Chiral matter wavefunctions in warped compactifications

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Based on:
arXiv:0812.2247,1012.2759 (F. Marchesano, PM, G. Shiu)

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Outline

1.Motivation and setup

2.The adjoint sector: 7-7 strings

3.The chiral sector: 7-7’ strings

4.Application: 4d effective field theories

5.Caveats and conclusions

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Warping and its benefits

- In this talk, I'll focus on warped geometries in string theory

$$\mathrm{d}s_{10}^2 = \mathrm{e}^{2\alpha}\mathrm{d}x_4^2 + \mathrm{e}^{-2\alpha}\,\mathrm{d}y_6^2$$

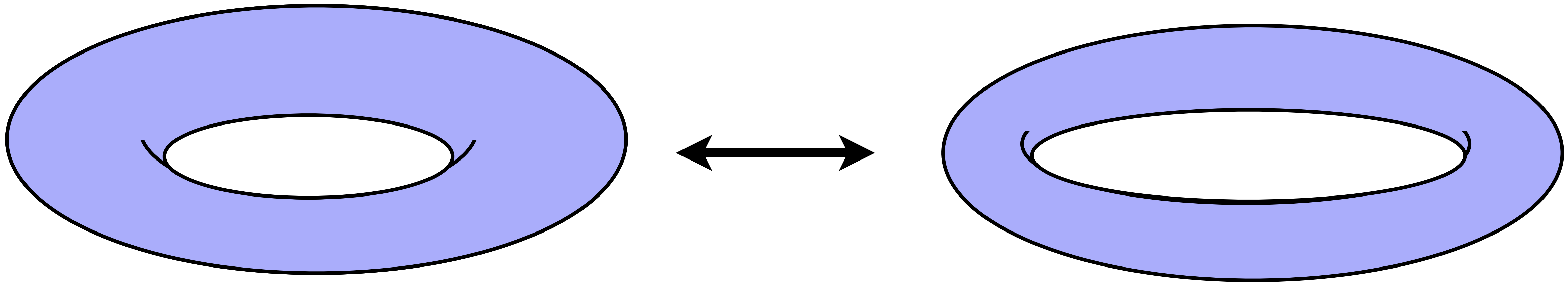
warp factor $\alpha = \alpha(y)$

- Such geometries have interesting phenomenology
  - Address Hierarchy problem [RS; GKP]
  - Late-time acceleration [KKLT;...]
  - Inflation [KKLMMT;...]
  - Sequestering [Luty, Sundrum; Kachru, McAllister, Sundrum; Berg, Marsh, McAllister, Pajer;...]
  - ...

Moduli stabilization

- Warping also a “generic” feature of string models

- String compactifications usually come with moduli: zero-energy deformations of the internal space



- Such moduli determine 4d effective field theory and result in “fifth” forces so must be lifted

Moduli stabilization (cont.)

- Lifting can be achieved by the addition of fluxes

$$W_{\mathrm{IIB}} = \int \Omega \wedge G^{(3)} \qquad W_{\mathrm{IIA}} = \int \Omega \wedge H^{(3)} + \int \mathrm{e}^{J} \wedge F$$

- Backreaction of fluxes complicates geometry (e.g. spoils Calabi-Yau)

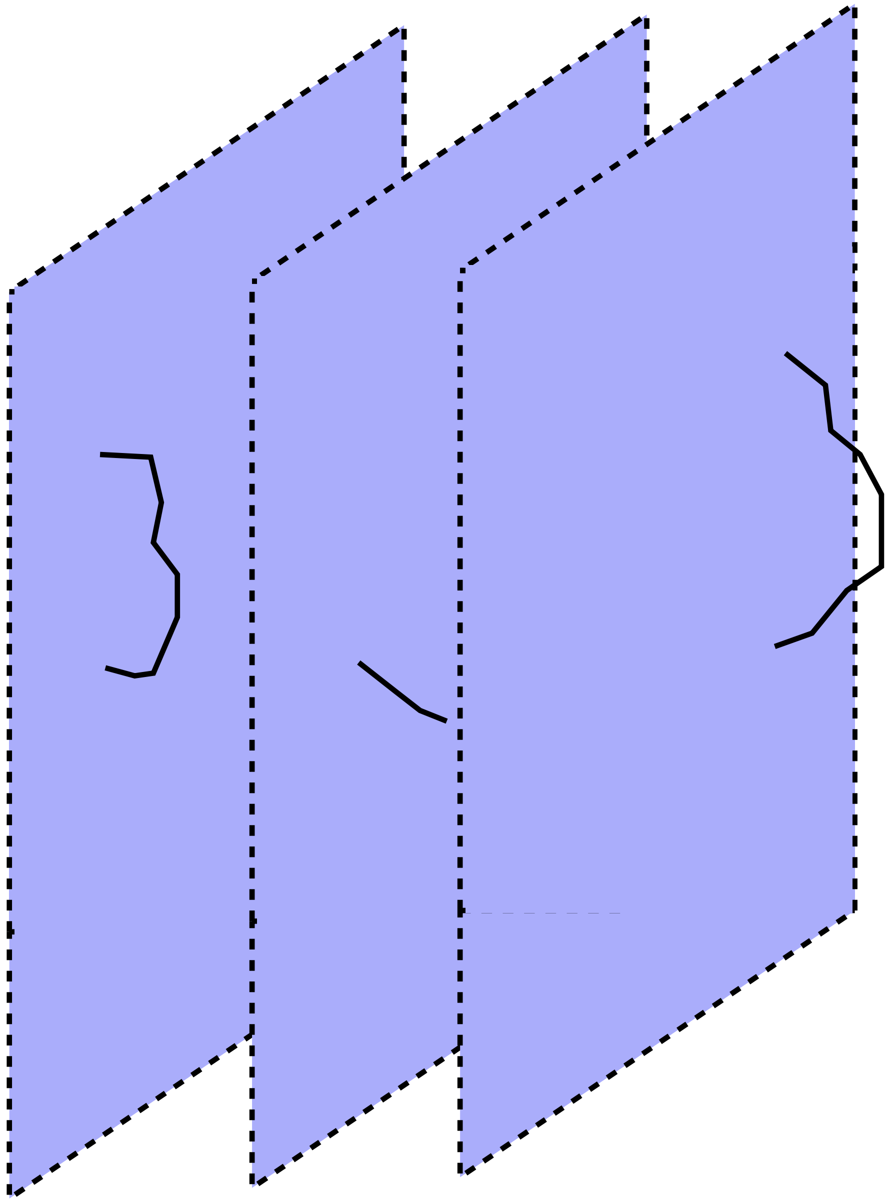
- Least amount of complication: type IIB with ISD flux and constant dilaton allows for warped Calabi-Yau

