Elementary/Composite Mixing in Randall-Sundrum Models *

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- arXiv:0706.0890
- arXiv:0710.1838

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5D Warped Dimension = 4D Strong Dynamics

- AdS/CFT duality: Extra dimension is a calculational tool
- Randall-Sundrum models $\Longleftrightarrow$ Standard Model partial compositeness
- How to quantify elementary/composite mixing?
  - Understand structure and phenomenology of 4D dual theory
- Answer: The Holographic Basis:

$$\Phi(x,y) = \varphi^s(x) g^s(y) + \sum_{n=1}^{\infty} \varphi^n_{CFT}(x) g^n(y)$$

Outline

- Randall-Sundrum models and geometrical hierachies
- AdS/CFT and Holography
- The Kaluza-Klein Basis
- The Holographic Basis
- Elementary/composite content of SM fields
- Explain warped phenomenology in a 4D language
  (e.g. RS GIM mechanism)

A slice of AdS

Randall, Sundrum '99

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

- Warped geometry $\Longrightarrow$
  Energy scales depend on location

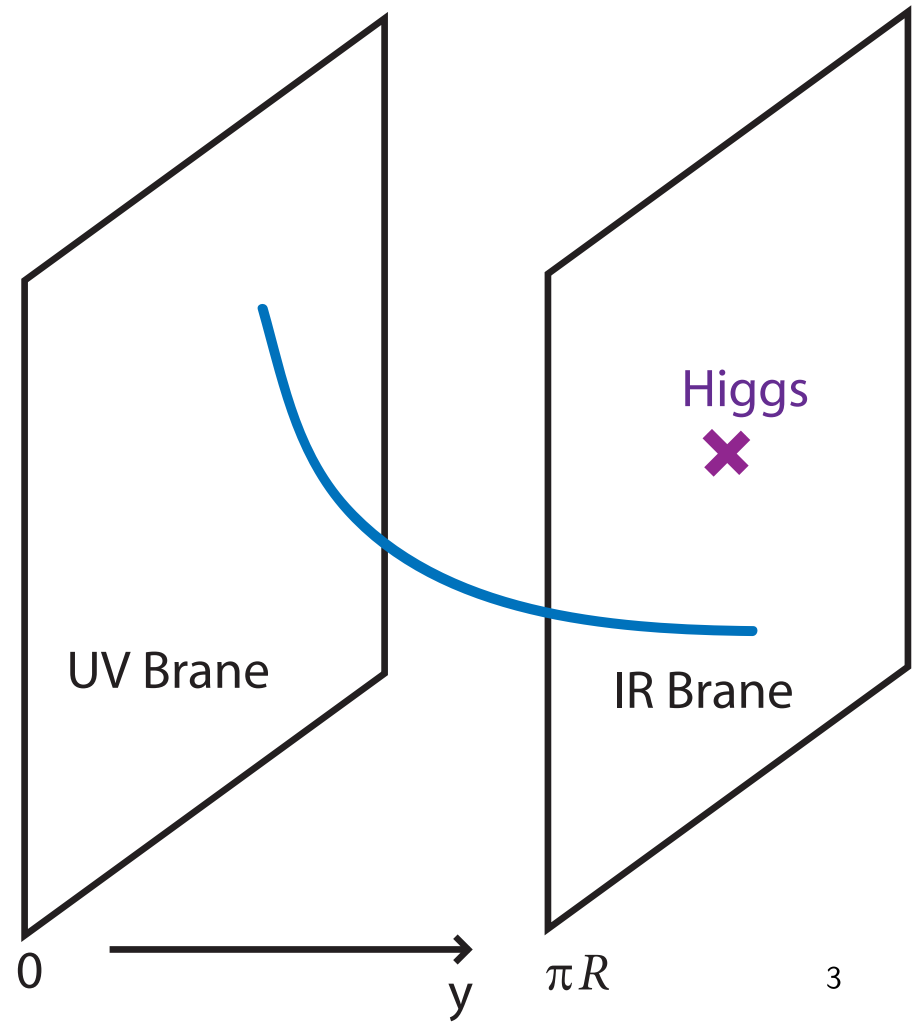
- Planck/Weak scale hierarchy:

