Elementary/Composite Mixing in Randall-Sundrum Models *

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5D Warped Dimension = 4D Strong Dynamics

- AdS/CFT duality: Extra dimension is a calculational tool
- Randall-Sundrum models ⇐⇒ Standard Model partial compositeness
- How to quantify elementary/composite mixing?
  - Understand structure and phenomenology of 4D dual theory
- Answer: The Holographic Basis:

\[
\Phi(x, y) = \varphi^s(x)g^s(y) + \sum_{n=1}^{\infty} \varphi^n_{CFT}(x)g^n(y)
\]
Outline

- Randall-Sundrum models and geometrical hierarchies
- AdS/CFT and Holography
- The Kaluza-Klein Basis
- The Holographic Basis
- Elementary/composite content of SM fields
- Explain warped phenomenology in a 4D language (e.g. RS GIM mechanism)
A slice of AdS

Randall, Sundrum '99

\[ ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \]

- Warped geometry \( \Rightarrow \) Energy scales depend on location

- Planck/Weak scale hierarchy:
  \[ \Lambda_{weak} \sim M_P e^{-\pi kR} \]
  \[ k \sim \mathcal{O}(M_P), \quad \pi kR \sim \mathcal{O}(30) \]

- \( R \) can be naturally stabilized

Goldberger, Wise '99

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Standard Model in the bulk

Davoudiasl, Hewett, Rizzo '99; Pomarol '99
Chang et al. '99; Gherghetta, Pomarol '00

Theory of Flavor!

- Natural Yukawa hierarchies
- FCNC suppressed
Bulk fields

Scalar field with tuned bulk and boundary masses

\[ S = \int d^5x \sqrt{-g} \left[ -\frac{1}{2} (\partial_M \Phi)^2 - \frac{1}{2} ak^2 \Phi^2 - b\kappa \Phi^2 \left( \delta(y) - \delta(y - \pi R) \right) \right] \]

Tuning:

\[ b = 2 \pm \sqrt{4 + a} \]

Why consider this toy model?

- Tuning allows for a localized zero mode: \( \tilde{f}^0(y) \sim e^{(b-1)ky} \)
  \[ \Rightarrow \] Holographic interpretation depends on \( b \)

- special values for \( b \) mimic bulk graviton and gauge boson
AdS/CFT duality

is for our purposes . . .

Weakly coupled gravity dual
in warped 5D

\[ \Longleftrightarrow \]

Strongly coupled gauge theory (CFT) in 4D*

* Large \( N_c \) gauge theory

\[
\left\langle \exp \left( - \int \varphi_0 \mathcal{O} \right) \right\rangle_{\text{CFT}} = \exp \left[ - \Gamma(\varphi_0) \right]
\]

Gubser, Klebanov, Polyakov '97; Witten '97
“Dictionary”

<table>
<thead>
<tr>
<th>5D</th>
<th>4D</th>
</tr>
</thead>
<tbody>
<tr>
<td>bulk field $\Phi(x, y)$</td>
<td>CFT operator $\mathcal{O}(x)$</td>
</tr>
<tr>
<td>BC $\Phi(x, y_0) = \varphi_0(x)$</td>
<td>source: $\varphi_0(x)\mathcal{O}(x)$</td>
</tr>
<tr>
<td>bulk mass</td>
<td>dimension $\Delta$ of $\mathcal{O}$</td>
</tr>
</tbody>
</table>

e.g.

Bulk gauge field $A_\mu(x, y)$ $\iff$ $J_{\mu}^{\text{CFT}}$

$m_A^2 = 0$ $\iff$ $\Delta_J = 3$
zero mode $\sim$ source field (elementary)

KK modes $\sim$ CFT bound states (composites)

but wait ... Mixing through operator $\varphi_0(x)O(x)$

$\implies$ Mass eigenstates are elementary/composite mixtures
The “Holographic Recipe”

Step 1: Evaluate bulk action for arbitrary boundary condition
\[ \Phi(x, y_0) = \varphi_0(x) \] to obtain \( \Gamma(\varphi_0) \)