- NB: non-perturbative effects needed to stabilize Kähler structure [KKLT] will break this condition [Koerber, Martucci; Heidenreich, McAllister, Torroba]

Open strings

- In type II theories, gauge groups and charged matter come from the open string excitations of D-branes

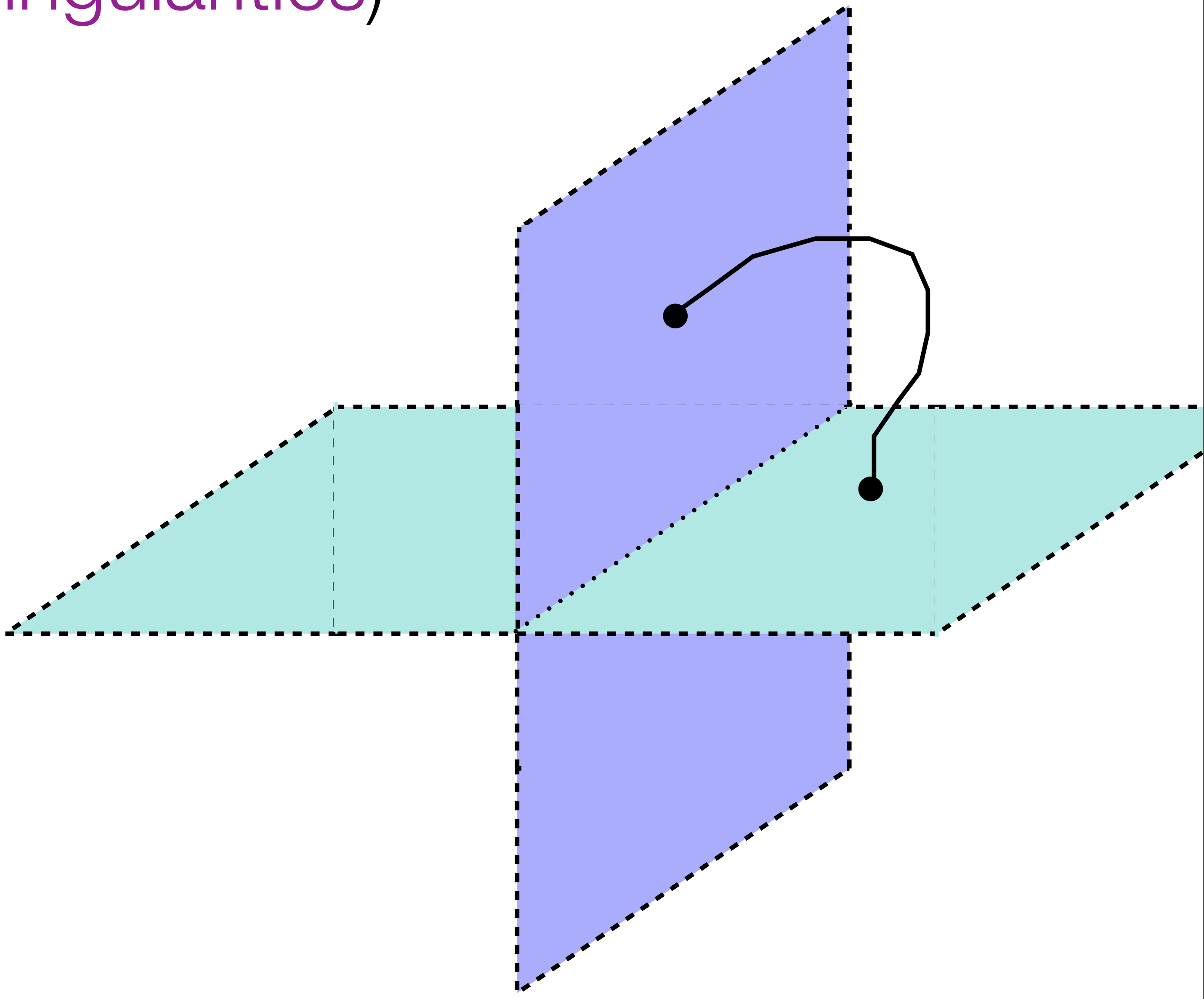
- Example: In flat space, the excitations of N coincident Dp-branes described by maximally supersymmetric (16 supercharges) U(N) Yang-Mills in (p+1)-dimensions



Open strings (cont.)

- Bifundamental matter comes from the intersection of D-branes (or from branes at singularities)

- Without fluxes, the number of families is the number of intersections of the branes