$$\Lambda_{weak} \sim M_P e^{-\pi k R}$$

$$k \sim \mathcal{O}(M_P), \quad \pi k R \sim \mathcal{O}(30)$$

- R can be naturally stabilized

Goldberger, Wise '99

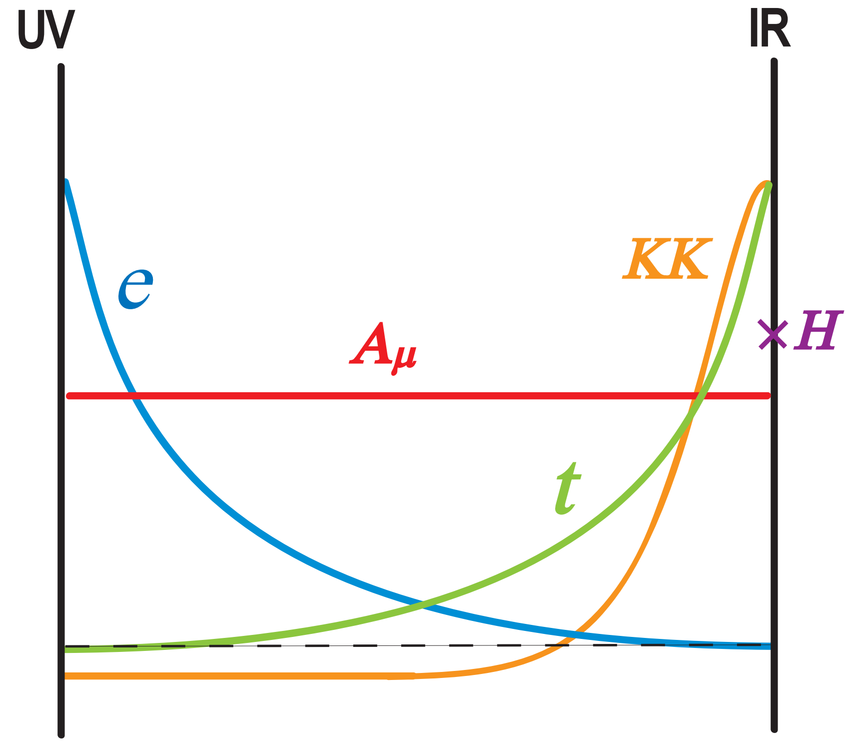


Standard Model in the bulk

Davoudiasl, Hewett, Rizzo '99; Pomarol '99

Grossman, Neubert '99

Chang et al. '99; Gherghetta, Pomarol '00



Theory of Flavor!

- Natural Yukawa hierarchies
- FCNC suppressed

Bulk fields

Scalar field with tuned bulk and boundary masses

$$S = \int d^5x\sqrt{-g}\left[-\frac{1}{2}(\partial_M\Phi)^2 - \frac{1}{2}ak^2\Phi^2 - bk\Phi^2\left(\delta(y) - \delta(y-\pi R)\right)\right]$$

Tuning:

$$b = 2 \pm \sqrt{4+a}$$

Why consider this toy model?

- Tuning allows for a localized zero mode: $\tilde{f}^0(y) \sim e^{(b-1)ky}$
  $\Longrightarrow$ Holographic interpretation depends on b
- special values for b mimic bulk graviton and gauge boson

AdS/CFT duality

Maldacena '97

is for our purposes . . .

Weakly coupled gravity in warped 5D	dual $\Longleftrightarrow$	Strongly coupled gauge theory (CFT) in 4D*

* Large N_c gauge theory

$$\left\langle \exp\left(-\int \varphi_0 \mathcal{O}\right) \right\rangle_{\mathrm{CFT}} = \exp\left[-\Gamma(\varphi_0)\right]$$

Gubser, Klebanov, Polyakov '97; Witten '97

"Dictionary"

5D		4D
bulk field $\Phi(x,y)$	$\Longleftrightarrow$	CFT operator $\mathcal{O}(x)$
BC $\Phi(x,y_0) = \varphi_0(x)$	$\Longleftrightarrow$	source: $\varphi_0(x)\mathcal{O}(x)$
bulk mass	$\Longleftrightarrow$	dimension Δ of $\mathcal{O}$

e.g.

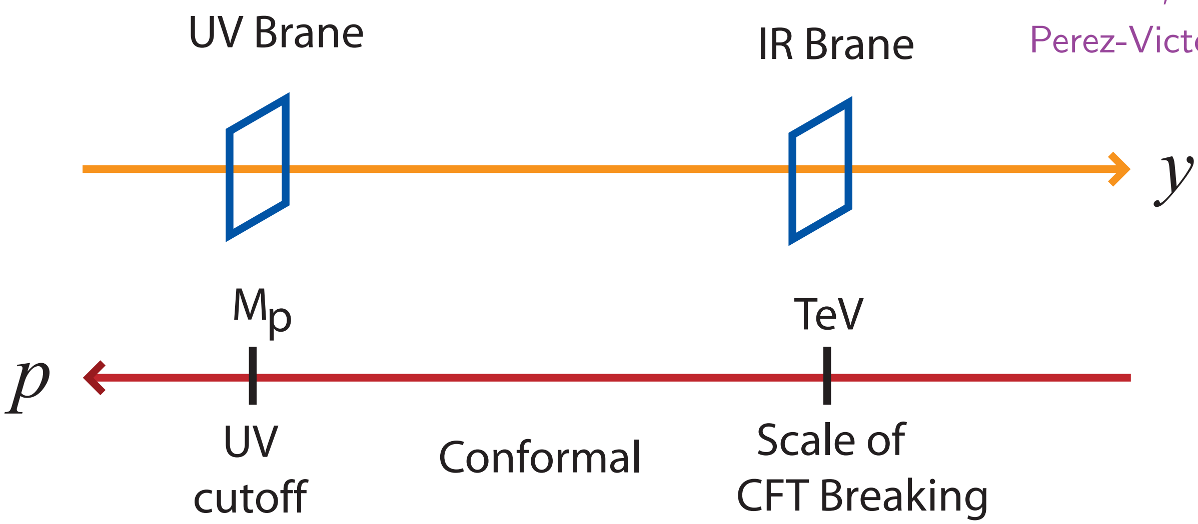
Bulk gauge field — global symmetry current

$$A_\mu(x,y) \quad \Longleftrightarrow \quad J_\mu^{CFT}$$

$$m_A^2 = 0 \quad \Longleftrightarrow \quad \Delta_J = 3$$

Holography for RS1

Arkani-Hamed, Porrati, Randall '00
Rattazzi, Zaffaroni '00
Perez-Victoria '00



zero mode	$\sim$	source field (elementary)
KK modes	$\sim$	CFT bound states (composites)

but wait . . .