Step 2: Take functional derivatives to compute correlation functions of CFT operators

\[
\langle \mathcal{O}\mathcal{O} \rangle(p) = \left\langle \exp \left( - \int \varphi_0 \mathcal{O} \right) \right\rangle_{\text{CFT}} = \frac{\delta^2}{\delta \varphi_0^2} \exp \left[ - \Gamma(\varphi_0) \right]
\]

\[
= \mp ip \left( J_{b-1} \left( \frac{ip}{k} \right) Y_{b-1} \left( \frac{ipe^{\pi k R}}{k} \right) - Y_{b-1} \left( \frac{ip}{k} \right) J_{b-1} \left( \frac{ipe^{\pi k R}}{k} \right) \right)
\]

Step 3: Interpret \( \langle \mathcal{O}\mathcal{O} \rangle(p) \)
Operator Dimension

\[ \Delta = 2 + \sqrt{4 + a} = 2 + |b - 2| \]

\[ \varphi_0(x) \quad \mathcal{O}(x) \]

\[ b < 1 \text{ or } b > 3 \quad \Rightarrow \quad \text{irrelevant mixing} \]

\[ 1 < b < 3 \quad \Rightarrow \quad \text{relevant mixing} \]
Two branches in dual theory

\[ \Delta = 2 + |b - 2| \]

- **\( b < 2 \):**
  - source field \( \varphi_0(x) \) massless
  - zero mode primarily elementary
  - Nearly all RS phenomenological examples are described by \( b < 2 \) (fermions too!)

- **\( b > 2 \):**
  - source field \( \varphi_0(x) \) massive \( M_0 \sim k \)
  - zero mode primarily composite
  - Higgs; perhaps \( t_R \) in some models
Partial compositeness of SM fields

- UV localized $\iff$ mostly elementary
- IR localized $\iff$ mostly composite

Can we quantify source/CFT (elementary/composite) mixing?
Kaluza-Klein mass eigenbasis

KK decomposition:

\[ \Phi(x, y) = \sum_{n=0}^{\infty} \phi^n(x) f^n(y), \]

BC : \((++)\)

\[ (\partial_5 - bk) f^n(y) \bigg|_{0, \pi R} = 0 \]

Localized massless mode:

\[ \tilde{f}^0(y) \sim e^{(b-1)ky}, \quad -\infty < b < \infty \]

The fields \( \phi^n(x) \) are the mass eigenstates

- Spectrum:

\[ J_{b-1} \left( \frac{m_n}{k} \right) Y_{b-1} \left( \frac{m_ne^{\pi kR}}{k} \right) - Y_{b-1} \left( \frac{m_n}{k} \right) J_{b-1} \left( \frac{m_ne^{\pi kR}}{k} \right) = 0 \]
Holographic basis

Basic idea:

Expand the bulk field directly in terms of a source field $\varphi^s(x)$ and composite CFT states $\varphi^n_{CFT}(x)$:

$$\Phi(x, y) = \varphi^s(x)g^s(y) + \sum_{n=1}^{\infty} \varphi^n_{CFT}(x)g^n(y)$$

- Leads to kinetic and mass mixing in 4D effective theory
- Mass eigenstates will be a mixture of $\varphi^s(x)$ and $\varphi^n_{CFT}(x)$
Source profile $g^s(y)$

$g^s(y)$ can be determined from mass of source

$$M_s^2 = \begin{cases} 
0 & \text{for } b < 2 \\
4(b - 2)(b - 3)k^2 & \text{for } b > 2
\end{cases}$$

$$\tilde{g}^s(y) \sim e^{-ky} e^{(4-\Delta)ky} = \begin{cases} 
e^{(b-1)ky} & \text{for } b < 2 \\
e^{(3-b)ky} & \text{for } b > 2
\end{cases}$$

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Source profiles mimic operator dimensions:

\[ \Delta = 2 + |2 - b| \]

- Indicates when mixing is relevant, marginal, or irrelevant
CFT composite profiles $g^n(y)$

CFT spectrum obtained from poles in 2-point function:

$$J_{b-2} \left( \frac{M_n}{k} \right) Y_{b-1} \left( \frac{M_n e^{\pi k R}}{k} \right) - Y_{b-2} \left( \frac{M_n}{k} \right) J_{b-1} \left( \frac{M_n e^{\pi k R}}{k} \right) = 0$$

Note different from KK spectrum!