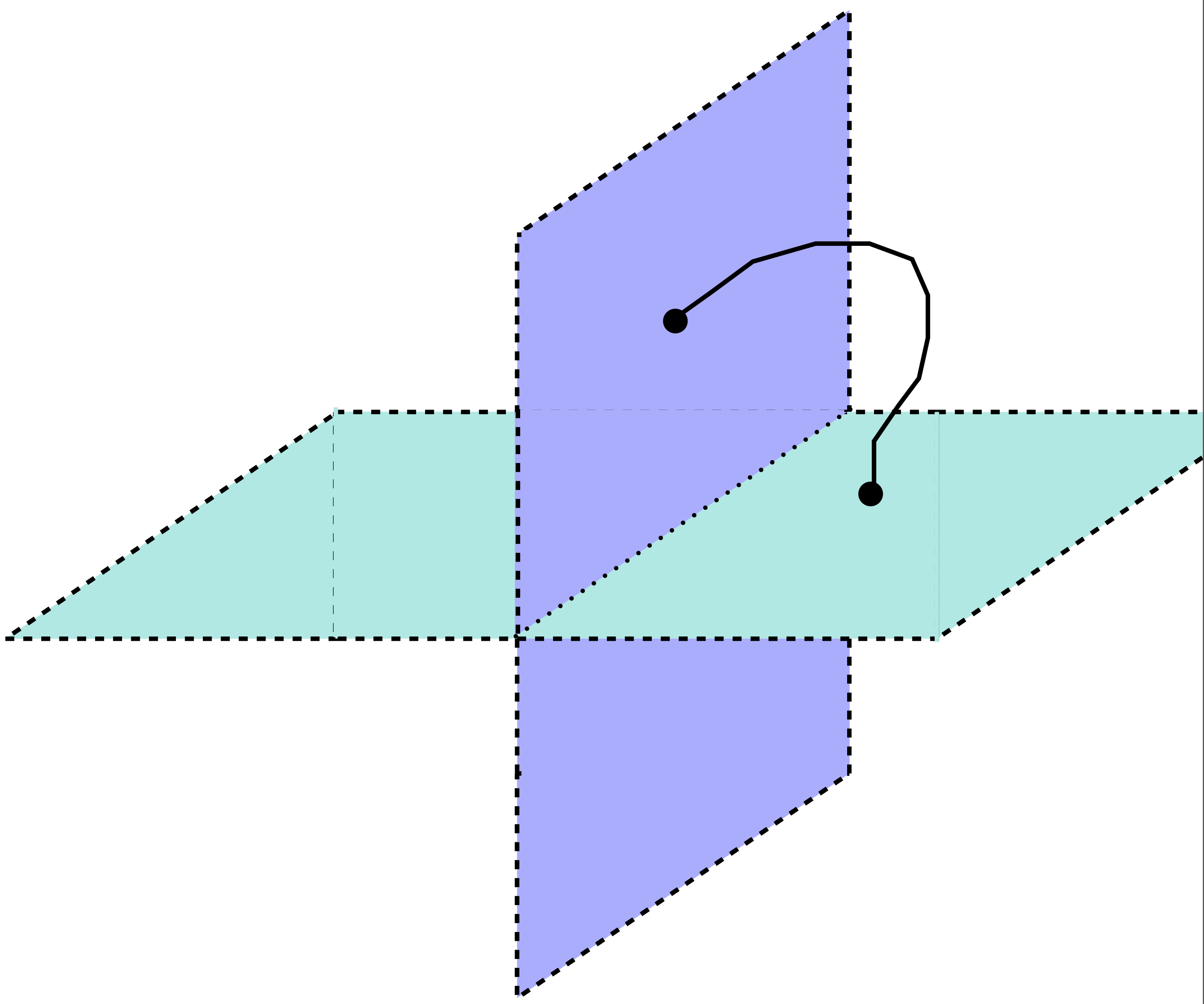


Chiral matter

- We will focus on D7-branes in warped IIB geometries

- Two D7s intersect on a 2-cycle in the 6d internal space and so to get a chiral spectrum in 4d, the branes need to be magnetized

$$\langle F^{(2)} \rangle \neq 0$$



4d effective field theories

- Intersecting, magnetized D7 branes in flux compactifications are thus promising for building realistic string models (though other ingredients needed)

- A 4d effective field theory is useful to discuss long wavelength phenomenology

- Two routes to an eft:
  - Conformal field theory techniques
  - Dimensional reduction

4d effective field theories (cont.)

- $\mathcal{N}_4 = 1$ warped compactifications in type IIB necessarily involve Ramond-Ramond fluxes:

$$\mathrm{d}s_{10}^2 = \mathrm{e}^{2\alpha}\mathrm{d}x_4^2 + \mathrm{e}^{-2\alpha}\,\mathrm{d}y_6^2$$

$$F^{(5)} = \big(1 + *\big)\mathcal{F}^{(5)} \qquad \mathcal{F}^{(5)} = \mathrm{d}\mathrm{e}^{4\alpha} \wedge \mathrm{dvol}_{R^{1,3}}$$

$$*_6 G^{(3)} = \mathrm{i}G^{(3)}$$

- This makes quantization of the string difficult and so the 4d eft cannot be easily extracted from stringy amplitudes

4d effective field theories (cont.)

- Alternate method: Higher dimensional actions are highly constrained so use dimensional reduction
- Requires solutions of higher-dimensional equations of motion

$$A_\mu(x,y) = A_\mu(x)\,s(y)$$

wavefunction

$$-\frac{1}{4g_8^2}\int_{\mathcal{W}} \sqrt{-g}F^2$$

$$\downarrow$$

$$-\frac{1}{4g_8^2}\int_{R^{1,3}} \mathrm{d}^4 x F^2 \int_{\Sigma^4} \mathrm{d}^4 y \sqrt{g}\, s^2$$

$$\downarrow$$

$$\boxed{\frac{1}{g_4^2} = \frac{\mathcal{V}_{\mathcal{S}_4}}{g_8^2}}$$

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Warped spherical cow

- Our goal is to learn something about the 4d eft from the 8d eft via dimensional reduction

- Since our focus is to learn about the effects of warping, I'll focus here on a simple background

$$\mathrm{d}s_{10}^2 = \mathrm{e}^{2\alpha}\mathrm{d}x_4^2 + \mathrm{e}^{-2\alpha}\,\underset{\substack{\nearrow \\ T^6}}{\mathrm{d}y_6^2}$$

$$F^{(5)} = \big(1 + *\big)\mathcal{F}^{(5)} \quad \mathcal{F}^{(5)} = \mathrm{d}\mathrm{e}^{4\alpha} \wedge \mathrm{dvol}_{R^{1,3}} \quad G^{(3)} = 0$$

- As a warmup, consider the adjoint matter on a single D7 wrapping $T^4 \subset T^6$

Bosonic modes

- The 8d bosonic degrees of freedom are

$$\begin{tikzcd}
A_\alpha & \text{8d gauge boson} \arrow[r] \arrow[dr] & A_\mu & \text{4d gauge boson} \\
\Phi^i & \text{transverse fluctuations} \arrow[r] & A_a, \Phi^i & \text{4d scalars}
\end{tikzcd}$$

- The bosonic action is the DBI+CS action

$$S_{\mathrm{D7}}^{\mathrm{DBI}} = -\tau_{\mathrm{D7}} \int_{\mathcal{W}} \mathrm{d}^8 x \sqrt{\left|\det\left(\mathrm{P}\left[g_{\alpha\beta}\right] + \lambda F_{\alpha\beta}\right)\right|}$$

$$S_{\mathrm{D7}}^{\mathrm{CS}} = \tau_{\mathrm{D7}} \int_{\mathcal{W}} \mathrm{P}\big[C^{(4)}\big] \wedge \mathrm{e}^{\lambda F^{(2)}} \qquad \lambda = 2\pi\alpha'$$

$$\mathrm{P}\big[g_{\alpha\beta}\big] = g_{\alpha\beta} + \lambda^2 g_{ij} \partial_\alpha \Phi^i \partial_\beta \Phi^j$$

Bosonic modes (cont.)

