The Cosmological Moduli Problem - Revisited

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Reevaluating the Cosmological Origin of Dark Matter.
e-Print: arXiv:0912.3003 (and references within)
Conclusions

A non-thermal history provides a viable alternative to the well motivated thermal scenario.

Unlike the thermal case, a non-thermal history would imply a direct connection to fundamental theory and an observational window on the properties of the early universe.

Working directly with fundamental theories non-thermal models can lead to predictions which are falsifiable in current and near term experiments.

{ *Provocative*  A non-thermal history may be the first robust prediction of string theory.  (very much work in progress) }
Precision Cosmology

Cosmic Energy Budget Today

- Dark Energy 72%
- Dark Matter 23%
- Baryons 5%
- Early universe remarkably homogeneous
- Very small density contrast (1:100,000) at time of decoupling of CMB

All suggest physics beyond the standard model.
Dark Matter Abundance from Thermal Production

\[ \Omega_{dm} \equiv \frac{\rho_{dm}}{\rho_c} = 0.23 \times \left( \frac{10^{-26} \text{cm}^3 \cdot s^{-1}}{\langle \sigma v \rangle} \right) \]
Thermal History

Dark Matter Abundance from Thermal Production

\[ \Omega_{dm} \equiv \frac{\rho_{dm}}{\rho_c} = 0.23 \times \left( \frac{10^{-26}\text{cm}^3 \cdot s^{-1}}{\langle \sigma v \rangle} \right) \]

Cosmological Measurement

Weak Scale Physics

Dark Matter WIMPs?
Thermal Dark Matter

\[ \dot{n}_\chi + 3Hn_\chi = -\langle \sigma v \rangle (n_\chi^2 - n_{eq}^2) \]

Number Density (comoving)

Relativistic
\[ m < T \]
\[ n \sim n_{eq} \sim T^3 \]

Non-relativistic
\[ m > T \]
\[ n \sim n_{eq} \sim e^{-m/T} \]

“Freeze-out”
\[ H > n\langle \sigma v \rangle \]
\[ n(T_f) \sim \frac{H(T_f)}{\langle \sigma v \rangle_f} \]
Thermal History

Dark Matter Abundance from Thermal Production

\[ \Omega_{dm} \equiv \frac{\rho_{dm}}{\rho_c} = 0.23 \times \left( \frac{10^{-26} \text{cm}^3 \cdot \text{s}^{-1}}{\langle \sigma v \rangle} \right) \]

Cosmological Measurement

Weak Scale Physics

Dark Matter WIMPs?

No UV sensitivity!
Are things so simple?
Thermal Microscopic History

**Dark Matter Abundance from Thermal Production**

\[ \Omega_{dm} \equiv \frac{\rho_{dm}}{\rho_c} = 0.23 \times \left( \frac{10^{-26} \text{cm}^3 \cdot \text{s}^{-1}}{\langle \sigma v \rangle} \right) \]

*Assumed thermal equilibrium was reached
*Assumed radiation dominated universe at freeze-out
*Assumed no entropy production after freeze-out
*Assumed no other sources of cdm (e.g. late decays)
Cosmic History

- Quantum Gravity
- Inflation
- GUTs
- Baryogenesis
- (p)reheating
- Radiation dominated?
- $m_p$
- $10^{15}$ GeV
- TeV
- GeV
- MeV
- eV
- CMB
- First Stars
- Structure formation
- Dark Energy
Microscopic History

Dynamical Symmetry Breaking (e.g. SUSY)
Higgs / Strongly coupled dynamics?
EWSB Phase Transition
Dark Matter WIMPs
QCD Phase Transition

Inflation
BBN
CMB

$10^{15}$ GeV
TeV
GeV
MeV
eV

$m_p$
Are other cosmic histories possible?

