The Non-Relativistic Limit of AdS/CFT

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References

- **"Galilean Conformal Algebras and AdS/CFT"**
  A. Bagchi and R. Gopakumar
  JHEP 0907:037,2009 [arxiv: 0902.1385 (hep-th)].

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- **"Supersymmetric Extension of the GCA"**
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- **"GCA in 2D"**
  A. Bagchi, R. Gopakumar, I. Mandal, A. Miwa.
  To appear.

Why a non-relativistic limit?

Introduction and motivation

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- Applications to real life systems in condensed matter
- Possible new tractable limit of parent conjecture

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The usual route:

- NR symmetry group = Schrödinger symmetry group $\longrightarrow$ the symmetries of free Schrödinger equations
- Relevant to study of cold atoms.
- Gravity dual proposed with these symmetries in two higher dimensions. (Son 2008; Balasubramanian, McGreevy 2008.)

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We are interested in a *different* limit. $\rightarrow$ **Why?**

- Systematic construction of NR algebra from the parent Conformal symmetry: Inönu-Wigner contractions
- No reason to believe that all NR systems would have the symmetries of free Schrödinger equations.
- Wish to find a holographic description in the standard one higher dimension.

Plan of the talk

Galilean Conformal Algebra and Schrödinger Algebra

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Infinite extension of GCA

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Outline

Inonu-Wigner Contractions: A Simple Example

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- Infinitesimal generators: $X_{ij} = x_i \partial_j - x_j \partial_i$
- Algebra: $[X_{ij}, X_{rs}] = X_{is}\delta_{jr} + X_{jr}\delta_{is} - X_{ir}\delta_{js} - X_{js}\delta_{ir}$

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$$P_i = \lim_{R\to\infty} \frac{1}{R} X_{i,3} = \lim_{R\to\infty} \frac{1}{R}(x_i\partial_3 - x_3\partial_i) \to -\partial_i \tag{2}$$

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Redefined algebra: $$[Y_{12}, P_i] = P_1\delta_{2i} - P_2\delta_{1i}, \quad [P_1, P_2] = 0 \quad (3)$$

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This is the $ISO(2)$ group. Expected! $\Rightarrow$ At North Pole, with $R \to \infty$, S^2 looks like R_2.
We will use this technique to investigate the non-relativistic limit of the conformal algebra.

Relativistic Conformal Algebra

- **Poincare generators** $(\mu, \nu = 0, 1 \ldots d)$

$$\begin{aligned} J_{\mu\nu} &= -(x_\mu \partial_\nu - x_\nu \partial_\mu), \quad B_i = J_{0i} \\ P_\mu &= \partial_\mu, \quad P_0 = H \end{aligned} \tag{4}$$

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- **Algebra**:

$$\begin{aligned} [J_{ij}, J_{rs}] &= so(d) \\ [J_{ij}, B_r] &= -(B_i \delta_{jr} - B_j \delta_{ir}) \\ [J_{ij}, P_r] &= -(P_i \delta_{jr} - P_j \delta_{ir}), \quad [J_{ij}, H] = 0 \\ [P_i, P_j] &= 0, \quad [H, P_i] = 0, \quad [H, B_i] = -P_i \end{aligned} \tag{5}$$

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$$[B_i, B_j] = -J_{ij}, \quad [B_i, P_j] = \delta_{ij} H \tag{6}$$

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- **Other generators**: Dilatations(D) and SCT(K_μ):

$$D = -(x \cdot \partial) \qquad K_\mu = -(2x_\mu(x \cdot \partial) - (x \cdot x)\partial_\mu) \tag{7}$$

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- **Remaining algebra**:

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[K, K_i] &= 0, \quad [K, B_i] = K_i, \quad [K, P_i] = 2B_i \\
[J_{ij}, K_r] &= -(K_i\delta_{jr} - K_j\delta_{ir}), \quad [J_{ij}, K] = 0 \\
[J_{ij}, D] &= 0, \quad [K_i, K_j] = 0, \quad [H, K_i] = -2B_i, \\
[D, K_i] &= -K_i, \quad [D, B_i] = 0 \quad [D, P_i] = P_i \\
[D, H] &= H, \quad [H, K] = -2D, \quad [D, K] = -K.
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$$[K_i, B_j] = \delta_{ij} K, \quad [K_i, P_j] = 2J_{ij} + 2\delta_{ij} D \tag{9}$$

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- **Non-relativistic limit:** Scale (with $\epsilon \to 0$):

$$t \to \epsilon^r t \qquad x_i \to \epsilon^{r+1} x_i \tag{10}$$

Equivalent to $v_i \sim \epsilon$ $(c = 1)$. Set $r = 0$ for simplicity.

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- **Contracted Generators:**
  Galilean generators:

$$\begin{aligned} J_{ij} &= -(x_i \partial_j - x_j \partial_i), \quad H = -\partial_t \\ P_i &= \partial_i, \quad B_i = t\partial_i. \end{aligned} \tag{11}$$

Galilean conformal generators:

$$\begin{aligned} D &= -(x_i \partial_i + t\partial_t), \quad K_i = t^2 \partial_i, \\ K &= K_0 = -(2t x_i \partial_i + t^2 \partial_t). \end{aligned} \tag{12}$$

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Changes in Algebra: Galilean Conformal Algebra

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Changes in Algebra: Galilean Conformal Algebra

- RHS of Red Equations: (6), (9) $= 0$.
- Rest of algebra $\Rightarrow$ same.

Schrödinger Algebra

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- Group of symmetries of the free Schrödinger equation.

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- For massive systems, consider rest energy > kinetic energy. Replace $\partial_0 \to -im_0 + \partial_t; m_0 \to \frac{m}{\epsilon^2}; x_i \to \epsilon x_i$.
- Klein Gordon equation reduces to Schrodinger equation

$$(\partial_0^2 - \partial_i^2 + m_0^2)\phi = 0 \to (i\partial_t + \frac{1}{2m}\partial_i^2)\phi = 0. \tag{13}$$

The parameter $\epsilon \sim \frac{v}{c}$ signifies taking the nonrelativistic limit

Schrödinger Algebra ...

Algebra:

- Galilean sub-group: $\{J_{ij}, B_i, P_i, H\}$

$$[B_i, P_j] = m\delta_{ij} \tag{14}$$

$m \longrightarrow$ central extension, NR mass. Rest: same as in GCA.

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- Other generators: $\{\tilde{D}, \tilde{K}\}$
  $\tilde{D} = -(2t\partial_t + x_i\partial_i) \longrightarrow$ scales space and time differently.
  Action: $(x_i, t) \longrightarrow (\lambda x_i, \lambda^2 t)$.
  $\tilde{K} = -(tx_i\partial_i + t^2\partial_t) \longrightarrow$ like temporal SCT.
  Action: $(x_i, t) \longrightarrow (\frac{x_i}{(1+\mu t)}, \frac{t}{(1+\mu t)})$.

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- Non-zero commutators:

$$\begin{aligned} [\tilde{K}, P_i] &= B_i, \quad [\tilde{K}, B_i] = 0, \quad [\tilde{D}, B_i] = -B_i \\ [\tilde{D}, \tilde{K}] &= -2\tilde{K}, \quad [\tilde{K}, H] = -\tilde{D}, \quad [\tilde{D}, H] = 2H. \end{aligned} \tag{15}$$

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- SA: Dilatation operator scales space and time differently.
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- SA: Allows mass, central extn between momentum and boosts.
  GCA: No mass (Jacobi id don't allow it).
  Symmetry group of massless/gapless NR system.