- In this case, warping does not effect the zero modes

$$\Box_{R^{1,3}}\Phi^i + e^{4\alpha}\Box_{T^4}\Phi^i = 0$$

$$\Box_{R^{1,3}}A_\mu + e^{4\alpha}\Box_{T^4}A_\mu = 0$$

$$\Box_{R^{1,3}}A^a + e^{2\alpha}\partial_b F^{ba} + e^{2\alpha}\epsilon^{abcd}\partial_b\left(e^{4\alpha}F_{cd}\right) = 0$$

- The zero modes satisfy

$$\Box_{R^{1,3}}X = 0$$

and so are constant

Fermionic modes

- The 8d fermionc degrees of freedom are encoded in a pair of 10d Majorana-Weyl fermions subject to a gauge redundancy called κ-symmetry (c.f. GS superstring)

$$\Theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \quad \Theta \sim \Theta + P_-^{\mathrm{D7}} \kappa \quad P_\pm^{\mathrm{D7}} = \frac{1}{2} \begin{pmatrix} 1 & \pm \mathrm{i}\Gamma_{(8)} \\ \mp \mathrm{i}\Gamma_{(8)} & 1 \end{pmatrix}$$

- The 4d degrees of freedom are fermionic superpartners of the gauge boson and complexified scalars

$\psi_0 \leftrightarrow A_\mu$	gaugino
$\psi_{1,2} \leftrightarrow A_{1,2}$	Wilsonini
$\psi_3 \leftrightarrow \Phi$	modulino

Fermionic modes (cont.)

- The fermionic action is [Martucci, Rosseel, Van denBleeken, Van Proeyen]

$$S^{\mathrm{f}}_{\mathrm{D7}} = \tau_{\mathrm{D7}} \int_{\mathcal{W}} \sqrt{|\det(g_{\alpha\beta})|}\,\bar{\Theta} P^{\mathrm{D7}}_{-} \Gamma^{\alpha} \Big(\nabla_{\alpha} + \frac{1}{16} \not{F}^{(5)} \Gamma_{\alpha} \mathrm{i}\sigma_2\Big)\Theta$$

- Then

4-cycle chirality operator

$$\Big(\not{\partial}_{R^{1,3}} + \not{\partial}_{T^4} + \frac{1}{2}\not{\partial}_{T^4}\alpha\left[1 + 2\Gamma_{T^4}\right]\Big)\theta = 0$$

- For the zero modes:

$$\psi_0 = \mathrm{e}^{-3\alpha/2}\eta_0$$
$$\psi_{1,2} = \mathrm{e}^{\alpha/2}\eta_{1,2}$$
$$\psi_3 = \mathrm{e}^{-3\alpha/2}\eta_3$$

constant spinors

Fermionic modes (cont.)

- Since the $\Gamma-$matrices are warped, these are consistent with supersymmetry

$$\delta A_\mu \sim \bar{\epsilon}\Gamma_\mu \psi_0$$
$$\delta A_a \sim \bar{\epsilon}\Gamma_a \psi_a$$
$$\delta \Phi \sim \bar{\epsilon}\Gamma^i \psi_3$$

$$\epsilon = \mathrm{e}^{\alpha/2}\eta_0$$

Killing spinor

- This analysis can be generalized (see `0812.2247`)
  - Abelian magnetic flux
  - Calabi-Yau
  - Bulk fluxes (see also [Cámara, Marchesano])

- We'll return to the above wavefunctions later

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Bifundamental matter

- Chiral matter is much more phenomenologically interesting, but more involved.

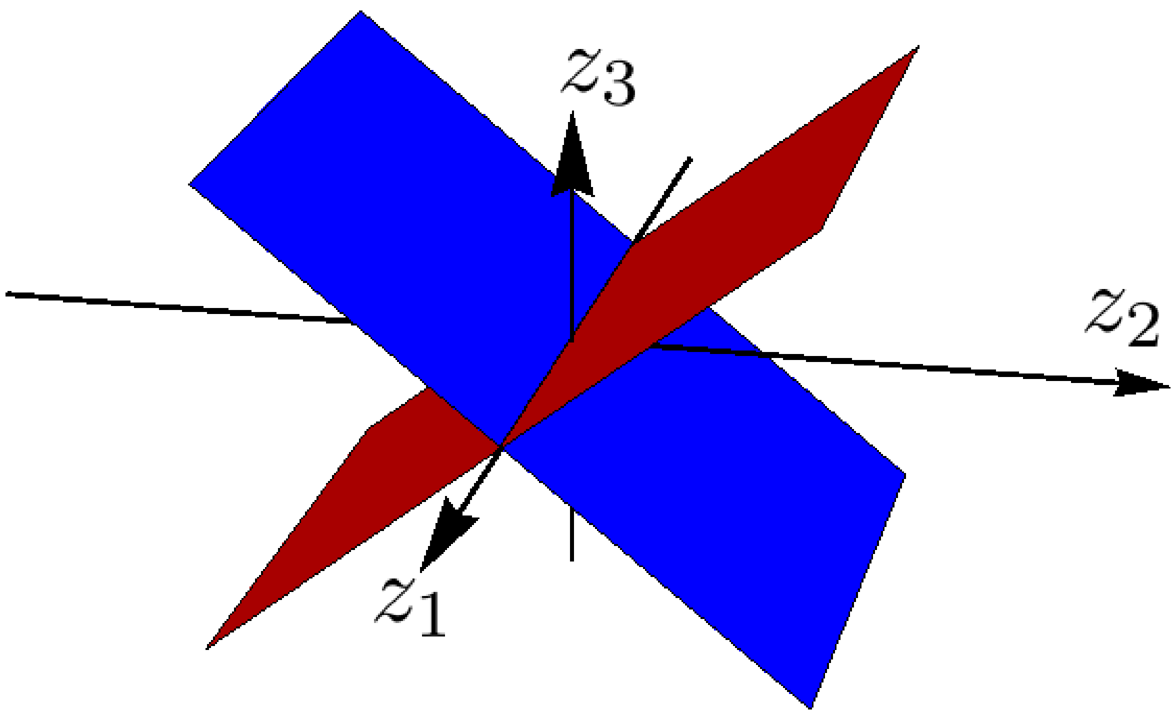
- Consider first vector-like bifundamental matter:

$$T^6 = T_1^2 \times T_2^2 \times T_3^2$$

matter curve

$$\mathrm{D7}_1 : z^3 = M_3 z^2$$

$$\mathrm{D7}_2 : z^3 = -M_3 z^2$$



Intersections as Higgsing

- When the branes are coincident, the symmetry is enhanced to $\mathrm{U}\left(2\right)$. The transverse fluctuations are promoted to an adjoint-valued scalar Φ