*If dark matter is not produced thermally, how is it produced?*
Complete Theories of EWSB

Light scalars are a generic prediction of physics beyond the standard model

- Some have a geometric interpretation (e.g. extra dimensions), others are scalar partners of standard model fermions (SUSY)

- Low energy parameters become dynamical fields in early universe

\[ \langle h \rangle \rightarrow h(t, \vec{x}) \quad m, g \rightarrow m(h), g(h) \]

- Many of these fields pass through cosmological phases where they have little or no potential: “Approximate Moduli”
Approximate Moduli

Moduli Potential

\[ V_\varphi(T, H, \varphi) = 0 \]
Approximate Moduli

Moduli Potential

\[ V_\varphi(T, H, \varphi) = 0 + V_{soft} \]
Approximate Moduli

Moduli Potential

\[ V_\varphi(T, H, \varphi) = 0 + V_{soft} + \frac{1}{M^{2n}} \varphi^{4+2n} \]
Approximate Moduli

Moduli Potential

\[ V_\phi(T, H, \phi) = 0 + V_{soft} + \frac{1}{M^{2n}} \phi^{4+2n} + V_{SUGRA} \]
Approximate Moduli

Moduli Potential

\[ V_\varphi(T, H, \varphi) = 0 + V_{soft} + \frac{1}{M^{2n}} \varphi^{4+2n} + V_{SUGRA} + V_{np}. \]
Approximate Moduli

Moduli Potential

\[ V_\varphi(T, H, \varphi) = 0 + V_{soft} + \frac{1}{M^{2n}} \varphi^{4+2n} + V_{SUGRA} + V_{np} + V_{thermal} \]
Approximate Moduli

Moduli Potential

\[ V_\varphi(T, H, \varphi) = 0 + V_{soft} + \frac{1}{M^{2n}} \varphi^{4+2n} + V_{SUGRA} + V_{np} + V_{thermal} \]

Example:

\[ V(T, H, \varphi) = 0 + m_{soft}^2 \varphi^2 - H^2 \varphi^2 + \frac{1}{M^{2n}} \varphi^{4+2n} \]

\[ \langle \varphi \rangle \sim M \left( \frac{H}{M} \right)^{\frac{1}{n+1}} \quad H \gg m_{3/2} \sim \text{TeV} \]

\[ \langle \varphi \rangle \approx 0 \quad H \ll M \]

\[ \Delta \Phi \rightarrow \Delta E \quad \rightarrow \quad \text{Scalar Condensate} \]
Effect of Decaying Scalars

Dark Matter from Scalar Decay:

- Moduli generically displaced in early universe
- Energy stored in scalar condensate
  \[ \Delta \Phi \rightarrow \Delta E \]
- Typically decays through gravitational coupling
  \[ T_r \simeq \left( \frac{m_\phi}{10 \text{ TeV}} \right)^{3/2} \text{ MeV} \]
- Large entropy production dilutes existing dark matter of thermal origin
  \[ \Omega_{cdm} \rightarrow \Omega_{cdm} \left( \frac{T_r}{T_f} \right)^3 \text{ Thermal abundances diluted} \]
Non-thermal Dark Matter from Light Scalars

\[ \Phi \rightarrow X \]  Additional source of Dark Matter (after freeze-out)

Critical yield \[ n_c = \left. \frac{3H}{\langle \sigma v \rangle} \right|_{T_r} \]

Two possibilities:

Sub-critical
\[ n_X < n_c \]  No annihilations take place (yield preserved)

Super-critical
\[ n_X > n_c \]  Rapid annihilation down to fixed point
**Additional Source of Dark Matter from Scalar Decay**

**Super-critical case (attractor)**

Given $T_r < T_f$ then dark matter populated non-thermally

$$\Omega_{cdm} \sim \frac{m_x}{T} \left( \frac{H}{T^2 \langle \sigma v \rangle} \right) \bigg|_{T = T_f} T = T_r$$

$$\Omega_{cdm}^N = 0.23 \times \left( \frac{10^{-26}\text{cm}^3/\text{s}}{\langle \sigma v \rangle} \right) \left( \frac{T_f}{T_r} \right)$$

**Freeze-out temp**

**Reheat temp**

$T_f \sim \text{GeV} \quad T_r \sim \text{MeV}$

Can vary over 3 orders of magnitude -- Allowed values still imply weak-scale physics “WIMP Miracle” survives
Non-thermal Production of Dark Matter

Initial Radiation Phase

Moduli Domination begins \( H \sim m_\phi \)

Standard Thermal WIMP freeze-out

Moduli Decay and Reheat \( H \sim \Gamma_\phi \)

- Dark matter from direct decay
- Entropy produced (dilute relic densities)
- Radiation dominated universe
- Baryons?
Are other cosmic histories possible?