Mixing through operator $\varphi_0(x)\mathcal{O}(x)$

$\Longrightarrow$ Mass eigenstates are elementary/composite mixtures

Step 1: Evaluate bulk action for arbitrary boundary conditon $\Phi(x, y_0) = \varphi_0(x)$ to obtain $\Gamma(\varphi_0)$

Step 2: Take functional derivatives to compute correlation functions of CFT operators

$$\begin{aligned}\langle \mathcal{O}\mathcal{O}\rangle(p) &= \frac{\delta^2}{\delta\varphi_0^2}\left\langle \exp\left(-\int \varphi_0 \mathcal{O}\right)\right\rangle_{\mathrm{CFT}} = \frac{\delta^2}{\delta\varphi_0^2}\exp\left[-\Gamma(\varphi_0)\right] \\ &= \mp ip\, \frac{J_{b-1}\left(\frac{ip}{k}\right) Y_{b-1}\left(\frac{ipe^{\pi kR}}{k}\right) - Y_{b-1}\left(\frac{ip}{k}\right) J_{b-1}\left(\frac{ipe^{\pi kR}}{k}\right)}{J_{b-2}\left(\frac{ip}{k}\right) Y_{b-1}\left(\frac{ipe^{\pi kR}}{k}\right) - Y_{b-2}\left(\frac{ip}{k}\right) J_{b-1}\left(\frac{ipe^{\pi kR}}{k}\right)}\end{aligned}$$