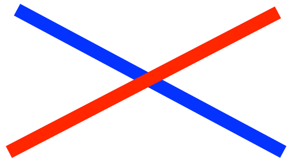
Position of $\mathrm{D7}_1$ Position of $\mathrm{D7}_2$

$$\Phi = \begin{pmatrix} \phi^a & \phi^- \\ \phi^+ & \phi^b \end{pmatrix}$$

7_1-7_2 strings (bifundamental)

- vevs for $\phi^{a,b}$ correspond to background D7 positions

$$\langle \phi^a \rangle = \lambda^{-1} M_3 z^2 \quad \langle \phi^b \rangle = -\lambda^{-1} M_3 z^2$$



small angle needed to neglect α' corrections

Non-Abelian bosonic action

- Must use the non-Abelian action [Myers]

$$S_{\mathrm{D7}}^{\mathrm{DBI}} = -\tau_{\mathrm{D7}} \int_{\mathcal{W}} \mathrm{d}^8 x \, \mathrm{Str} \left\{ \sqrt{\left| \det \left(M_{\alpha\beta} \right) \det \left(Q^i_j \right) \right|} \right\}$$

$$S_{\mathrm{D7}}^{\mathrm{CS}} = \tau_{\mathrm{D7}} \int_{\mathcal{W}} \mathrm{Str} \left\{ \mathrm{P} \left[\mathrm{e}^{\mathrm{i}\lambda \iota_\Phi \iota_\Phi} C^{(4)} \right] \wedge \mathrm{e}^{\lambda F^{(2)}} \right\}$$

symmetrization interior product

$$\mathrm{P}\left[v_\alpha\right] = v_\alpha + \lambda D_\alpha \Phi^i v_i$$

$$M_{\alpha\beta} = \mathrm{P}\left[g_{\alpha\beta} + g_{\alpha i} \left(Q^{-1} - \delta \right)^{ij} g_{j\beta} \right] + \lambda F_{\alpha\beta}$$

$$Q^i_j = \delta^i_j - \mathrm{i}\lambda \left[\Phi^i, \Phi^k \right] g_{kj}$$

D7 bosonic action (cont.)

- Bulk fields given as a non-Abelian Taylor expansion

adjoint valued

$$\Psi\left[\Phi\right] = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} \Phi^{i_1} \cdots \Phi^{i_n} \partial_{i_1} \cdots \partial_{i_n} \Psi_0$$

neutral

$$= \Psi_0 + \mathcal{O}\left(\lambda\right)$$

- Leading order in α', action is warped Yang-Mills

- Equations of motion are second order and hard to solve in general

Non-Abelian fermionic action

- Abelian analogue of Martucci action not known

$$S_{\mathrm{D7}}^{\mathrm{F}} = \frac{1}{g_8^2}\int \mathrm{d}^8x\,\bar{\theta}\Big\{\mathrm{e}^{-\alpha}\not{\partial}_{R^{1,3}} + \mathrm{e}^{\alpha}\not{\partial}_{T^4} + \mathrm{e}^{\alpha}\frac{1}{2}\not{\partial}_{T^4}\alpha\big(1 + 2\Gamma_{T^4}\big)\Big\}\theta$$

- To leading order in α' [Wynants]

$$\not{\partial} \to \not{D} \qquad\qquad \delta\mathcal{L} = -\mathrm{i}\bar{\theta}\mathrm{e}^{-\alpha}\Gamma_i\left[\Phi^i, \theta\right]$$

Equations of motion

- For the bifundamental zero modes, take the ansatz

$$\psi^{\mp}_{0,3} = \mathrm{e}^{-3\alpha/2}\chi^{\mp}_{0,3} \qquad \psi^{\mp}_{1,2} = \mathrm{e}^{\alpha/2}\chi^{\mp}_{1,2}$$

gaugino, modulino

wilsonini

- Equations of motion from Fermionic action (no flux yet)

$$\begin{aligned}
0 =& \partial_1\chi_1^{\mp} + \partial_2\chi_2^{\mp} + \mathrm{e}^{-4\alpha}D_3^{\mp}\chi_3^{\mp} \\
0 =& \partial_1\chi_0^{\mp} + \partial_2^*\chi_3^{\mp} - D_3^{\pm *}\chi_2^{\mp} \\
0 =& \partial_1^*\chi_3^{\mp} - \partial_2\chi_0^{\mp} - D_3^{\pm *}\chi_1^{\mp} \\
0 =& \partial_1^*\chi_2^{\mp} - \partial_2^*\chi_1^{\mp} + \mathrm{e}^{-4\alpha}\hat{D}_3^{\mp}\chi_0^{\mp}
\end{aligned}$$

$$D_3^{\mp} = \mp \mathrm{i} M_3 \bar{z}^{\bar{2}}$$

BPS conditions

- For a single D7 brane, the equations of motion follow from F- and D-flatness conditions: [Jockers, Louis; Martucci]

$$W = \int_{T^4} \mathrm{P}\big[\gamma\big] \wedge \mathrm{e}^{\lambda F^{(2)}} \qquad D = \int_{T^4} \mathrm{P}\big[\mathrm{Im}\,\eta\big] \wedge \mathrm{e}^{\lambda F^{(2)}}$$

fundamental 3-form

$$\mathrm{d}\gamma = \Omega \wedge \mathrm{e}^{B^{(2)}} \qquad \eta = \mathrm{e}^{2\alpha} \mathrm{Im}\,\tau\, \mathrm{e}^{\mathrm{i}J} \wedge \mathrm{e}^{B^{(2)}}$$

warped Kähler form

- Comparing to the CS-action, the non-Abelian version should be (see also [Butti et. al.])