Yes.
Is a non-thermal history an exotic or a robust possibility?
What were the key ingredients?

1. “Light” Scalar
   \[ m_\phi \approx 10 \text{ TeV} \]

2. Gravitationally coupled
   \[ \Gamma_\phi \sim \frac{m^3_\phi}{M_p^2} \]

3. Stable dark matter particle
   \[ m_x \approx 100 \text{ GeV} \]
What were the key ingredients?

1. “Light” Scalar
   \[ m_\phi \approx 10 \text{ TeV} \]
   
   Light enough for decay after freeze-out, Heavy enough to evade BBN bounds

3. Stable dark matter particle
   \[ m_x \approx 100 \text{ GeV} \]
What do we expect from top-down model building?
Guidance from Fundamental Theory

What is needed from a top-down approach:

• 4D Effective theory with perturbative couplings
• Stabilize hierarchy $M_{EW\,SB} \ll M_p$ (and explain it?)
• Spontaneously broken SUSY (or alternative)
• Small and Positive Vacuum Energy

In String theory, all these problems are related and are essentially a problem of stabilizing scalars.

Additional challenge is embedding of SM (treat separately)
String Models that adequately meet these goals
String Models that adequately meet these goals

(joke)
Summary of Progress in Moduli Stabilization

• Moduli at Enhanced Symmetry Points (ESPs) are stable. (Dine)

• EPSs are cosmological attractors (Liam, et. al., S.W., Greene, S.W., ...).

• At least one modulus will not be stabilized at ESPs (Cremonini, S.W., also Dine, et. al.). (SM perturbative, Discrete R sym, 10D gauge symmetries --> 4D shift symmetry)

• Type II flux compactifications one finds bulk scalars with masses parametrically below the KK scale (e.g. Kachru, et. al., Silverstein).

• **Upshot:** Moduli can be stabilized, but string theory (consistent with pheno) robustly predicts at least one light scalar (mass from SUSY breaking).

**Warning:** There are models with no moduli! (e.g., Dine and Silverstein), but these models are not viable phenomenologically.
The Cosmological Moduli Problem


“Model Independent properties and cosmological implications of the dilaton and moduli sectors of 4-d strings”
Carlos, Casas, and Quevedo -- Phys. Lett. B318, 1993

\[ V = e^{\frac{K}{m_p^2}} |DW|^2 - 3m_{3/2}^2 m_p^2 \]

Shift symmetry
\[ \Phi = \phi + i\alpha \quad \rightarrow \quad W \neq W(\Phi) \]

Zero vacuum energy, stabilize scalar, break SUSY (spontaneously)
\[ \Delta V(\Phi) = m_{3/2}^2 m_p^2 f \left( \frac{\Phi}{m_p} \right) \]

\[ m_\phi \sim m_{3/2} \sim \text{TeV} \]
“Quasi-Realistic Models”
(focus on moduli sector)
Recipe for string vacuum (IIB)

Step One:

Flux provides stabilizing potential for many of the scalars in the theory (e.g. dilaton and structure moduli)

String scale masses

\[ m_z \approx M_s \approx 10^{17} \text{ GeV} \]

At low scales most string scale physics decouples

\[ W = W_0 \]
Recipe for string vacuum (IIB)

Step One:

\[ W = W_0 \]

Want: \( m_{3/2} \approx \text{TeV} \)

\[
m_{3/2} = \frac{|W_0|}{M_p^2 V_6}
\]