Step 3: Interpret $\langle \mathcal{O}\mathcal{O}\rangle(p)$

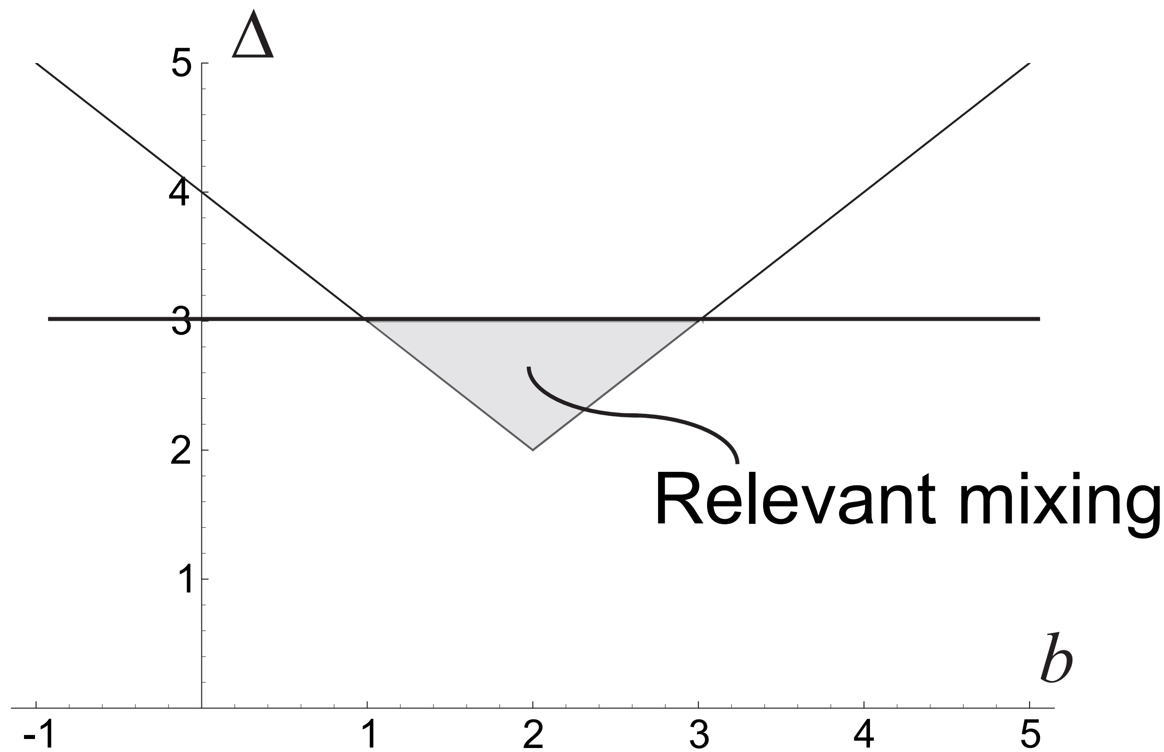
Operator Dimension

$$\Delta = 2 + \sqrt{4+a} = 2 + |b-2|$$

$$\overbrace{\varphi_0(x)}^{1} \quad \overbrace{\mathcal{O}(x)}^{2+|b-2|}$$

$b < 1$ or $b > 3$
$\Longrightarrow$ irrelevant mixing

$1 < b < 3$
$\Longrightarrow$ relevant mixing

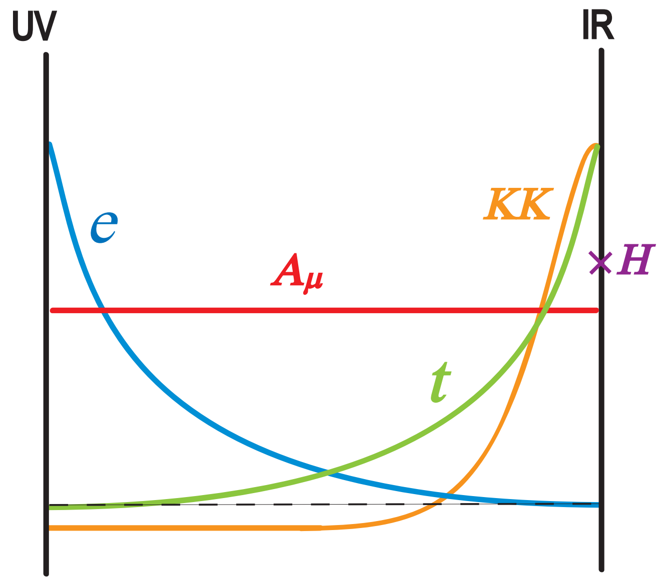


Two branches in dual theory $\Delta = 2 + |b - 2|$

- $b < 2$:
  - source field $\varphi_0(x)$ massless
  - zero mode primarily elementary
  - Nearly all RS phenomenological examples are described by $b < 2$ (fermions too!)

- $b > 2$:
  - source field $\varphi_0(x)$ massive $M_0 \sim k$
  - zero mode primarily composite
  - Higgs; perhaps t_R in some models

Partial compositeness of SM fields



- UV localized $\Longleftrightarrow$ mostly elementary

- IR localized $\Longleftrightarrow$ mostly composite

Can we quantify source/CFT (elementary/composite) mixing?

Kaluza-Klein mass eigenbasis

KK decomposition:

$$\Phi(x,y) = \sum_{n=0}^{\infty} \phi^n(x) f^n(y), \qquad \mathrm{BC:}\ \ (++) \quad (\partial_5 - bk) f^n(y)\big|_{0,\pi R} = 0$$

Localized massless mode:

$$\tilde{f}^0(y) \sim e^{(b-1)ky}, \qquad -\infty < b < \infty$$

The fields $\phi^n(x)$ are the mass eigenstates

- Spectrum:

$$J_{b-1}\left(\frac{m_n}{k}\right) Y_{b-1}\left(\frac{m_n e^{\pi k R}}{k}\right) - Y_{b-1}\left(\frac{m_n}{k}\right) J_{b-1}\left(\frac{m_n e^{\pi k R}}{k}\right) = 0$$

Holographic basis

Basic idea:

Expand the bulk field directly in terms of a source field $\varphi^s(x)$ and composite CFT states $\varphi^n_{CFT}(x)$:

$$\Phi(x,y) = \varphi^s(x) g^s(y) + \sum_{n=1}^{\infty} \varphi^n_{CFT}(x) g^n(y)$$

- Leads to kinetic and mass mixing in 4D effective theory
- Mass eigenstates will be a mixture of $\varphi^s(x)$ and $\varphi^n_{CFT}(x)$

Source profile $g^s(y)$

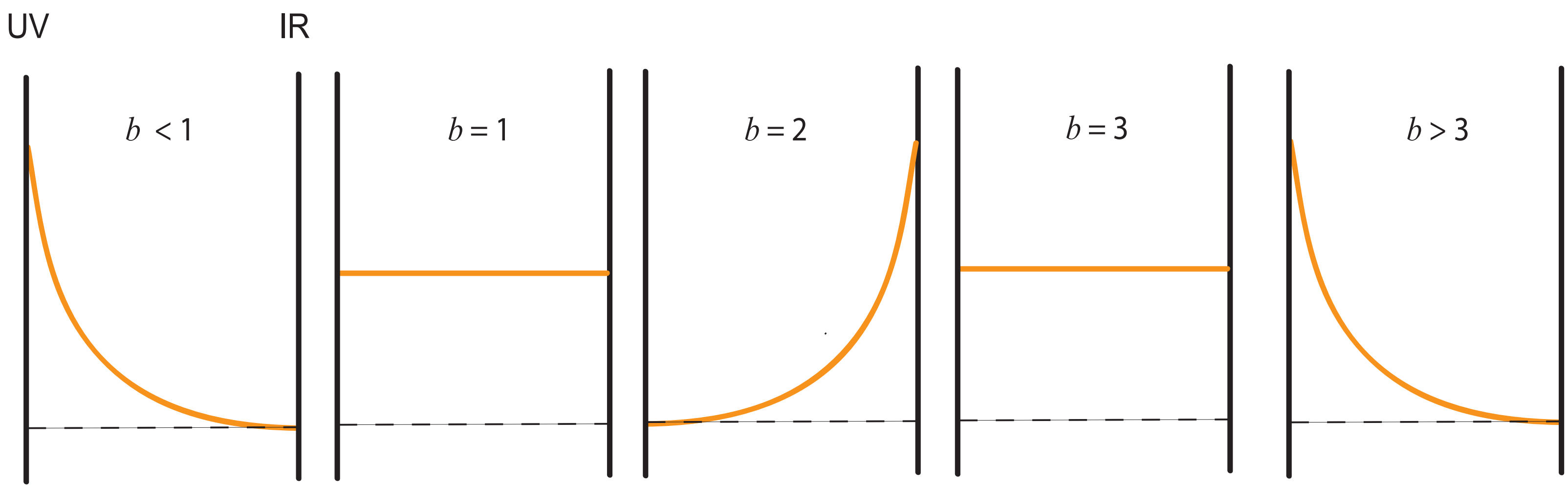
$g^s(y)$ can be determined from mass of source

$$M_s^2 = \begin{cases} 0 & \text{for} \quad b < 2 \\ \\ 4(b-2)(b-3)k^2 & \text{for} \quad b > 2 \end{cases}$$

$$\Longrightarrow \quad \tilde{g}^s(y) \sim e^{-ky} e^{(4-\Delta)ky} = \begin{cases} e^{(b-1)ky} & \text{for} \quad b < 2 \\ \\ e^{(3-b)ky} & \text{for} \quad b > 2 \end{cases}$$