Identical to the spectrum obtained with the following BC for $g^n(y)$:

$$BC : \quad (+-)$$

$$g^n(y) \bigg|_0 = 0$$

$$(\partial_5 - bk) g^n(y) \bigg|_{\pi R} = 0$$
Effective 4D Lagrangian in the holographic basis

\[ \mathcal{L} = \frac{1}{2} \phi^T Z \Box \phi - \frac{1}{2} \phi^T M^2 \phi, \]

where \( \phi^T = (\phi^s, \phi^{1}_{CFT}, \phi^{2}_{CFT}, \cdots) \)

\[
Z = \begin{pmatrix}
1 & z_1 & z_2 & z_3 & \cdots \\
z_1 & 1 & 0 & 0 & \cdots \\
z_2 & 0 & 1 & 0 & \cdots \\
z_3 & 0 & 0 & 1 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}, \quad M^2 = \begin{pmatrix}
M^2_s & \mu^2_1 & \mu^2_2 & \mu^2_3 & \cdots \\
\mu^2_1 & M^2_1 & 0 & 0 & \cdots \\
\mu^2_2 & 0 & M^2_2 & 0 & \cdots \\
\mu^2_3 & 0 & 0 & M^2_3 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\]

Notice kinetic mixing \( \Rightarrow \) nonorthogonal basis

\( z_n \) and \( \mu^2_n \) computed from wavefunction overlap integrals

Diagonalization leads to KK basis
\[ \gamma - \rho \text{ mixing in SM} \]

Vector Meson Dominance

\[ \mathcal{L} = -\frac{1}{4} (F_{\mu
u})^2 - \frac{1}{4} (\rho_{\mu\nu})^2 - \frac{1}{2} z\gamma_\rho F_{\mu
u}\rho^{\mu\nu} - \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu \]

“Physical” photon can be viewed as partly composite.
Graviton $h_{\mu\nu}$

\[
\tilde{f}^0(y) \sim e^{-ky}
\]

\[b = 0; \ \Delta = 4 \implies \text{irrelevant mixing}\]

\[
\begin{pmatrix}
  h^0 \\
  h^1 \\
  \vdots
\end{pmatrix}
= 
\begin{pmatrix}
  1 & \sim e^{-\pi kR} & \cdots \\
  0 & \sim -1 & \cdots \\
  \vdots & \vdots & \ddots
\end{pmatrix}
\begin{pmatrix}
  h^s \\
  h^1(\text{CFT}) \\
  \vdots
\end{pmatrix}
\]

- 4D graviton $h_{\mu\nu}^0(x) \sim$ elementary source; compositeness negligible
- KK modes are purely composite
Gauge field \( A_\mu \)

\[
\tilde{f}^0(y) = \frac{1}{\sqrt{\pi R}}
\]

\( b = 1; \Delta = 3 \implies \text{marginal mixing} \)

\[
\begin{pmatrix}
A_0^0 \\
A_1^0 \\
A_2^0 \\
\vdots
\end{pmatrix} = \begin{pmatrix}
1 & -0.19 & 0.13 & \cdots \\
0 & -0.98 & -0.03 & \cdots \\
0 & 0.01 & -0.99 & \cdots \\
\vdots & \vdots & \vdots & \ddots
\end{pmatrix} \begin{pmatrix}
A_0^s \\
A_1^{1(CFT)} \\
A_2^{1(CFT)} \\
\vdots
\end{pmatrix}
\]

- massless eigenstate \( A_\mu^0(x) \) is primarily elementary
- KK modes are purely composite
Bulk Fermions

- Bulk mass \( m_\psi = c k \)

- KK mass eigenbasis:

\[
\psi_\pm(x, y) = \sum_{n=0}^{\infty} \psi^n_\pm(x) f^n_\pm(y),
\]

- Chiral zero mode \( \psi^0_+(x) \); wavefunction \( \widetilde{f^0}_+(y) \sim e^{(\frac{1}{2} - c) ky} \)
Fermion holography

- Operator dimension:
  \[ \Delta_- = \frac{3}{2} + \left| c + \frac{1}{2} \right| \]

- If \( \Delta_- < \frac{5}{2} \) \( \implies \) relevant mixing

- Holographic basis

  \[
  \psi_+(x, y) = \psi^s(x)g^s(y) + \sum_{n=1}^{\infty} \lambda^n_+(x)g^n_+(y), \\
  \psi_-(x, y) = \chi(x)g^\chi(y) + \sum_{n=1}^{\infty} \lambda^n_-(x)g^n_-(y),
  \]
Two branches in dual theory

\[ \Delta_- = \frac{3}{2} + \left| c + \frac{1}{2} \right| \]