\( W_0 \ll 1 \) \hspace{2cm} \( V_6 \gg 1 \)

(KKLT)

or

“Large Volume”

\( V_6 \approx 10^{14} \)
Recipe for string vacuum (IIB)

Step Two:

Some scalars naturally remain light
(Axionic shift symmetry / No scale structure)

Stabilize by non-perturbative dynamics

\[ W = W_0 + A e^{-a\phi} \]

**SUSY restored**, Anti-deSitter Minimum

\[ V \ll 0 \]
Recipe for string vacuum (IIB)

Final Step:

Uplift (anti-brane / charged matter / string corrections) minimum to dS, SUSY broken

Result:

If $W_0$ appropriately tuned (exponential and discrete) to preserve hierarchy:

$$m_\phi \sim \log \left( \frac{m_p}{m_{3/2}} \right) m_{3/2}$$
Other models with possible non-thermal contribution:

- Large Volume Compactifications
  e.g. Conlon and Quevedo -- arXiv:0705.3460

- F-theory
  Heckman, Tavanfar, and Vafa-- arXiv:0812.3155

- M-theory on G2 manifolds
  Acharya, et. al. -- arXiv:0804.0863
Other models with possible non-thermal contribution:

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• M-theory on G2 manifolds  
  Acharya, et. al. -- arXiv:0804.0863

Remarks

• Many open questions:  
  Embedding visible sector, uplifting, path to 4d, SUSY breaking

• Gaugino (dark matter ) has three robust patterns  

• Light scalar may be robust prediction  
  “A Non-thermal WIMP Miracle”,  Acharya, et. al. -- 0908.2430
“Quasi-Quasi-Realistic Models”
The G2 MSSM

Acharya, et. al. -- arXiv:0804.0863

M-theory compactified to 4D on 7D manifold of G2 Holonomy

Defining properties: (Witten, Acharya, Cvetic, ...)

- N=1 SUSY in 4D

- Co-dimension 4 singularities give non-Abelian sectors localized on 3 cycles (hidden sector gauge fields)

- Co-dimension 7 conical singularities give chiral matter
Claim: All the moduli are geometric and protected by shift symmetries (parametrize volume of 3 cycles)

\[ W = 0 + m_p^3 \left( C_1 e^{-b_1 f_1} + C_2 e^{-b_2 f_2} \right) \]

\[ b_1 = \frac{2\pi}{P}, \quad b_2 = \frac{2\pi}{Q} \]

\[ f_1 = f_2 = t_i + is_i \]

Hierarchy preserved

Thus, all moduli can be stabilized by non-perturbative effects (e.g. gaugino condensation in a hidden sector) without turning on flux.

\[ V_7 = \prod_{i=1}^{N} s_i^{a_i} \quad \sum_i a_i = \frac{7}{3} \]

\[ V_Q = \sum_{i=1}^{N} N_i s_i \]

Perturbative theory \( V_7 \gg V_Q \gg 1 \)
Cautionary Remarks

✴ No explicit examples of metrics (only non-singular G2’s -- Joyce)

✴ Symmetries known (Witten, Atiyah, Acharya)

\[
K = -3m_p^2 \ln \left(4\pi^{1/3}V_7\right) \quad V_7 = \prod_{i=1}^{N} s_i^{a_i} \quad \sum_{i} a_i = \frac{7}{3}
\]

✴ M2 / M5 and flux?

✴ Blow up modes and structure of singularities?
G2 MSSM

Minimum is SUSY preserving!
Break SUSY / dS uplift via chiral matter (Nilles)

\[
K/m_p^2 = -3 \ln(4\pi^{1/3} V_7) + \bar{\phi} \phi, \quad V_7 = \prod_{i=1}^{N} s_i^{a_i}
\]

\[
W = m_p^3 \left( C_1 P \phi^{-(2/P)} e^{i b_1 f_1} + C_2 Q e^{i b_2 f_2} \right); \quad b_1 = \frac{2\pi}{P}, \quad b_2 = \frac{2\pi}{Q}
\]

\[
f_1 = f_2 = f_{\text{hid}} = \sum_{i=1}^{N} N_i z_i; \quad z_i = t_i + is_i.
\]