Source profiles mimic operator dimensions:

$$\Delta = 2 + |2 - b|$$



- Indicates when mixing is relevant, marginal, or irrelevant

CFT composite profiles $g^n(y)$

CFT spectrum obtained from poles in 2-point function:

$$J_{b-2}\left(\frac{M_n}{k}\right) Y_{b-1}\left(\frac{M_n e^{\pi kR}}{k}\right) - Y_{b-2}\left(\frac{M_n}{k}\right) J_{b-1}\left(\frac{M_n e^{\pi kR}}{k}\right) = 0$$

Note different from KK spectrum!

Identical to the spectrum obtained with the following BC for $g^n(y)$:

$$\begin{aligned} \mathrm{BC}: &\qquad (-+) \\ g^n(y)\Big|_0 &= \quad 0 \\ (\partial_5 - bk) g^n(y)\Big|_{\pi R} &= \quad 0 \end{aligned}$$

Effective 4D Lagrangian in the holographic basis

$$\mathcal{L} = \frac{1}{2}\vec{\varphi}^{\mathrm{T}}\mathbf{Z}\Box\vec{\varphi} - \frac{1}{2}\vec{\varphi}^{\mathrm{T}}\mathbf{M}^2\vec{\varphi},$$

where $\vec{\varphi}^{\mathrm{T}} = (\varphi^s, \varphi^1_{CFT}, \varphi^2_{CFT}, \cdots)$

$$\mathbf{Z} = \begin{pmatrix} 1 & z_1 & z_2 & z_3 & \cdots \\ z_1 & 1 & 0 & 0 & \cdots \\ z_2 & 0 & 1 & 0 & \cdots \\ z_3 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad \mathbf{M}^2 = \begin{pmatrix} M_s^2 & \mu_1^2 & \mu_2^2 & \mu_3^2 & \cdots \\ \mu_1^2 & M_1^2 & 0 & 0 & \cdots \\ \mu_2^2 & 0 & M_2^2 & 0 & \cdots \\ \mu_3^2 & 0 & 0 & M_3^2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Notice kinetic mixing $\Longrightarrow$ nonorthogonal basis

z_n and μ_n^2 computed from wavefunction overlap integrals

Diagonalization leads to KK basis

$\gamma - \rho$ mixing in SM

Vector Meson Dominance

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 - \frac{1}{4}(\rho_{\mu\nu})^2 - \frac{1}{2}z_{\gamma_\rho}F_{\mu\nu}\rho^{\mu\nu} - \frac{1}{2}m_\rho^2\rho_\mu\rho^\mu$$

"Physical" photon can be viewed as partly composite

Graviton $h_{\mu\nu}$

$$\tilde{f}^0(y) \sim e^{-ky}$$

$b = 0$; $\Delta = 4 \Longrightarrow$ irrelevant mixing

$$\begin{pmatrix} h^0 \\ h^1 \\ \vdots \end{pmatrix} = \begin{pmatrix} 1 & \sim e^{-\pi k R} & \cdots \\ 0 & \sim -1 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} h^s \\ h^{1(CFT)} \\ \vdots \end{pmatrix}$$

- 4D graviton $h^0_{\mu\nu}(x) \sim$ elementary source ; compositeness negligible
- KK modes are purely composite

Gauge field A_μ

$$\tilde{f}^0(y) = \frac{1}{\sqrt{\pi R}}$$

$b = 1;\ \Delta = 3 \Longrightarrow$ marginal mixing

$$\begin{pmatrix} A_\mu^0 \\ A_\mu^1 \\ A_\mu^2 \\ \vdots \end{pmatrix} = \begin{pmatrix} 1 & -0.19 & 0.13 & \cdots \\ 0 & -0.98 & -0.03 & \cdots \\ 0 & 0.01 & -0.99 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} A_\mu^s \\ A_\mu^{1(CFT)} \\ A_\mu^{1(CFT)} \\ \vdots \end{pmatrix}$$

- massless eigenstate $A_\mu^0(x)$ is primarily elementary
- KK modes are purely composite

Bulk Fermions

- Bulk mass $\quad m_\psi = ck$

- KK mass eigenbasis:

$$\psi_\pm(x,y) = \sum_{n=0}^{\infty} \psi_\pm^n(x) f_\pm^n(y),$$

- Chiral zero mode $\psi_+^0(x)$; wavefunction $\widetilde{f}_+^0(y) \sim e^{(\frac{1}{2}-c)ky}$

Fermion holography

Contino, Pomarol '04

- Operator dimension:

$$\Delta_- = \frac{3}{2} + \left| c + \frac{1}{2} \right|$$

- If $\Delta_- < 5/2 \Longrightarrow$ relevant mixing
- Holographic basis

$$\begin{aligned}\psi_+(x,y) &= \psi^s(x)g^s(y) + \sum_{n=1}^{\infty} \lambda_+^n(x) g_+^n(y), \\ \psi_-(x,y) &= \chi(x)g^\chi(y) + \sum_{n=1}^{\infty} \lambda_-^n(x) g_-^n(y),\end{aligned}$$

