The Higgs Cubic
and
The Viability of Electroweak Baryogenesis

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After sifting through the astrophysical evidence . . .
The Baryogenesis Challenge

Even though matter and anti-matter are *nearly* symmetric in the SM, the universe appears to be dominated by matter.

Is there a dynamical mechanism in the evolution of the universe that could account for this asymmetry?
A Precise Target

\[ \eta \equiv \frac{n_b - n_{\bar{b}}}{n_{\gamma}} \]

\[ = (273.9 \times 10^{-10}) \Omega_B h^2 \]

\[ 5.9 < \eta \times 10^{10} < 6.4 \]

(Simha and Steigman 2008)
For $t \lesssim 10^{-6}$ s, \[
\frac{n_q - n_{\bar{q}}}{n_q} \sim \frac{3}{100,000,000}
\]
Many Creative Ideas

- Planck Scale Baryogenesis
- GUT Baryogenesis
- Electroweak Baryogenesis (EWBG)
- Leptogenesis
- Affleck-Dine Baryogenesis

Many nice reviews: Cohen, Kaplan, Nelson 1993
                   Trodden 1998, Riotto and Trodden 1999
                   Dine and Kusenko 2003
The Higgs Cubic Coupling

Our claim: The higgs cubic provides a model-independent collider probe of the viability of EWBG.

\[ \lambda_3 \equiv \frac{1}{6} \left. \frac{d^3 V_{\text{eff}}}{dh^3} \right|_{h=v} \]

\[ \left( \text{e.g.} \quad \lambda_{3,SM} = \frac{m_h^2}{2v} \right) \]

- ILC measurement:
  20\% precision for \( m_h < 140\text{GeV} \) and \( 1\text{ab}^{-1} \).

- Comparable precision at the SLHC/VLHC for \( m_h < 200\text{GeV} \).
Outline

• Overview of EWBG.

• The Higgs Effective Potential.

• The Higgs Cubic and EWBG.
Sakharov’s Criteria

A successful mechanism for Baryogenesis must include:

• Violation of B.
• Violation of C and CP.
• Nonequilibrium dynamics.
SM: Violation of B

(t’Hooft 1976)

Anomalous violation of B and L:

\[ \partial_\mu j_\mu^B = \partial_\mu j_\mu^L \sim N_f \frac{g^2}{32\pi^2} W \tilde{W} \]

Multiple vacua related by topologically non-trivial gauge transformations.

\[ V_{\text{eff}} \]

\[ \text{sphaleron} \]

\[ E_{\text{sp}} = \frac{2m_w}{\alpha_w} \]

\[ \sim 10\text{TeV} \]

\[ \Delta B \rightarrow \Delta L = N_f \]

\[ W^a, \Phi, \psi_i \]
Conserves $B - L$

Violates $B + L$:

$$\mathcal{O}_{B+L} \sim \prod_{i=1}^{N_f} u_i L_i u_i \bar{d}_i L_i e_i$$

$T = 0$:

$$\frac{\Gamma}{V} \sim e^{-\frac{2\pi}{\alpha_w}} \sim 10^{-80}, \quad \tau \gg t_{\text{universe}}$$

$T \neq 0$:

$$\frac{\Gamma}{V} \sim T^4 e^{-E_{sp}(T)/T} \quad \text{broken phase}$$

$$\sim T^4 (\alpha_w T)^4 \quad \text{symmetric phase}$$
SM: Violation of C and CP

• Maximal violation of C under SU(2)$_L$.

• Insufficient CP violation to achieve $\eta \sim 10^{-10}$.

\[
\begin{align*}
\delta & \lesssim 10^{-20} \quad \text{from CKM} \\
\theta & \lesssim 10^{-9} \quad \text{from QCD instantons}
\end{align*}
\]
One possibility: A First Order Phase Transition (FOPT) in the breaking of the electroweak symmetry,

\[ SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM} \]

Second Order Transition:

\[ V_{eff}(h, T) \]
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First Order Transition:

$V_{eff}(h, T)$

$h$
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First Order Transition:

\[ V_{eff}(h, T) \]

\[ h \]

\[ T_1 \]

\[ T_2 \]
SM: Nonequilibrium Dynamics

One possibility: A First Order Phase Transition (FOPT) in the breaking of the electroweak symmetry,

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Higgs Phase Diagram:

Transition is second order for $m_h > 114$GeV.