- **\( c > -1/2 \):**
  - source field \( \psi^{s}(x) \) chiral; \( \chi(x) \) absent from theory
  - zero mode primarily elementary
  - Nearly all bulk fermions described by \( c > -1/2 \)

- **\( c < -1/2 \):**
  - field \( \chi(x) \) marries with source field \( \psi^{s}(x) \) to become massive \( M_0 \sim k \)
  - zero mode primarily composite
  - perhaps \( t_R \) in some models
Right-handed top $t_R$

$$\tilde{f}^0(y) = e^{(\frac{1}{2} - c)ky} \quad m_\psi = ck$$

Take e.g. $c = -0.7; \Delta = 1.7 \implies$ relevant mixing

$$\begin{pmatrix}
  t^{(0)}_R \\
  t^{(1)}_R \\
  t^{(2)}_R \\
  t^{(3)}_R \\
  \vdots
\end{pmatrix}
= 
\begin{pmatrix}
  0.9796 & \sim -1 & \sim 0 & \sim 0 & \cdots \\
  -0.1816 & \sim 0 & \sim -1 & \sim 0 & \cdots \\
  0.0514 & \sim 0 & \sim 0 & \sim -1 & \cdots \\
  0.0471 & \sim 0 & \sim 0 & \sim 0 & \cdots \\
  \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\begin{pmatrix}
  t^s_R \\
  t^{CFT(1)}_R \\
  t^{CFT(2)}_R \\
  t^{CFT(3)}_R \\
  \vdots
\end{pmatrix}$$

- massless eigenstate $t^0_R(x)$ roughly equal mixture of source/CFT
- KK modes contain elementary component
Example: RS GIM mechanism

Important point on inverse transformation:
- Source field contains zero mode

\[ \psi^s(x) = \psi_0^0(x) + \sum_{m=1}^{\infty} \omega_{s}^{m} \psi_{+}^{m}(x) \]

- Composite modes do not; entirely composed of KK modes

\[ \lambda_{+}^{n}(x) = \sum_{m=1}^{\infty} \omega_{+}^{nm} \psi_{+}^{m}(x) \]
Gauge interactions $g^A_{\psi\psi}$

Contain SM fermions

$g^{s}_{ss}$ $g^{*n}_{ss}$ $g^{s}_{ns}$

$g^{s}_{nn}$ $g^{*m}_{sn}$ $g^{*m}_{ln}$
• Light fermions have exponentially suppressed couplings to composites
• 3-source vertex dominates
RS GIM mechanism

For light fermions, \( c > 1/2 \), 3-source vertex dominates:

\[
\begin{align*}
ss & \quad g_{ss}^s \\
0 & \quad g \\
0 & \quad g_{\sqrt{\pi R} f^1(0)}
\end{align*}
\]

KK gauge boson couplings are approx. universal for light fermions

\[\rightarrow\text{ FCNCs suppressed}\]
Flavor violation

Important to track nonuniversal contribution to coupling

\[ g_{ss}^* = \int_0^{\pi R} e^{ky} g^s(y) g^* n(y) g^s(y) \]

Sum over all composite modes \[ \Rightarrow \]

\[ g_{nonuniversal}^1 = \sum_{n=1}^{\infty} g_{ss}^* \omega^n = g^1 - g_{universal}^1 \]
Flavor violation - cont’d

Near $c \sim 1/2$, first composite mode saturates nonuniversal piece:

$$g_{\text{nonuniversal}}^1 \approx g_{ss}^* \omega^{11}$$

$$\approx g \sqrt{2\pi k R} \left( \frac{2c - 1}{2 - 2c} \right) e^{(1 - 2c)\pi k R}.$$

- Works well - order few % - for $c < 0.6$ (e.g. tau, muon)

- Deviates for $c > 0.6$ (e.g. electron), but nonuniversal contributions smaller anyway*

* Thanks to K. Agashe for discussions
Conclusions

- **Holographic basis:** bulk field expanded in source and CFT resonances
- **Quantitatively** describe elementary/composite mixing in warped duals
- Explain warped physics in terms of strong gauge dynamics

**Things to do:**

- Other applications: Higgsless models, warped SUSY, Gauge-Higgs models (QCD?)
- Loop diagrams
  - Important for EWPT, gauge coupling unification etc.
- Brane localized kinetic terms - could modify composite content
- More general geometries?