Outline

Redefining Finite algebra

- Redefine generators of finite algebra:

$$\begin{aligned} L^{(-1)} &= H, \quad L^{(0)} = D, \quad L^{(+1)} = K, \\ M_i^{(-1)} &= P_i, \quad M_i^{(0)} = B_i, \quad M_i^{(+1)} = K_i. \end{aligned} \tag{16}$$

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- The finite dimensional GCA (with $m, n = 0, \pm 1$):

$$\begin{aligned} [L^{(m)}, L^{(n)}] &= (m-n)L^{(m+n)}, \\ [L^{(m)}, M_i^{(n)}] &= (m-n)M_i^{(m+n)}, \quad [M_i^{(m)}, M_j^{(n)}] = 0, \\ [J_{ij}, L^{(n)}] &= 0, \quad [J_{ij}, M_k^{(m)}] = -(M_i^{(m)}\delta_{jk} - M_j^{(m)}\delta_{ik}). \end{aligned}$$

Infinite GCA

- Define for *arbitrary* integer n, vector fields:

$$\begin{aligned} L^{(n)} &= -(n+1)t^n x_i \partial_i - t^{n+1}\partial_t, \quad M_i^{(n)} = t^{n+1}\partial_i \\ J_a^{(n)} &\equiv J_{ij}^{(n)} = -t^n(x_i\partial_j - x_j\partial_i) \end{aligned} \tag{17}$$

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- Commuting generators $M_i^{(n)}$ function like generators of a global symmetry.
- Can consistently set these generators to zero and add usual Virasoro-Kac-Moody central terms.

Physical Significance of the GCA

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- These two together generate what is sometimes called the Coriolis group: the biggest group of "isometries" of "flat" Galilean spacetime.
- $L^{(n)}$ seem to be generators of a conformal "isometry" of Galilean spacetime.

$$t \to f(t), \qquad x_i \to \frac{df}{dt} x_i \tag{20}$$

Realisation of the GCA: NR Conformal Hydrodynamics

Similar algebra also looked by Bhattacharya *et al* (2008).

- NR limit of quantum field theories at finite temperature in the hydrodynamic limit $\rightarrow$ recover Navier-Stokes eqns.

$$\partial_t v_i(x,t) + v_j \partial_j v_i(x,t) = -\partial_i p(x,t) + \nu_0 \partial_j \partial_j v_i(x,t) \tag{21}$$

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- Has all symmetries of finite GCA, except D which is broken (by the viscous term) because of the choice of temperature.
- Has all $M_i^{(n)}$ as symmetries!
- Navier-Stokes eqn should describe the hydrodynamic limit of all NR field theories.
  A part of the infinitely extended GCA exists as its symmetries!

Realisation of the GCA: NR Conformal Hydrodynamics ...

- Navier-Stokes equation with viscosity set to zero = Incompressible Euler equations

$$\partial_t v_i(x,t) + v_j \partial_j v_i(x,t) = -\partial_i p(x,t) \tag{22}$$

Entire finite GCA is a symmetry (since D is now also a symmetry).

This shows that one can readily realise "gapless" non-relativistic systems in which space and time scale in the same way!

Outline

Representations

- States would be labeled under L_0 (dilatations) and M_0 (boosts).

$$L_0|\Delta,\xi^i\rangle = \Delta|\Delta,\xi^i\rangle, \quad M_0^i|\Delta,\xi^i\rangle = \xi^i|\Delta,\xi^i\rangle \tag{23}$$

- L_n, M_n^i lower the dilatation eigenvalue (Δ) and L_{-n}, M_{-n}^i raise it.
- Primary states: there must exist states with a lower bound on the Δ eigenvalue, so that for them Δ cannot be lowered further.

$$L_n|\Delta,\xi^i\rangle_p = 0, \quad M_n|\Delta,\xi^i\rangle_p = 0 \quad \forall n > 0 \tag{24}$$

- Can construct representations by acting on the primary state by raising operators.