Two branches in dual theory

$$\Delta_- = \frac{3}{2} + \left| c + \frac{1}{2} \right|$$

- $c > -1/2$:
  - source field $\psi^s(x)$ chiral; $\chi(x)$ absent from theory
  - zero mode primarily elementary
  - Nearly all bulk fermions described by $c > -1/2$

- $c < -1/2$:
  - field $\chi(x)$ marries with source field $\psi^s(x)$ to become massive $M_0 \sim k$
  - zero mode primarily composite
  - perhaps t_R in some models

Right-handed top t_R

$$\tilde{f}^0(y) = e^{(\frac{1}{2}-c)ky} \qquad m_\psi = ck$$

Take e.g. $c = -0.7$; $\Delta = 1.7 \Longrightarrow$ relevant mixing

$$\begin{pmatrix} t_R^{(0)} \\ t_R^{(1)} \\ t_R^{(2)} \\ t_R^{(3)} \\ \vdots \end{pmatrix} = \begin{pmatrix} 0.9796 & \sim -1 & \sim 0 & \sim 0 & \cdots \\ -0.1816 & \sim 0 & \sim -1 & \sim 0 & \cdots \\ 0.0514 & \sim 0 & \sim 0 & \sim -1 & \cdots \\ 0.0471 & \sim 0 & \sim 0 & \sim 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} t_R^s \\ t_R^{CFT(1)} \\ t_R^{CFT(2)} \\ t_R^{CFT(3)} \\ \vdots \end{pmatrix}$$

- massless eigenstate $t_R^0(x)$ roughly equal mixture of source/CFT
- KK modes contain elementary component

Example: RS GIM mechanism

Important point on inverse transformation:

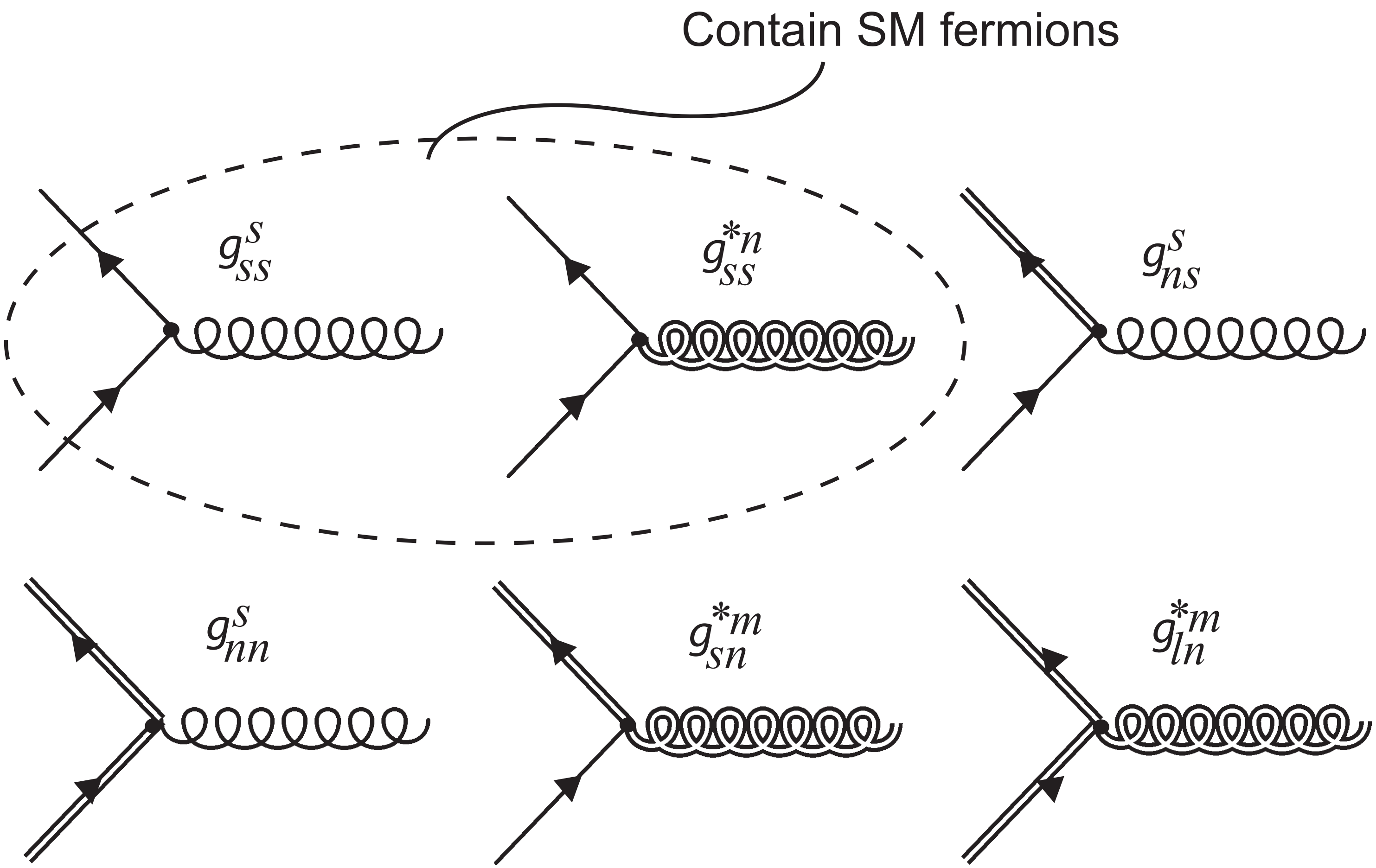
- Source field contains zero mode

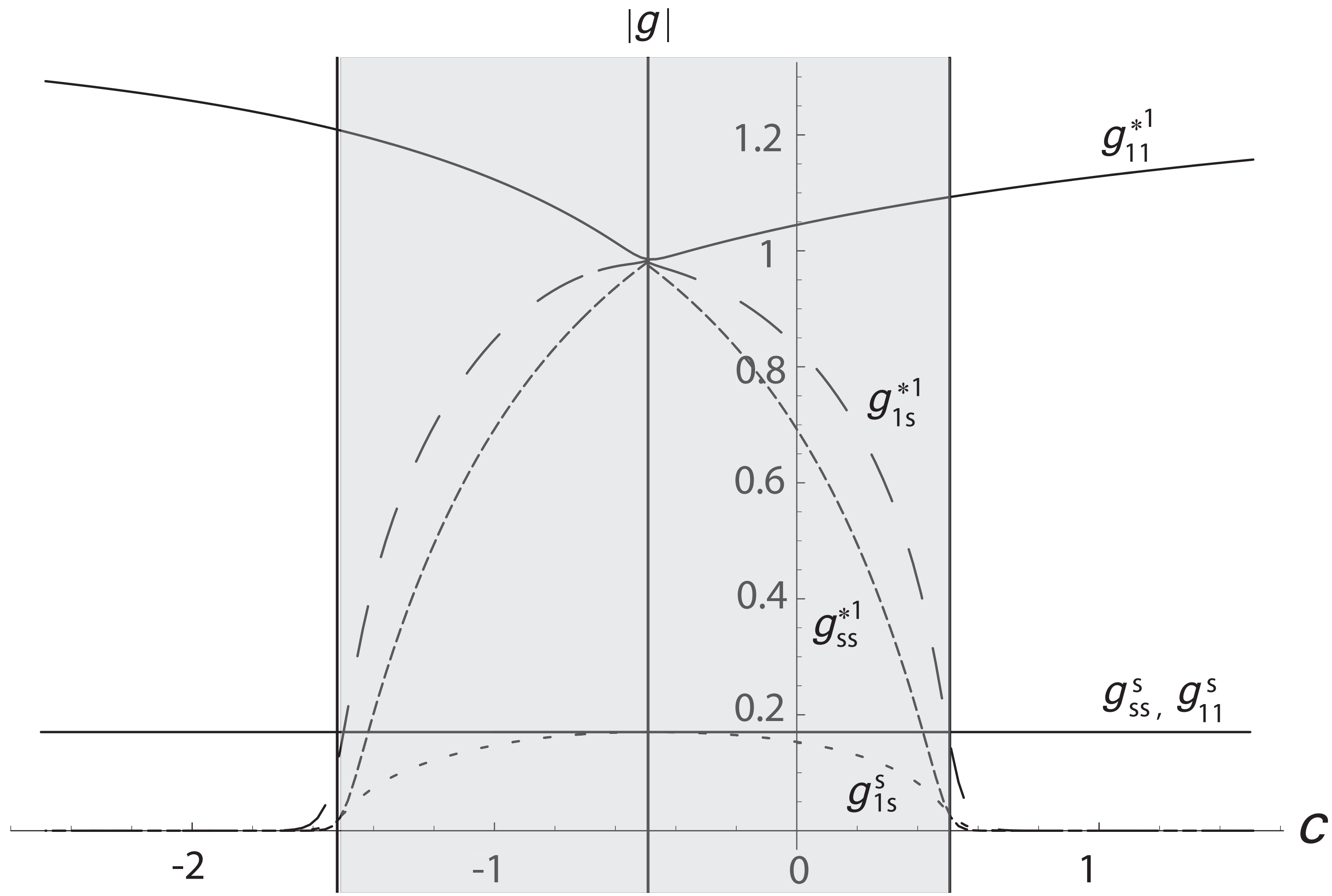
$$\psi^s(x) = \psi_+^0(x) + \sum_{m=1}^{\infty} \omega_+^{sm} \psi_+^m(x)$$