$$W = \int_{T^4} \mathrm{Str}\Big\{\mathrm{P}\big[\mathrm{e}^{\mathrm{i}\lambda\iota_\Phi\iota_\Phi}\gamma\big] \wedge \mathrm{e}^{\lambda F^{(2)}}\Big\} \quad D = \int_{T^4} \mathrm{S}\Big\{\mathrm{P}\big[\mathrm{e}^{\mathrm{i}\lambda\iota_\Phi\iota_\Phi}\mathrm{Im}\,\eta\big] \wedge \mathrm{e}^{\lambda F^{(2)}}\Big\}$$

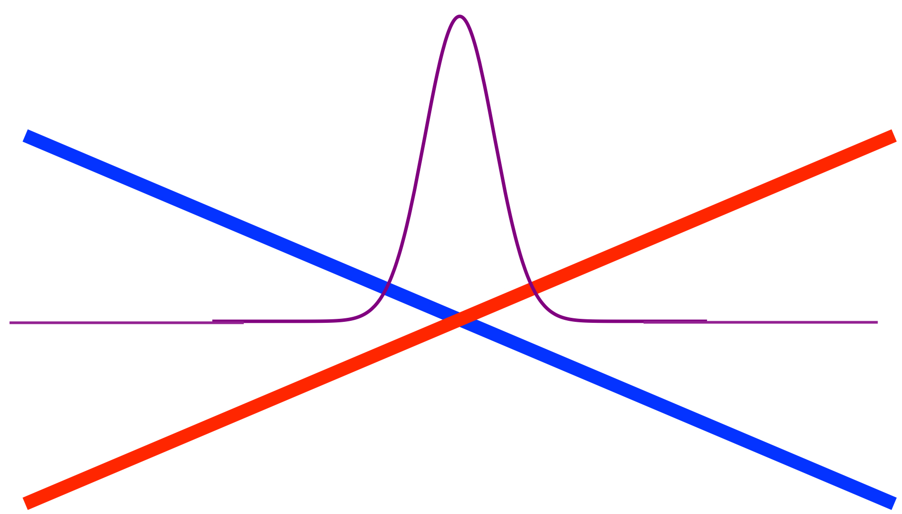
These yield the previous equations of motion with $\psi_0 = 0$

Unwarped zero mode

- In the absence of warping, the zero modes are exponentially localized on the intersection

$$
\begin{aligned}
\psi^{\mp}_{0,1} &= 0 \\
\psi^{\mp}_{2} &= \sigma^{\mp}(x^{\mu})\mathrm{e}^{-M_3\left|z^2\right|^2} \\
\psi^{\mp}_{3} &= \pm\,\mathrm{i}\sigma^{\mp}(x^{\mu})\mathrm{e}^{-M_3\left|z^2\right|^2}
\end{aligned}
$$

4D field



- Mixture of deformation modulus and Wilson lines of the un-Higgsed theory

Warped zero mode

- For arbitrary warping, no simple analytic solution

- In the weak warping case, can treat the warping as a perturbation

$$\mathrm{e}^{-4\alpha} = 1 + \epsilon\beta \qquad\qquad \epsilon \ll 1$$

- Can then expand the warped zero mode in terms of the unwarped massive modes

Unwarped spectrum

- The equation of motion for the massive modes is

$$\mathbf{D}^{\mp}\mathbf{X}^{\mp}_{\lambda} = m_{\lambda}\mathbf{X}^{\pm *}_{\lambda} \qquad \mathbf{D}^{\mp} = \begin{pmatrix} 0 & \partial_1 & \partial_2 & D_3^{\mp} \\ -\partial_1 & 0 & D_3^{\pm *} & -\partial_2^* \\ -\partial_2 & -D_3^{\pm *} & 0 & \partial_1^* \\ -D_3^{\mp} & \partial_2^* & -\partial_1^* & 0 \end{pmatrix} \quad \mathbf{X}^{\mp}_{\lambda} = \begin{pmatrix} \chi_0^{\mp} \\ \chi_1^{\mp} \\ \chi_2^{\mp} \\ \chi_3^{\mp} \end{pmatrix}$$

- Easiest to work in a rotated basis $\mathbf{X}'^{\mp} = \mathbf{J}^{-1}\mathbf{X}^{\mp}$

$$\mathbf{J} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1/\sqrt{2} & \mathrm{i}/\sqrt{2} \\ & & \mathrm{i}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$$\begin{aligned} \partial_1 &\to \partial_1 \\ \partial_2 &\to \hat{D}_2'^{\mp} = \frac{1}{\sqrt{2}}(\partial_2 \pm M_3 \bar{z}^{\bar{2}}) \\ \partial_3 &\to \hat{D}_3'^{\mp} = \frac{\mathrm{i}}{\sqrt{2}}(\partial_2 \mp M_3 \bar{z}^{\bar{2}}) \end{aligned}$$

Unwarped spectrum (cont.)

- Boundary conditions:
  - Periodicity along matter curve T_1^2
  - Localized on intersection

- $-$ - sector modes built from ladder operators (giving two simple harmonic oscillator algebras) and Fourier modes

lowering

$$\mathrm{i}\hat{D}_2'^{-} \qquad \mathrm{i}\left(\hat{D}_3'^{+}\right)^*$$

$$\mathrm{i}\left(\hat{D}_2'^{+}\right)^* \qquad \mathrm{i}\hat{D}_3'^{-}$$

raising

$$\varphi^-_{mnpl} = h_{mn}\left(z^1, \bar{z}^{\bar{1}}\right)\left[\mathrm{i}\left(\hat{D}_2'^{+}\right)\right]^l \left(\mathrm{i}\hat{D}_3'^{-}\right)^p \mathrm{e}^{-M_3\left|z^2\right|^2}$$

Fourier mode

Unwarped spectrum (cont.)