Stabilization in dS vacuum

\[
\langle s_i \rangle \sim f(V_Q) \quad m_i \sim f(V_Q) m_{3/2}
\]
Take home message

Everything controlled by 3 cycle volume!

\[ \langle s_i \rangle \sim f(V_Q) \quad m_i \sim f(V_Q)m_{3/2} \]

- Perturbative control
- dS minimum
- Moduli masses
- Hierarchy problem

LOTS of work to be done, but promising first results.
Non-thermal Dark Matter from Light Scalars

Additional source of Dark Matter (non-thermal origin)

Critical yield \[ n_c = \frac{3H}{\langle \sigma v \rangle} \bigg|_{T_r} \]

Two possibilities:

Sub-critical
\[ n_X < n_c \]  
No annihilations take place (yield preserved)

Super-critical
\[ n_X > n_c \]  
Rapid annihilation down to fixed point
No annihilations take place (yield preserved)

\[ n_X < n_c \]

\[ Y_X = B_\phi \Delta_\phi^{-1} Y_\phi(0) \sim \frac{B_\phi n_\phi^{(0)}}{T_r^3} \]

- Amount of Dark matter
- Branching ratio
- Entropy dilution
- Amount of moduli

G2 example, branching ratio and reheat temperature both depend on 3 cycle volume (we are measuring geometry)!

Contrast to Thermal Case (no UV connection!)

\[ \Omega_{cdm} \sim \frac{m_x}{T} \left( \frac{H}{T^2 \langle \sigma v \rangle} \right) \bigg|_{T=T_f} \]
Super-critical

\[ n_X > n_c \]

Rapid annihilation down to fixed point

\[ \Omega_{cdm} \sim \frac{m_x}{T} \left( \frac{H}{T^2 \langle \sigma v \rangle} \right) \bigg|_{T=T_f} \quad T = T_r \]

\[ \Omega_{cdm}^{NT} = 0.23 \times \left( \frac{10^{-26}\text{cm}^3/\text{s}}{\langle \sigma v \rangle} \right) \left( \frac{T_f}{T_r} \right) \]

Freeze-out temp

Reheat temp

\[ T_f \sim \text{GeV} \quad T_r \sim \text{MeV} \]

Recall: \[ T_r \sim \sqrt{\Gamma_\phi m_p} \]

G2 example (UV sensitivity possible)

\[ \Omega_{LSP} h^2 \approx 0.27 \left( \frac{m_{LSP}}{100\text{GeV}} \right)^3 \left( \frac{10.75}{g_*(T_r)} \right)^{1/4} \left( \frac{3.26 \times 10^{-7}\text{GeV}^{-2}}{\langle \sigma v \rangle} \right) \left( \frac{4}{D_{X_i}} \right)^{1/2} \left( \frac{2m_{3/2}}{m_{X_i}} \right)^{3/2} \left( \frac{100\text{ TeV}}{m_{3/2}} \right)^{3/2} \]
What we would like to say...

• Non-thermal dark matter is a generic consequence of string theory.

• Thermal histories are difficult to achieve due to the existence of light scalars associated with SUSY breaking.

• Models are highly constrained by both theory and experiment

Some reasons why we can’t:

• Need better understanding of SUSY breaking and dS uplift

• Visible / MSSM sector?

• Are we looking at very special places in the landscape (lamppost problem)?
A Non-thermal WIMP Miracle


In gravity mediation if scalars stabilized without reintroducing electroweak hierarchy and accounting for small and positive vacuum energy this typically implies:

\[ m_\phi \approx m_{3/2} \approx \text{TeV} \]

A new “WIMP” miracle

- Scalar decays into Dark Matter and radiation  \[ \phi \rightarrow X \]

- Initial abundances diluted  \[ \Omega_{cdm} \rightarrow \Omega_{cdm} \left( \frac{T_r}{T_f} \right)^3 \]

- Non-thermal history for dark matter
Some Phenomenological Implications of a Non-thermal history
SUSY Model Constraints Enforcing WMAP (blue)

Ellis, et. al. 2005
SUSY Model Constraints *Without* Enforcing WMAP (blue)

\[ \tan \beta = 10, \mu > 0 \]

Gelmini, Gondolo, Soldatenko, Yaguna hep-ph/0605016
PAMELA -- Indirect Evidence for WIMPs?