(Csikor, Fodor, Heitker 1999)
Once \( \frac{\Gamma_n}{H} \sim \frac{m_{pl}}{T_n} e^{-F/T} > 1 \),

bubbles of true vacuum nucleate and percolate to fill all space.
Non-local, Thin-Wall EWBG

\[ \frac{d(n_b - n_b^\text{\bar{}})}{dt} \sim \frac{\Gamma_{sp}}{T} \mu_B \]

\[ \frac{\Gamma_{sp}}{V} \sim T^4 e^{-E_{sp}(T)/T} \quad \text{broken phase} \]

\[ \frac{\Gamma_{sp}}{V} \sim (\alpha_w T)^4 \quad \text{symmetric phase} \]

\[ v \text{wall} \]
Non-local, Thin-Wall EWBG
(Cohen, Kaplan, Nelson 1992)

\[ \frac{d(n_b - n_{\bar{b}})}{dt} \sim \frac{\Gamma_{sp}}{T} \mu_B \]

\[ \delta_{wall} \sim \text{mean free path} \sim 4/T \]

\[ \Gamma_{sp} \rightarrow 0 \]

A baryon asymmetry is generated in front of the bubble wall then consumed. If \( E_{sp}(T) \gg T \), \( \Gamma_{sp} \rightarrow 0 \) inside the bubble, and washout can be avoided.
Non-local, Thin-Wall EWBG

(Cohen, Kaplan, Nelson 1992)

\[ V_{eff}(h, T) \]

\[ T_1 \]

\[ v(T_1) \]

\[ E_{sp}(T_1) \sim \frac{2m_w(T_1)}{\alpha_w} = \frac{4\pi}{g} v(T_1) \gg T \]

\[ \xi \equiv \frac{v(T_1)}{T_1} \gtrsim 1 \]
MSSM: A Narrow Window
(Carena, Quiros, Wagner 1998)

- Violation of B: Inherited from SM.

- Violation of C: Inherited from SM.
  Violation of CP: $O(1)$ from gaugino masses, $\mu$, etc.

- Nonequilibrium dynamics:
  For $m_h < 120\text{GeV}$ and $m_{\tilde{t}_R} < m_t$, the phase transition can be first order due to an enhancement in the cubic coupling of the effective potential.
Generic BSM Scenario

• Violation of B: Inherited from SM.

• Violation of C: Inherited from SM.
  Violation of CP: $\mathcal{O}(1)$ a possibility in many models.

• Nonequilibrium dynamics:
  The enlarged parameter space may allow for a first order phase transition.
EWBG Phenomenology

• A precision measurement of the full TeV Lagrangian (masses, couplings, mixings, etc.) would allow us to calculate the viability of various EWBG mechanisms.

• Lacking that, how much can we determine from the least data?

• New CP violating sectors are highly model dependent and difficult to probe.

• How about signatures of nonequilibrium dynamics?
  - Astrophysics: Gravitational relics may be accessible to LISA. (Grojean and Servant, 2006)
  - Collider Physics: Search for simple observables correlated to the order of the phase transition.
The Higgs Effective Potential
Zero Temperature

\[ Z[j] \equiv \int [\mathcal{D}\phi] \exp \left[ i(S[\phi] + j\phi) \right] \]

\[ S_{eff}[\phi_{cl}] \equiv -i \log Z[j] - j\phi_{cl}, \quad \text{where} \quad \phi_{cl} \equiv \langle \Omega | \phi(x) | \Omega \rangle_J \]

\[ S_{eff}[\phi_{cl}] \equiv \int d^4x \left[ -V_{eff}(\phi_{cl}) + A(\phi_{cl})(\partial_\mu \phi_{cl})^2 + \cdots \right] \]

\[ \left. \frac{\delta V_{eff}(\phi_{cl})}{\delta \phi_{cl}} \right|_{J=0} = 0 \]