Two and Three point functions of GCA

Look at correlation functions of quasi-primary operators.
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Two pt function: $G^{(2)} = C_{(12)}\delta_{\Delta_1,\Delta_2}\delta_{\xi_1^i,\xi_2^i}\, t_{12}^{2\Delta_1} \exp\left(\frac{2\xi_i x_{12}^i}{t_{12}}\right)$

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Three pt function:

$$G^{(3)} = C^{(3)} t_{13}^{\Delta^{132}} t_{23}^{\Delta^{231}} t_{12}^{\Delta^{123}} \exp\left(-\frac{\xi_i^{132} x_{13}^i}{t_{13}} - \frac{\xi_i^{231} x_{23}^i}{t_{23}} - \frac{\xi_i^{123} x_{12}^i}{t_{12}}\right)$$

where $\Delta^{lmn} = -(\Delta^l + \Delta^m - \Delta^n)$ and similarly ξ_i^{lmn}.

2 and 3-pt functions of Conformal and Schrödinger Algebras

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- Schrödinger algebra:

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\begin{aligned}
G^{(2)} &= C_{(12)} \delta_{h_1,h_2} \delta_{m_1,m_2} t_{12}^{h_1} \exp\{ m_1 \frac{{x_{12}^i}^2}{2t_{12}} \} \\
G^{(3)} &= C_{(123)} \delta_{m_1+m_2,m_3} t_{13}^{-h_{132}} t_{23}^{-h_{231}} t_{12}^{-h_{123}} \\
&\times \quad \exp\left(\frac{m_1 {x_{13}^i}^2}{2t_{13}} + \frac{m_2 {x_{23}^i}^2}{2t_{23}} \right) \Psi \left(\frac{[x_{13}^i t_{23} - x_{23}^i t_{13}]^2}{(t_{12} t_{23} t_{13})} \right)
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  - SA further constrained by mass selection rules. Not present in GCA: boosts act on co-ordinates unlike mass.

Outline

Mapping of the representations ...

- Representation of classical Virasoro algebra in 2d :

$$\mathcal{L}_n = -z^{n+1}\partial_z, \quad \bar{\mathcal{L}}_n = -\bar{z}^{n+1}\partial_{\bar{z}} \tag{25}$$

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- In space-time coord, $z = t + x$, $\bar{z} = t - x$.
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$$\mathcal{L}_n + \bar{\mathcal{L}}_n = -t^{n+1}\partial_t - (n+1)t^n x\partial_x + O(\epsilon^2) \tag{26}$$

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- As $\epsilon \to 0$, identification:

$$\mathcal{L}_n + \bar{\mathcal{L}}_n \longrightarrow L^{(n)}, \quad \epsilon(\mathcal{L}_n - \bar{\mathcal{L}}_n) \longrightarrow -M^{(n)} \tag{28}$$

Mapping of Quantum algebra

- 2d relativistic Virasoro Algebra:

$$[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\mathcal{L}_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n,0}$$
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- The linear combinations before taking the limits:

$$[\mathcal{L}_m + \bar{\mathcal{L}}_m, \mathcal{L}_n + \bar{\mathcal{L}}_n] = (m-n)(\mathcal{L}_{m+n} + \bar{\mathcal{L}}_{m+n}) + \frac{c+\bar{c}}{12}m(m^2-1)\delta_{m+n,0}$$
$$[\mathcal{L}_m + \bar{\mathcal{L}}_m, \mathcal{L}_n - \bar{\mathcal{L}}_n] = (m-n)(\mathcal{L}_{m+n} - \bar{\mathcal{L}}_{m+n}) + \frac{c-\bar{c}}{12}m(m^2-1)\delta_{m+n,0}$$
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Mapping of Quantum algebra ...

- After taking the limit

$$
\begin{aligned}
[L^{(m)}, L^{(n)}] &= (m-n)L^{(m+n)} + C_1 m(m^2-1)\delta_{m+n,0} \\
[L^{(m)}, M^{(n)}] &= (m-n)M^{(m+n)} + C_2 m(m^2-1)\delta_{m+n,0} \\
[M^{(m)}, M^{(n)}] &= 0.
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- Infinite extended GCA!