Expected Positron Flux

\[ \Phi \sim \frac{\langle \sigma v \rangle}{m^2_X} \times \rho^2(r) \]

Microphysics  Astrophysics

Important Considerations

- Astrophysical uncertainties: Halo profile, propagation, backgrounds
- Unknown astrophysical sources, e.g. Pulsars
- Proton contamination (10,000/1)

Taken alone probably not a compelling case for dark matter
Larger cross-section can address PAMELA excess

Figure by Ran Lu (grad student MCTP)
Fermi predictions

Figure by Ran Lu (grad student MCTP)
Photon-baryon heating during ionization from dark matter annihilation

Slatyer, Padmanabhan and Finkbeiner 0906.1197
Other concerns / constraints

- Isocurvature perturbations (require thermal equilibrium). (Weinberg, Martin, et.al.)

- Direct Detection (no signal for Wino Neutralino)

- Gravitino problem (model dependent)

- FCNC (model dependent?)

- Baryon asymmetry (generated at decay?)
Conclusions

A non-thermal history provides a viable alternative to the well motivated thermal scenario.

Unlike the thermal case, a non-thermal history would imply a direct connection to fundamental theory and an observational window on the properties of the early universe.

Working directly with fundamental theories non-thermal models can lead to predictions which are falsifiable in current and near term experiments.

{ *Provocative* A non-thermal history may be the first robust prediction of string theory. (very much work in progress) }
Experimental Result Leads to Excitement and Controversy
by Dennis Overbye

To the physicist, the above expression succinctly summarizes the recent surprising results coming from the Large Hadron Collider (LHC) located in Geneva, Switzerland. The equation symbolically represents the amount of dark matter in the universe, which from the initial findings of the experiment seem to fall short of expectations coming from cosmological observation.

\[ \Omega_{cdm} = 0.002 \]
Thank You.
Back up slides
Scalar Condensates

$V(\varphi)$

Scalar Condensate forms

$\Delta \Phi \to \Delta E$

Coherent Oscillations

$V(\Phi) \sim \Phi^\gamma, \quad p = \left( \frac{2\gamma}{2 + \gamma} - 1 \right) \rho.$

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<th>$\gamma$</th>
<th>$p$</th>
<th>Notes</th>
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<td>0</td>
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<td>4</td>
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Cosmological Moduli Problem


Decay Gravitationally

\[ \Gamma_\varphi \sim \frac{m^3_\varphi}{m_p^2} \]
Cosmological Moduli Problem


Decay Gravitationally

\[ \Gamma_\varphi \sim \frac{m_\varphi^3}{m_p^2} \]

Two possibilities:

Stable

\[ m_\varphi < T eV \quad \rightarrow \quad \rho_{mod} < \rho_c \rightarrow m_\varphi < 10^{-26} \text{ eV} \]
Cosmological Moduli Problem


Decay Gravitationally

\[ \Gamma_\varphi \sim \frac{m_\varphi^3}{m_p^2} \]

Two possibilities:

Stable

\[ m_\varphi < T eV \quad \rightarrow \quad \rho_{\text{mod}} < \rho_c \quad \rightarrow \quad m_\varphi < 10^{-26} \text{ eV} \]

Decay

\[ m_\varphi > T eV \quad T_r > 1 \text{ MeV} \quad (BBN) \quad \rightarrow \quad m_\varphi > 10 \text{ TeV} \]

Concern: Decay to secondaries (model dependent) --> e.g. gravitino problem
Pamela anti-protons