From here on, \( h \equiv \phi_{cl} \).
Zero Temperature

\[ V_{eff}(h, T = 0) = V^t + V_0^l \]

\[ = -\frac{\mu^2}{2} h^2 + \frac{\lambda}{4} h^4 + \sum_i n_i \int \frac{d^4 k_E}{(2\pi)^4} \log \left( k_E^2 + m_i^2(h) \right) \]

\[ = -\frac{\mu^2}{2} h^2 + \frac{\lambda}{4} h^4 + \sum_i n_i \frac{m_i^4(h)}{64\pi^2} \left( \log \frac{m_i^2(h)}{\mu^2} + \text{const.} \right) \]

where \( i \in \{t, W, Z, h, G, BSM\} \)

\[ m_i^2(h) = m_{0i}^2 + ah^2 \] in a renormalizable theory
The Goldstones

Problem: $m_G^2(h) \leq 0$ for $h \leq v$.

Solution: Use on-shell renormalization conditions.

(Delaunay, Grojean, Wells, 2006)

\[
\frac{dV_{\text{eff}}(h, T = 0)}{dh} \bigg|_{h=v} = 0
\]

\[
\frac{d^2V_{\text{eff}}(h, T = 0)}{dh^2} \bigg|_{h=v} = m_h^2 - \Delta \Sigma
\]

\[
V_{\text{eff}}(h, T = 0) = -\frac{m_h^2}{4} h^2 + \frac{m_h^2}{8v^2} h^4
\]

\[
+ \sum_i \frac{n_i}{64\pi^2} \left( m_i^4(h) \left( \log \frac{m_i^2(h)}{m_i^2(v)} - \frac{3}{2} \right) + 2m_i^2(v)m_i^2(h) \right)
\]
Finite Temperature

Rotate to Euclidean time: \( x^0 = -i x^0_E \)

Compactify on a circle: \( 0 \leq x^0_E < 2\pi R \), where \( T \equiv 1/2\pi R \)

Require field configurations to be static.

\[
Z[j] = \int [\mathcal{D}\phi] \exp \left[ - \int d^4 x_E \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V_0(\phi) + j\phi \right) \right]
\]

\[
Z[j] = \int [\mathcal{D}\phi] \exp \left[ - \frac{1}{T} \int d^3 x \left( \frac{1}{2} \partial_i \phi \partial^i \phi + V_0(\phi) + j\phi \right) \right]
\]

\[
Z[j = 0] = \int [\mathcal{D}\phi] \, e^{-\frac{E[\phi]}{T}} \sim \sum_{S=\text{all states}} e^{-E_S/T}
\]
The Perscription

\[ \int \frac{dk_0}{2\pi} f(k_0) \to T \sum_{n=-\infty}^{\infty} f(k_0 = -i\omega_n) \]

Statistics on a circle of compactified time:

Bosons are periodic, so \( \omega_n = 2n\pi T \).

Fermions are anti-periodic, so \( \omega_n = (2n + 1)\pi T \).
The Potential

\[ V_{\text{eff}}(h, T) = -\frac{\mu^2}{2} h^2 + \frac{\lambda}{4} h^4 + \sum_i \frac{n_i T}{2} \sum_{n=-\infty}^{+\infty} \int \frac{d^3 k}{(2\pi)^3} \log \left( k^2 + \omega_n^2 + m_i^2(h) \right) \]

\[ = V_{\text{eff}}(h, T = 0) + \sum_i \frac{n_i T}{2\pi} \int dk k^2 \log \left( 1 + \exp \left( -\frac{1}{T} \sqrt{k^2 + m_i^2(h)} \right) \right) \]

Pheno note:
The zero temperature potential completely determines the finite temperature potential.
Thermal IR Divergences

\[ \sim \lambda T \sum_n \int \frac{d^3 k}{(2\pi)^3} \frac{1}{\omega_n^2 + k^2 + m^2} \]

For boson loops, with \( m << T \), the integral diverges for \( n=0, k=0 \).