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- Infinite GCA which was first written by observation has now been derived as a simple limit of the algebra of 2d CFTs.

GCA: The large spin sector of 2D Virasoro

- More subtlety in the above limit. Can be understood by looking at correlation functions.
- Would not get the important exponential pieces of correlation fns if one took the simple NR limit.
- Also need to take $h + \bar{h} = \Delta$ and $h - \bar{h} = \frac{\xi}{\epsilon}$.
- Two point correlator of $2d$ relativistic CFT in this limit:

$$
\begin{aligned}
G_R^{(2)}(z_i, \bar{z}_i) &= \delta_{h_1,h_2}\delta_{\bar{h}_1,\bar{h}_2} C_{12}(t_{12} - x_{12})^{-2h}(t_{12} + x_{12})^{-2\bar{h}} \\
&= \delta_{h_1,h_2}\delta_{\bar{h}_1,\bar{h}_2} C_{12}(t_{12} - x_{12})^{-2(h-\bar{h})}(t_{12}^2 - x_{12}^2)^{-2\bar{h}} \\
&= \delta_{h_1,h_2}\delta_{\bar{h}_1,\bar{h}_2} C_{12}\ t_{12}^{\Delta}\ (1 - \frac{x_{12}}{t_{12}})^{2s}\ (1 - \frac{x_{12}^2}{t_{12}^2})^{-2\bar{h}}
\end{aligned}
$$

GCA: The large spin sector of 2D Virasoro ...

- Now, we want to take the limit of large spin along with the non-relativistic limit on the co-ordinates. Keeping in mind the identity $\lim_{N\to\infty}(1+\frac{x}{N})^N = e^x$, we find

$$G_R^{(2)} \to C_{12} \;\; \delta_{\Delta_1,\Delta_2} \;\; \delta_{\xi_1,\xi_2} \;\; t_{12}^{\Delta} \;\; \exp(\frac{2\xi x_{12}}{t_{12}}) = G_{GCA}^{(2)} \tag{31}$$

Outline

Bulk Dual of the GCA

- AdS_{d+2} in Poincare coordinates:

$$ds^2 = R^2 \frac{dt^2 - dz^2 - dx_i^2}{z^2} \tag{32}$$

- Bulk dual to a system with GCA should arise from some scaling limit of above metric.

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- The $SL(2,R)$ suggests an AdS_2 part: in the t and z.
- Boundary metric degenerates in the non-relativistic limit with the d spatial directions scaling as $x_i \propto \epsilon$ while $t \propto \epsilon^0$.
- Expect this feature to be shared by the bulk metric. Geometry on constant radial sections expected to have such a scaling.

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- Need to fix scaling of z.
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- $\Rightarrow$ In the bulk the time and radial directions of the metric *both* survive the scaling.
  $\Rightarrow$ AdS_2 sitting inside the original AdS_{d+2}.
- So expect the dual spacetime to have the structure $AdS_2 \times R^d$ with degenerate metric on the R^d.

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- Degenerate nature of the metric might seem to imply that the gravitational dynamics is singular.
- However, similar situation in asymptotically flat space in recovering Newtonian gravity from Einstein gravity in the non-relativistic limit.
- The answer: there is a well-defined geometric theory of Newtonian gravitation - Newton-Cartan theory.

Newton-Cartan Theory ...

- The ingredients: A space time endowed with absolute time function t, a non-dynamical *spatial* (Euclidean) metric, a dynamical *non-metric* connection $\Gamma^i_{00} \propto \partial_i \Phi$.
- Einstein's equations reduce to $R_{00} \propto \rho$ which is Poisson's equation for Φ.