- Composite modes do not; entirely composed of KK modes

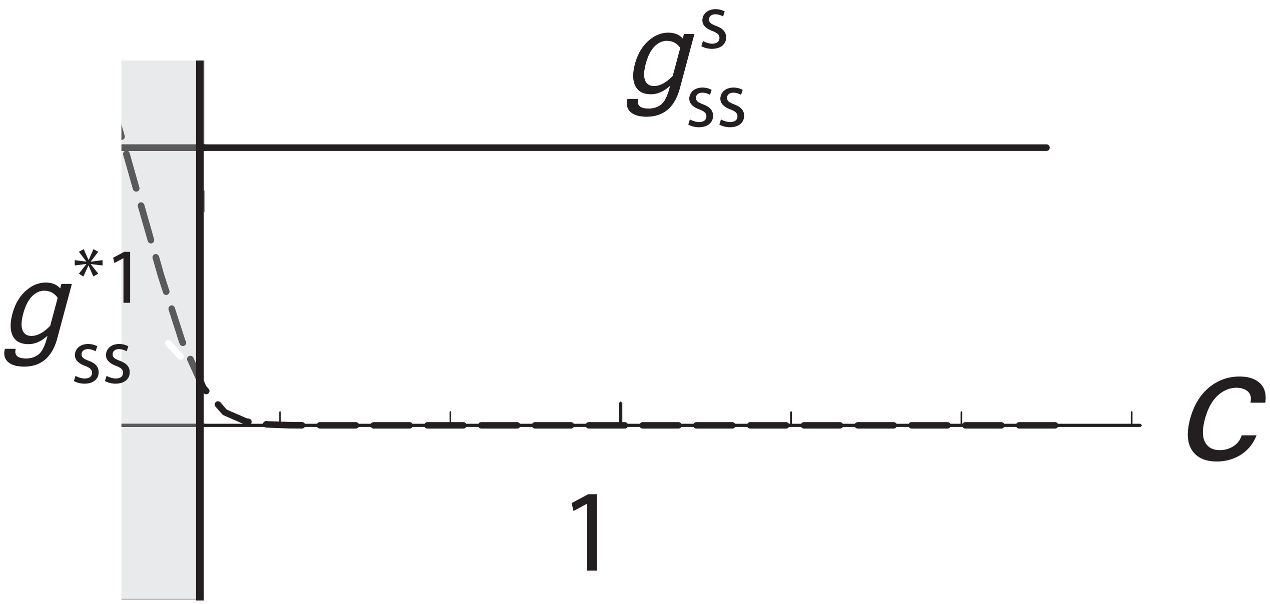
$$\lambda_+^n(x) = \sum_{m=1}^{\infty} \omega_+^{nm} \psi_+^m(x)$$

Gauge interactions $g^A_{\psi\psi}$





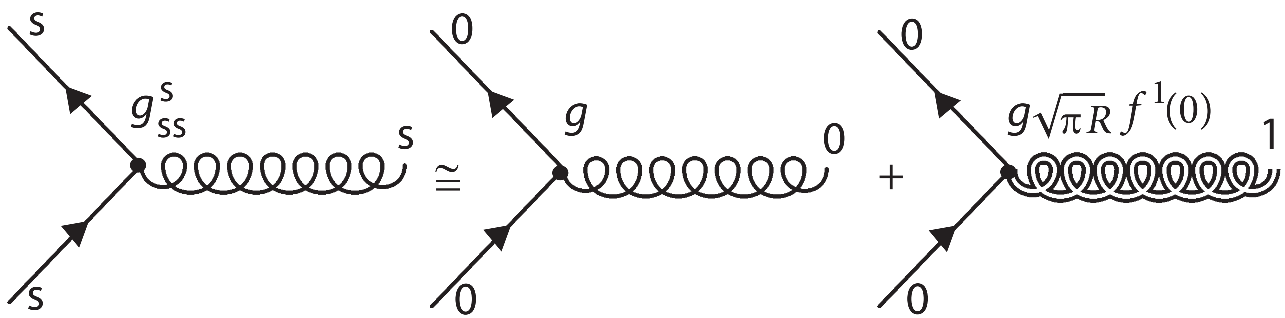
- Light fermions have exponentially suppressed couplings to composites



- 3-source vertex dominates

RS GIM mechanism

For light fermions, $c > 1/2$, 3-source vertex dominates:

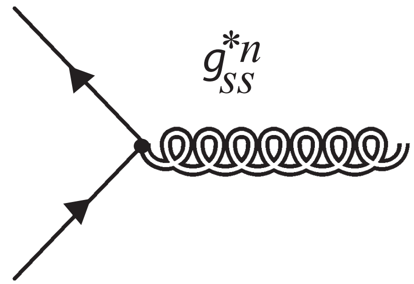


KK gauge boson couplings are approx. universal for light fermions

$\Longrightarrow$ FCNCs suppressed

Flavor violation

Important to track nonuniversal contribution to coupling



$$g_{ss}^{*n} = \int_0^{\pi R} e^{ky} g^s(y) g^{*n}(y) g^s(y)$$

Sum over all composite modes $\Longrightarrow$

$$g^1_{nonuniversal} = \sum_{n=1}^{\infty} g_{ss}^{*n} \omega^{n1} = g^1 - g^1_{universal}$$

Flavor violation - cont'd

Near $c \sim 1/2$, first composite mode saturates nonuniversal piece:

$$\begin{aligned} g^1_{nonuniversal} &\simeq g^{*1}_{ss}\omega^{11} \\ &\simeq g\,\sqrt{2\pi kR}\,\left(\frac{2c-1}{2-2c}\right)e^{(1-2c)\pi kR}. \end{aligned}$$

- Works well - order few % - for $c < 0.6$ (e.g. tau, muon)
- Deviates for $c > 0.6$ (e.g. electron), but nonuniversal contributions smaller anyway*

* Thanks to K. Agashe for discussions

Conclusions

- Holographic basis: bulk field expanded in source and CFT resonances
- Quantitatively describe elementary/composite mixing in warped duals
- Explain warped physics in terms of strong gauge dynamics
- Things to do:
  - Other applications: Higgsless models, warped SUSY, Gauge-Higgs models (QCD?)
  - Loop diagrams
    - important for EWPT, gauge coupling unification etc.
  - Brane localized kinetic terms - could modify composite content
  - More general geometries?