchinski and Sully `09; Fitzpatrick, Katz, Poland, Simmons-Duffin `10

Santa Barbara group: calculate 4-point function and project onto each individual two-particle partial wave;
Boston group: in global AdS, calculate perturbations of the dilatation operator using the old-fashioned perturbation theory.
• Physical interpretation

\[ d = 2 \quad \gamma(n) = \frac{\mu}{8\pi} \frac{1}{2\Delta + 2n - 1}, \]

\[ d = 4 \quad \gamma(n) = \frac{\mu}{16\pi^2} \frac{(n + 1)(\Delta + n - 1)(2\Delta + n - 3)}{(2\Delta + 2n - 1)(2\Delta + 2n - 3)} \]

\[ n \gg 1 \quad \frac{\phi^4}{\Lambda^{d-3}} \quad \gamma(n) \sim n^{d-3} \]
- Physical interpretation

\[
\frac{\phi^4}{\Lambda^{d-3}} \quad \gamma(n) \sim n^{d-3}
\]

\[
\frac{\mathcal{O}}{\Lambda^p} \quad A \sim E^p
\]

Eg: Euler-Heisenberg Lagrangain

\[
F^4 \quad A \sim E^4
\]

\[
\gamma(n) \leftrightarrow A \\
n \leftrightarrow E
\]
Three cases

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<th>AdS UV b.c</th>
<th>IR b.c</th>
<th>CFT</th>
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<td>Regular at $z_0 \to \infty$</td>
<td>Standard quantization $\Delta &gt; \frac{d}{2}$</td>
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<td>Mixed Neuman/</td>
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<td>CFT w/ a double-trace deformation:</td>
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<tr>
<td>Dirichlet b.c</td>
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<td>$\text{CFT}^{\text{UV}} \to \text{CFT}^{\text{IR}}$</td>
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<tr>
<td>Dirichlet b.c</td>
<td>$z_0 = z_{\text{IR}}$</td>
<td>Spontaneous breaking of CFT</td>
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</table>
CFT with double-trace deformation

- CFT: \( \mathcal{L}_{\text{CFT}} + \tilde{f} O^2 \)
- Dual to mixed boundary condition: Witten 01';
  \[ f \phi(k) + \epsilon \partial_{z_0} \phi(k) \big|_{z_0=\epsilon} = 0 \]

\( \text{UV: } f = 0 \quad \Delta_- = d/2 - \nu \quad \Delta[O^2] < d \quad \text{relevant perturbation} \)
\( \text{IR: } f \to \infty \quad \Delta_+ = d/2 + \nu \)

Some possible pheno applications:
- (unsuccessful) attempt to explain QCD confinement
  D.B. Kaplan, Lee, Son, Stephanov 09;
- Non-susy theory w/ natural light scalar:
  Strassler 03;
- Split SUSY: Sundrum 09;
• Repeat calculation with the new boundary conditions and corresponding propagators

For instance, for the toy example of mass perturbation,

\[
\begin{align*}
\text{UV} & \quad f \to 0 \quad \gamma = -\frac{\delta m^2}{2\nu}, \\
\text{IR} & \quad f \to \infty \quad \gamma = \frac{\delta m^2}{2\nu},
\end{align*}
\]

\[
\begin{align*}
\Delta^- &= \frac{d}{2} - \sqrt{\left(\frac{d}{2}\right)^2 + m^2 + \delta m^2} \\
&= \Delta_0 - \frac{\delta m^2}{2\nu} + \mathcal{O}((\delta m^2)^2)
\end{align*}
\]
Spontaneously broken CFT

- Supposing the existence of a Wilsonian scheme, the renormalization of CFT should not be sensitive to the interior boundary condition.