- Unwarped spectrum:

$$m_\lambda^2 = m^2 + n^2 + M_3\big(l + p + 1\big) \qquad \Phi_\lambda'^- = \big(\varphi_{mnlp}^-, 0, 0, 0\big)^{\mathrm{T}}$$

$$m_\lambda^2 = m^2 + n^2 + M_3\big(l + p + 1\big) \qquad \Phi_\lambda'^- = \big(0, \varphi_{mnlp}^-, 0, 0\big)^{\mathrm{T}}$$

$$m_\lambda^2 = m^2 + n^2 + M_3\big(l + p\big) \qquad \Phi_\lambda'^- = \big(0, 0, \varphi_{mnlp}^-, 0\big)^{\mathrm{T}}$$

$$m_\lambda^2 = m^2 + n^2 + M_3\big(l + p + 2\big) \qquad \Phi_\lambda'^- = \big(0, 0, 0, \varphi_{mnlp}^-\big)^{\mathrm{T}}$$

Expanding the warped zero mode

- Write the warped zero mode as

$$\mathbf{X}^- = \mathbf{\Phi}_0^- + \sum_\lambda c_\lambda \mathbf{\Phi}_\lambda^-$$

unwarped modes

- To leading order

$$c_\lambda = \frac{1}{m_\lambda^2} \int_{\mathcal{S}_4} \mathrm{d}^4 y \left(\mathbf{\Phi}_\lambda^-\right)^* \cdot \left(\mathbf{D}_0^+\right)^* \beta \mathbf{K}^- \mathbf{\Phi}_0^-$$

where

$$\mathbf{D}^- = \mathbf{D}_0^- + \epsilon \beta \mathbf{K}^- + \mathcal{O}\left(\epsilon^2\right)$$

- Examples given in [1012.2759]

Chirality

- Without magnetic flux, the spectrum is vector-like
- In order to have a chiral theory, the intersection must be magnetized

$$\frac{1}{2\pi}\int_{T^2} F^{(2)} = M_1 \sigma_3$$

- SUSY requires [Marino, Minasian, Moore, Strominger; ...]

$$F^{(2)} = -*_4 F^{(2)}$$

Hodge-$*$ on $\mathcal{S}_4$

Unwarped zero modes

- For example, if $M_1 > 0$, only the $-$-sector has zero modes

- Due to magnetic flux, wavefunction are quasi-periodic
[Cremades, Ibáñez, Marchesano;...]

$$\varphi_0^{j,-} = \mathrm{e}^{-\kappa \left|z^2\right|^2} \mathrm{e}^{2\pi \mathrm{i} M_1 z^1 \mathrm{Im}\, z^1} \vartheta \begin{bmatrix} j/2M_1 \\ 0 \end{bmatrix} \left(2M_1 z^1, \mathrm{i}2M_1\right)$$

$$\kappa = \sqrt{\left(\tfrac{M_1}{2}\right)^2 + M_3^2} \qquad j = 0, \ldots, 2M_1 - 1$$

families orthogonal: $\displaystyle\int_{T^4} \mathrm{d}^4 y \left(\varphi_0^{j,-}\right)^* \varphi_0^{k,-} = \delta^{kj}$

- Each family is a Gaussian peak at a different location on the matter curve

Warped zero modes

- As in unmagnetized case, warped zero mode has no general simple analytic solution
- Again, expand in unwarped massive modes
- Spectrum built from three QSHO algebras

$$\varphi_{nlp}^{j,-} = \left(\mathrm{i}D_1'^{-}\right)^n \left[\mathrm{i}\left(D_2'^{+}\right)^\dagger\right]^l \left(\mathrm{i}\hat{D}_3'^{-}\right)^p \varphi_0^{j,-}$$

$$D_1'^{\mp} = \partial_1 \mp M_1 \bar{z}^{\bar{1}}$$
$$D_2'^{\mp} \propto \partial_2 \pm \kappa \bar{z}^{\bar{2}}$$
$$D_3'^{\mp} \propto \mathrm{i}\left(\partial_2 \mp \kappa \bar{z}^{\bar{2}}\right)$$

Warped zero modes (cont.)

- Expand warped zero mode in terms of unwarped massive modes

$$\mathbf{X}^{j,-} = \mathbf{\Phi}_0^{j,-} + \sum_{k,\lambda} c_\lambda^k \mathbf{\Phi}_\lambda^{k,-}$$

unwarped modes

then

$$c_\lambda^k = \frac{1}{m_\lambda^2} \int_{\mathcal{S}_4} \mathrm{d}^4 y \left(\mathbf{\Phi}_\lambda^{k,-}\right)^* \cdot \left(\mathbf{D}_0^+\right)^* \beta \mathbf{K}^- \mathbf{\Phi}_0^{j,-}$$

family mixing is generic

- Examples given in [1012.2759]

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Warped kinetic terms

- The warped wavefunctions are useful in deducing the warped effective field theory

- Example: kinetic terms for modulino

$$\theta_3\left(x^\mu, y^a\right) = \mathrm{e}^{-3\alpha/2}\, \psi_3\left(x^\mu\right) \otimes \eta_3\left(y^a\right)$$

$$\int_{\mathcal{W}} \mathrm{d}^8 x\, \sqrt{g}\bar{\theta}_3 \Gamma^\mu \partial_\mu \theta_3$$

$$= \int_{R^{1,3}} \mathrm{d}^4 x \bar{\psi}_3 \partial\!\!\!/ \psi_3 \int_{\mathcal{S}^4} \mathrm{d}^4 y\, \sqrt{\tilde{g}} \mathrm{e}^{-4\alpha} \eta_3^\dagger \eta_3$$

$$\Gamma^\mu \sim \mathrm{e}^{-\alpha} \qquad \sqrt{g} = \sqrt{\tilde{g}}$$

Warped kinetic terms (cont.)

- Can infer that the Kähler metric behaves as

$$\mathcal{K}_{3\bar{3}} \sim \frac{\mathcal{V}_4^{\mathrm{w}}}{\mathcal{V}_6^{\mathrm{w}}} \qquad \mathcal{V}_6^{\mathrm{w}} = \int_{Y^6} \mathrm{d}^6 y \sqrt{\tilde{g}}\, \mathrm{e}^{-4\alpha}$$

$$\mathcal{V}_4^{\mathrm{w}} = \int_{\mathcal{S}^4} \mathrm{d}^4 y \sqrt{\tilde{g}}\, \mathrm{e}^{-4\alpha}$$

- Agrees with bosonic analysis:

$$\Phi\left(x^\mu, y^a\right) = \mathrm{const} \times \sigma\left(x^\mu\right)$$

$$\int_{\mathcal{W}} \mathrm{d}^8 x \sqrt{g} g^{\mu\nu} g_{ij} \partial_\mu \Phi^i \partial_\nu \Phi^j$$
$$\sim \int_{R^{1,3}} \mathrm{d}^4 x\, \eta^{\mu\nu} \partial_\mu \sigma \partial_\mu \sigma^* \int_{\mathcal{S}^4} \mathrm{d}^4 y \sqrt{\tilde{g}} \mathrm{e}^{-4\alpha}$$

Warped kinetic terms (cont.)