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- Einstein's equations reduce to $R_{00} \propto \rho$ which is Poisson's equation for Φ.
- One can write all this in a more covariant form: A d dimensional fibre R^d over a base R which is parametrised by the time t.
- Nothing singular about this classical geometric description, only unusual.

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- Thus the bulk boundary relation is some kind of an AdS_2/CFT_1 duality.
- The correlators on the boundary theory also point to the same thing. "Conformal" pieces in t. But non-trivial pieces from the fibre-bundle structure.

GCA from Bulk Killing Vectors

- AdS_{d+2} in radially infalling coordinates for null geodesics $(t' = t + z, z' = z)$ (tranforming from Poincare co-ordinates)

$$ds^2 = \frac{R^2}{z'^2}(-dt'(2dz' - dt') - dx_i^2). \tag{34}$$

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- Contracted Killing vectors given by

$$\begin{aligned} P_i &= -\partial_i, \quad B_i = -(t' - z')\partial_i, \quad K_i = -(t'^2 - 2t'z')\partial_i \\ H &= \partial_{t'}, \quad D = t'\partial_{t'} + z'\partial_{z'} + x_i\partial_i, \\ K &= t'^2\partial_{t'} + 2(t' - z')(z'\partial_{z'} + x_i\partial_i). \end{aligned} \tag{35}$$

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- More compactly (for $m, n = 0, \pm 1, l = 0$).

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- Reduces at the boundary ($z = 0$) to the generators of the contracted conformal algebra. And satisfies the *same* algebra.
- In fact, these bulk vector fields (for arbitrary m, n, l) reduce to that of the extended Kac-Moody algebra at the boundary.

Interpretation of Bulk Vector Fields

- The $M_i^{(n)}$ and $J_a^{(n)}$ act only on the R^d. Rotation and translation of the spatial slices which depend on t, z.
  Isometries of the spatial metric. Act trivially on the AdS_2.

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  Isometries of the spatial metric. Act trivially on the AdS_2.
- The Virasoro generators act as the generators of asymptotic symmetries of the AdS_2.
- Under its action (with infinitesimal parameter a_n)

$$\begin{aligned} z \to \tilde{z} &= z[1 + a_n(n+1)(t^n - nzt^{n-1})] \\ t \to \tilde{t} &= t[1 + a_n t^n] \\ x_i \to \tilde{x}_i &= x_i[1 + a_n(n+1)(t^n - nzt^{n-1})]. \end{aligned} \tag{37}$$

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- On taking the scaling limit:

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  Near the boundary $z = 0$ the diffeomorphisms generated by these vector fields leave the metric unchanged upto a factor which has a falloff like z^2.
- Action of the $L^{(n)}$ on the spatial metric on the slices of constant t, z is again an isometry.
- $L^{(n)}, J_a^{(n)}, M_i^{(N)}$ together generate (asymptotic) isometries of the spatial and AdS_2 metrics γ^{ij} and g_{ab}.
- Therefore it seems natural to consider the action of these generators on the Newton-Cartan like geometry.

Summary

- GCA is obtained by a parametric group contraction of the relativistic conformal algebra.
- It can be given an infinite lift for any spacetime dimensions.
- Finite algebra realised as invariant symmetry algebra of the Euler equations.
- Novel structures for the 2 and 3 pt functions
- For d=2, a mapping exists from the Virasoro to the GCA.
- Bulk dual is in standard one higher dimension and is a Newton-Cartan like $AdS_2 \times R^d$.
- GCA is obtained by contracting bulk killing vectors and can be interpretted as the asympototic isometry of the NC structure.

Future Directions

- Interpreting and using the infinite symmetry in conformal theories with $d > 2$.
- Infinite symmetry = integrability ?
- Understanding the bulk better: bulk-bdy dictionary, N-C structure.
- Embeddings in string theory.
- Other cond mat systems with GCA. (e.g. aging systems, quantum hall systems.)
- A host of other questions!

Thank You!!