Underlying problems:
1. We have a double expansion in both \( \lambda \) and \( \lambda \frac{T}{M} \),
2. We lose perturbative control in the high-\( T \) limit.
Resumming these “ring” or “daisy” diagrams, the leading two-loop contributions to the effective potential, cancels imaginary, and unphysical, contributions of the Goldstones to the finite temperature potential.

\[ \sum_{i} \frac{n_i T}{12\pi} \left( m_i^3(h) - (m_i^2(h) + \Pi_i(T^2))^{3/2} \right) \]
Low-T Expansion: \( m \gg T \)

\[
V_{\text{eff}}(h, T) = V_{\text{eff}}(h, T = 0) + \sum_i n_i T^4 \left( \frac{m_i(h)}{2\pi T} \right)^{3/2} e^{-m_i(h)/T}
\]

For a phase transition at \( T \sim 100 \text{GeV} \), only weak-scale states will effect the dynamics.
High-T Expansion: $m \ll T$

$$V_{eff}(h, T) = D(T^2 - T_2^2)h^2 - ETh^3 + \frac{\lambda(T)}{4}h^4$$

$$\xi \equiv \frac{v(T_1)}{T_1} = \frac{2E}{\lambda(T_1)}$$

If new scalar d.o.f. couple to the Higgs such that

$$m_i^2(h) = m_0^2 + ah^2$$

their contributions to $V_{eff}(h, T \neq 0)$ enhance $E$, and hence

$$\xi$$

while their loop contributions to $V_{eff}(h, T = 0)$ enhance

$$\lambda_3$$
The Higgs Cubic And EWBG
A Proposal for EWBG Pheno

- Phenomenologically interesting BSM physics scenarios replace the ad hoc SM Higgs potential with a realistic mechanism for EWSB.
- This new Higgs physics modifies the shape of $V_{eff}(h, T)$ at the EW phase transition and may allow for a strong first order phase transition, i.e. one where $\xi \gtrsim 1$.
- The same new physics modifies $V_{eff}(h, T = 0)$, leading to deviations in $\lambda_3$ from its SM value.

Our Claim: Models possessing a strong, first order Electroweak Phase Transition (EWPT) exhibit large (typically 20-100%) deviations of the Higgs cubic coupling from its SM value.
Our Evidence

We demonstrate the correlation between $\xi$ and $\lambda_3$ by analyzing a series of toy models that can be matched onto a broad range of realistic BSM Higgs scenarios with weakly coupled physics at the TeV scale.

- **Toy Model I:**
  Loop Modified, Unmixed Higgs.

- **Toy Model II:**
  Tree-Level Modified, Unmixed Higgs.

- **Toy Model III:**
  Tree-Level Modified, Mixed Higgs.
Add a single BSM real scalar field (inspired by Little Higgs models).

\[ \Delta V_{SM} = \frac{1}{2} M_{0,S}^2 S^2 + a |H|^2 S^2 \]

\[ \Delta V_{eff}(h, T = 0) = \frac{1}{64\pi^2} m_S^4(h) \log \frac{m_S^2(h)}{m_S^2(v)} + \cdots \]

- \( M_{0,S}^2 > 0 \) ensures \( \langle S \rangle = 0 \).

- Most general interaction after imposing a symmetry \( S \rightarrow -S \) to prevent mixing.
‘Bumpy’ Higgs Potentials

BSM couplings may induce a ‘bump’ in the zero temperature potential. This bump generally persists at finite temperature, allowing for a strong EWPT.

\[ V_{\text{eff}}(h, T = 0) \]

\[ V_{\text{eff}}(h, T = 0) \]
I: Loop Modified, Unmixed $h$

Add a single BSM real scalar field.

Expt. Prospects:
20% for a $<140\text{GeV}$ Higgs at a $500\text{GeV}$ ILC  \(^{(\text{Djouadi, et. al., 2007})}\)
20-30% for $160-180\text{GeV}$ Higgs at SLHC  \(^{(\text{Baur, et. al., 2002})}\)
8-25% for $150-200\text{GeV}$ Higgs at $200\text{TeV}$ VLHC  \(^{(\text{Baur, et. al., 2002})}\)
I: Loop Modified, Unmixed h

Multiple BSM scalars.