- Impose an IR cutoff surface with Dirichlet b.c., for mass perturbations, the answers do not change!
Spontaneously broken CFT (continued)

- Impose an IR cutoff surface with Dirichlet b.c., for mass perturbations, the answers do not change!

\[
K(z_0, k) = \left( \frac{z_0}{\epsilon} \right)^{d/2} \frac{K_\nu(kz_0) + aI_\nu(kz_0)}{K_\nu(k\epsilon) + aI_\nu(k\epsilon)}
\]

\[
\langle \phi_0(x)\phi_0(0) \rangle = \int \frac{dz_0 dz^d \delta m^2 \phi(z_0, z)^2}{z_0^{d+1}} |z|^\Delta \cdot \cdots + \epsilon^{2\nu-d} \log \epsilon \frac{2\delta m^2}{\pi^{d/2}} \left( \frac{\Gamma(\Delta)}{\Gamma(\Delta - d/2)} + 2a \frac{\Gamma(\Delta)}{\Gamma(\Delta - d/2 + 1) \Gamma(\nu) \Gamma(-\nu)} \right) |\vec{x}|^{-2\Delta}
\]

\[
\langle \mathcal{O}(x)\mathcal{O}(0) \rangle^{(0)} = \frac{2\nu}{\pi^{d/2}} \left( \frac{\Gamma(\Delta)}{\Gamma(\Delta - d/2)} + 2a \frac{\Gamma(\Delta)}{\Gamma(\Delta - d/2 + 1) \Gamma(\nu) \Gamma(-\nu)} \right) |\vec{x}|^{-2\Delta}
\]

\[
\gamma = -\epsilon \frac{\partial \log Z}{\partial \epsilon} = \frac{\delta m^2}{2\nu}
\]
• For contact interactions, however,

• Single-trace:

  the loop momentum integration is dependent of the particular choice of the bulk propagator;

  UV divergences: determined by short-distance physics; unaffected by the IR boundary;

  Finite correction: sensitive to the interior b.c..

• Double-trace: similar to mass perturbation, independent of IR condition
Conclusion

- I present a simple prescription to calculate the anomalous dimensions of CFT operators from bulk interactions.
- The key ingredient is to use the radial position as the regulator.
Open questions

- RGE of local boundary operators beyond leading order in $N$:
  Lewandowski `04; However, part of his answer does not look Wilsonian-like as it depends on the IR b.c.’s;
- Understand correction to non-RG quantity, e.g., OPE coefficients
- Theory with gauge interactions and fermions.
Thank you!
Boundary action

Cutoff surface Sliding brane

\[ \varepsilon_0 \quad \varepsilon \quad \varepsilon_0 \]

\[ S_\varepsilon = S^{local}_\varepsilon + S^{non-local}_\varepsilon \]

\[ S = S_\varepsilon + \int d^d x \int_{z_0 > \varepsilon} d\varepsilon_0 \mathcal{L}_{bulk} \]
\[ \psi(kz_0) \equiv z_0^{d/2} K_\nu(kz_0), \]

\[ K(k, z_0) = \frac{\psi(kz_0)}{\tilde{f}\psi(k\epsilon) + \partial\psi(k\epsilon) \cdot \vec{n}}. \]

\[ \langle \mathcal{O}(k)\mathcal{O}(k') \rangle_f^{(0)} = -\epsilon^{-d} \delta^d(k + k') \frac{\psi(k\epsilon)}{\tilde{f}\psi(k\epsilon) + \partial\psi(k\epsilon) \cdot \vec{n}} \]

\[ = -\epsilon^{-d} \delta^d(k + k') \frac{1}{-f \epsilon^{2\nu} \left(2\pi^{d/2} \frac{\Gamma(1-\nu)}{\Gamma(\Delta_-)}\right) + \epsilon^{2\nu} (k^2)^{2\nu} \frac{\Gamma(-\nu)}{\Gamma(\nu)} (2\nu)}, \]

\[ \langle \mathcal{O}(k)\mathcal{O}(k') \rangle_f^{(1)} = -\epsilon^{-d} \delta^d(k + k') \delta m^2 \int_\epsilon \frac{dz_0}{z_0} \left( \frac{\psi(kz_0)}{\tilde{f}\psi(kz_0) + \partial\psi(kz_0) \cdot \vec{n}} \right)^2 \]

\[ = -\epsilon^{-d} \delta^d(k + k') \delta m^2 \int_\epsilon \frac{dz_0}{z_0} \left( \frac{2\nu^{-1} \Gamma(\nu)(kz_0)^{-\nu} + 2^{-\nu-1} \Gamma(-\nu)(kz_0)^\nu}{-f \epsilon^{\nu\pi^{d/2} 2\nu} \frac{\Gamma(1-\nu)\Gamma(\nu)}{\Gamma(\Delta_-)} k^{-\nu} - 2^{-\nu} \Gamma(1-\nu)(k\epsilon)^\nu} \right)^2 \]