- For a more general Calabi-Yau,

$$\Phi(x^\mu, y^a) = \sigma^A(x^\mu) s_A(y) + \text{c.c.}$$

$$\mathcal{K}_{A\bar{B}} \sim \frac{1}{\mathcal{V}_6^{\mathrm{w}}} \int_{\mathcal{S}^4} \mathrm{e}^{-4\alpha}\, m_A \wedge m_{\bar{B}} \qquad m_A = \iota_{S_A} \Omega$$

- Then borrowing from warped closed string results [Shiu, Torroba, Underwood, Douglas] and unwarped closed-open results [Jockers, Louis]

$$\mathcal{K} = -\log\left[-\mathrm{i}(S - \bar{S}) - 2\mathrm{i}\mathcal{L}_{A\bar{B}} \sigma^A \bar{\sigma}^{\bar{B}}\right]$$

$$S = \tau - \mathcal{L}_{A\bar{B}} \sigma^A \bar{\sigma}^{\bar{B}}$$

Wilson lines

- Similarly, for Wilson lines

$$\mathcal{K}_{a\bar{b}} \sim \frac{1}{\mathcal{V}_6^{\mathrm{w}}} \int_{\mathcal{S}^4} \sqrt{\tilde{g}}\,\tilde{g}^{a\bar{b}}$$

- In the Calabi-Yau case

harmonic (1,0)-forms

$$A_{\text{int}}^{(1)} = w_I\big(x^\mu\big) W^I\big(y\big) + \text{c.c.}$$

$$\mathcal{K}_{I\bar{J}} \sim \frac{1}{\mathcal{V}_6^{\mathrm{w}}} \int_{\mathcal{S}^4} \mathrm{P}\big[\tilde{J}\big] \wedge W^I \wedge \bar{W}^{\bar{J}}$$

unwarped Kähler form

- Kähler potential can be found in special cases

Bifundamentals

- Warping modifications for chiral matter more complex - much less is known about Kähler potential even in the unwarped toroidal case

- For non-chiral bifundamental matter

$$S = -\frac{1}{g_8^2}\int_{\mathcal{W}} \mathrm{d}^8 x\,\sqrt{\tilde{g}}\,\mathrm{tr}\left\{\frac{1}{2}\eta^{\mu\nu}\tilde{g}^{ab}F_{\mu a}F_{\nu b} + \mathrm{e}^{-4\alpha}\eta^{\mu\nu}\tilde{g}_{ij}\partial_\mu\Phi^i\partial_\nu\Phi^j\right\}$$

$$\mathcal{K}^{\mp}_{i\bar{i}} \sim \frac{1}{\mathcal{V}_6^{\mathrm{w}}}\int_{\mathcal{S}^4} \mathrm{d}^4 y\,\sqrt{\tilde{g}}\left(\mathbf{X}^{\mp}\right)^* \cdot \mathrm{e}^{\#\alpha}\,\mathbf{X}^{\mp}$$

$$\mathrm{e}^{\#\alpha} = \mathrm{diag}\left(\mathrm{e}^{-4\alpha}, 1, 1, \mathrm{e}^{-4\alpha}\right)$$

warped zero mode

Bifundamentals (cont.)

- Recall the bifundamental wavefunctions are exponentially localized in the weakly warped case
- Approximating the Gaussians as δ-functions, for weak warping

$$\mathcal{K}_{i\bar{i}} \sim \frac{\mathcal{V}_1 + \mathcal{V}_1^{\mathrm{w}}}{\mathcal{V}_6^{\mathrm{w}} M_3}$$

Wilson line contribution

Modulus contribution

$$\mathcal{V}_2^{\mathrm{w}} = \int_{\Sigma^2} \mathrm{d}^2 y \, \sqrt{\tilde{g}} \mathrm{e}^{-4\alpha}$$

Chiral matter

- For chiral matter, warping induces off-diagonal entries in Kähler metric

$$\mathcal{K}^{\mp}_{j\bar{k}} \sim \frac{1}{\mathcal{V}_6^{\mathrm{w}}} \int_{\mathcal{S}^4} \mathrm{d}^4 y \, \sqrt{\tilde{g}} \big(\mathbf{X}^{k,\mp} \big)^{*} \cdot \mathrm{e}^{\#\alpha} X^{j,\mp}$$

- Even to leading order in weak warping, the family orthogonality is spoiled

$$\int_{T^4} \mathrm{d}^4 y \, \big(\varphi_0^{j,-}\big)^{*} \varphi_0^{k,-} \, \mathrm{e}^{-4\alpha} \neq \delta^{kj}$$

- Requires care for model building of this sort

Soft terms

- All of the above analysis was performed in the supersymmetric case

- Non-supersymmetric perturbations will induce soft susy-breaking terms that calculated using the above analysis (work in progress)

- Example (see also [Cámara, Ibáñez, Urangra;...])

$$\delta m_{1/2} \sim \int_{\mathcal{S}^4} *G^{(3)} \wedge \Omega + \cdots$$

Probe approximation

- All of the above was discussed in the probe approximation where the gravitational backreaction of D7 is ignored
- There are situations where this is ok (e.g. Sen limit, quenched approximation) but generically questionable
- Progress has been made for the adjoint case in the unwarped limit [Grimm]
- Such effects likely lead to problematic soft terms

Moduli stabilization

- We were lead to consider warping from the consideration of moduli stabilization (i.e. fluxes $\rightarrow$ warping) but effects were fluxes were largely neglected here

- Since fluxes generate a potential for D7 moduli as well, expect impact on wavefunctions and 4d physics

- Additionally, stabilization of Kähler moduli requires departure from conformally Calabi-Yau

Conclusions

- We analyzed the wavefunctions for open string fields coming from D7s and intersections of D7s in warped geometries
- Warping effect on adjoint matter simple, but not so for chiral matter
- Such wavefunctions are important for understanding the 4d effective field theory of such constructions
- Here, I talked about some of the effects on kinetic terms (more on kinetic terms and Yukawas detailed in `0812.2247`,`1012.2759`)
- More realistic models will also include:
  - Fluxes
  - 7-brane backreaction
  - Non-perturbative effects (for stabilizing Kähler moduli)

Thank you!