• The same conclusions apply to models with N real (or N/2 complex) identical scalars by a simple scaling argument.

• We checked that the pattern continues to hold for 2 non-identical scalars. A conjecture that it holds for N independent scalars seems reasonable.

• The one-loop analysis is independent of the scalars’ gauge charges. They could be stops in the MSSM decoupling limit (one unmixed Higgs), weak triplets, etc.
Add a BSM boson-fermion pair (as in SUSY).
We choose a Dirac fermion and four identical real scalars.

\[ \Delta V_{SM} = \sum_i \left( \frac{1}{2} M_{0,S}^2 S_i^2 + a |H|^2 S_i^2 \right) + (M_0,\Psi + \frac{a}{M_{0,\bar{\Psi}}} |H|^2) \Psi^\dagger \Psi \]

\[ \Downarrow \]

\[ \Delta V_{eff}(h, T = 0) = \sum_i \frac{n_i}{64\pi^2} m_i^4(h) \log \frac{m_i^2(h)}{m_i^2(v)} + \cdots \]
I: Loop Modified, Unmixed $h$

Add a BSM boson-fermion pair.

Accidental cancellations violate our claim!

For $M_{0,S} = M_{0,\Psi}$, the contributions of this supermultiplet to the zero temperature potential vanish, but not so in the finite temperature potential.
Consider the SM Higgs sector as an EFT and add the leading correction.

\[ \Delta V_{SM} = \frac{1}{\Lambda^2} |H|^6 \]

(Grojean, Servant, Wells, 2007)
II: Tree-Level, Unmixed $h$

Consider the SM Higgs sector as an EFT and add the leading correction.

\[ \xi \text{ vs } \lambda_3 \]

\[ \lambda_3 \text{ vs } m_h \text{ for } \xi > 1 \]

\[
\frac{\lambda_3^{\text{GSW}}}{\lambda_3^{\text{SM}}} = 1 + \frac{2v^4}{m_h^2 \Lambda^2}
\]

Blue = Bumpy at $T=0$
III: Tree-Level, Mixed h

Consider the most general, renormalizable potential with one additional scalar (as in the NMSSM or nMSSM).

\[ \Delta V_{SM} = \frac{a_1}{2} |H|^2 S + \frac{a_2}{2} |H|^2 S^2 + \frac{b_2}{2} S^2 + \frac{b_3}{3} S^3 + \frac{b_4}{4} S^4 \]

Mass eigenstates:

\[ h_1 = \sin \theta \ s + \cos \theta \ h \]
\[ h_2 = \cos \theta \ s - \sin \theta \ h \]

- Generically, H and S both acquire vevs, so the order parameter for the phase transition is a linear combination of two classical fields.
- Non-SM Yukawas.
- \( h_1 \) is the most doublet-like, so we consider its \( \lambda_3 \).
III: Tree-Level, Mixed h

- A partial scan of the 6-dimensional parameter space roughly consistent with EW precision constraints.
- Both suppression and enhancement of $\lambda_3$ is possible.
- Small $\lambda_3$ corrections only occur due to accidental cancellations of two large contributions.
III: Tree-Level, Mixed $h$

- All parameters are fixed except for the mixing coefficient $a_1$.
- If the Higgs is mixed, deviations from the SM Higgs production x-section and branching ratios would be observed well before $\lambda_3$ is measured. Nevertheless, the correlation between $\xi$ and $\lambda_3$ persists.
Conclusions

• Barring the possibility of accidental cancellations, there must be a large deviation in $\lambda_3$ from its SM value to achieve a strong first order EWPT and make EWBG viable.

• Large deviations in $\lambda_3$ are generic to BSM models exhibiting a strong EWPT.

• Typical deviations are large enough to be probed at the ILC and SLHC/VLHC.

• Future work: For specific models, could the order of the EWPT be determined from a small number of quantities measured to an accessible level of precision?