

Can Kinoform hard X-ray optics produce sub-10nm beams?

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- Brief review of Refractive optics
- Motivation for kinoforms
- Are there fundamental limits of these optics



Main points of this talk

- 1. We do not need to fabricate lenses with feature sizes comparable to the optics resolution. (Why?)
- 2. Numerical Aperture not limited by absorption
- 3. We can exceed the critical angle limit with compound lenses.

 $\Rightarrow We can get down to below 10nm$ (no new physics or technology, just improve what we are doing now)



Status: Local NSLS results

A 4 micron by 0.6 micron spot from a crossed lens

Horizontal lens



Horizontal knife edge; 4 microns

Status 1a: Local NSLS results

Submicron performance with 100micron Aperture



- •Knife edge consists of Cu metal grating with 2 micron period.
- •Figure on left shows a knife edge scan with and without a lens in the path.
- •Efficiency is greater than 60%



700 (ket) 600 400 300 200 -0.945 -0.945 -0.94 -0.935 -0.93Knife Edge Position (millimeters)

Detailed knife edge scan showing submicron performance. Distance between experimental points is 0.5 micron What is a kinoform

- A kinoform is a phase optic
- Assymetric computer generated profile
- Efficiency and resolution are metrics to consider
- Phase profile accuracy is important;
 => Elliptical shape for point to parallel refractive optic.

One can view the kinoform equivalently as

- a) A blazed zone plate
- b) An array of coherently interfering micro-lenses.

We report, you decide.....

For far field optics, resolution is λ /(Numerical Aperture) Limiting value of N.A. is 1

"State of the Art*" of the different microscopies						
Method	Wavelength	Resolution	Ratio			
Optical	200nm	200nm	1			
Electrons	0.05nm	0.1nm	2			
Soft X-rays	10nm	30nm	3			
Hard X-rays	0.1nm	50nm	500 (We need better optics)			

*Very crude

Why refractive optics were not initially considered

Refractive index $n = 1 - \delta - i\beta$, where $\delta is \sim 10^{-6}$

- •Roentgen: No refractive lenses for X-rays
- •Also real part of refractive index is less than one so lens is concave
- •Beta/delta gets more favorable as you increase energy up from soft
- •Inelastic compton scattering will be one of the limits (Lengeler)

Initial attempts

R. Gähler, J. Kalus, and W. Mampe, "An optical instrument for the search of a neutron charge," Journal of Physics E **13**, 546-548 (1980).

S. Suehiro, H. Miyaji, and H. Hayashi, "Refractive lens for X-ray focus," Nature **352**, 385-386 (1991).

Response by Michette:

A. Snigirev, V. Kohn, I. Snigireva *et al.*, "A compound refractive lens for focusing high-energy X-rays," Nature **384**, 49-51 (1996).



•A lens takes the diffracted beams from the sample and recombine them in the image plane, while maintaining the relative phases.

- •Lens resolution is ~ λ /(numerical aperture) ; limiting value is λ .
- •Either shorten the focal length or open up the aperture (preferably both)



 $R \sim (0.1m * 1e-6) = 0.1microns;$ aperture too small! N lenses reduce focal length: $f=f_0/N$ So reduce the curvature by N (open the aperture) and stack N lenses up

Can't make circles as small as you would want with drilling

A. Snigirev, V. Kohn, I. Snigireva *et al.*, "A compound refractive lens for focusing highenergy X-rays," Nature **384**, 49-51 (1996).

A commercial product: Refractive parabolic Beryllium lenses

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Another commercial product; plastic (Be) compound lenses

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Pure refractive results

Nanofocusing parabolic refractive x-ray lenses

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FIG. 1. Scanning electron micrograph of an array of parabolic refractive x-ray lenses made of silicon. The shaded areas (a) and (b) delineate an individual and a compound NFL, respectively. The optical axis of the NFL is shown as a white dashed line.



A 10 micron diameter pinhole is focussed down to 210nm

My entry point: Deep RIE etching of Bragg-Fresnel optics



- Since we have complete control of the profile with the electron-beam writer, why not
- 1. Minimize the "dead" regions?
- 2. write the curvature as small as we want, instead of using a compound lens?
- 3. What is the best shape anyway?

From Fermats theorem for n<1 the best shape for a point to parallel converter is an ellipse.



Clearly, the ellipse and parabola are similar near the optical axis



Hecht

Physical intuition tells you a parabola is not correct



A consequence of the ideal shape

The resolution is independent of focal length; is this a fundamental limit?



This observation is a central issue in a "controversy".

*K. Evans-Lutterodt et al., "Single-element elliptical hard x-ray micro-optics", Optics Express 11 (8) 919-926, 21 April 2003.

.....One implication of the elliptical shape is that for a given focal length and refractive index, the diffraction-limited resolution given by the Rayleigh criterion $f \lambda / (aperture) = f \lambda / 2b \sim \lambda / \sqrt{2\delta}$

is dependent only on the choice of material and the wavelength, even for lossless material and in the refractive limit. For $\delta = 10^{-6}$, one gets a resolution of $\sim 10^{3}\lambda$. This is not a fundamental limit; by using more than one element i.e. a compound lens, one can exceed this limit.

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PHYSICAL REVIEW LETTERS

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Focusing X-Ray Beams to Nanometer Dimensions

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We address the question: what is the smallest spot size to which an x-ray beam can be focused? We show that confinement of the beam within a narrowly tapered waveguide leads to a theoretical minimum beam size of the order of 10 nm (FWHM), the exact value depending only on the electron density of the confining material. This limit appears to apply to all x-ray focusing devices. Mode mixing and interference can help to achieve this spot size without the need for ultrasmall apertures.

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A way out: make compound lens

$$\frac{1}{f} = \left(\frac{1}{f_1} + \frac{1}{f_2} + \dots\right)$$

N lenses reduce focal length: $f=f_0/N$

So reduce the curvature by N (open the aperture) and stack N lenses up Ideally use varying shape for each one

If there is no loss, this will work. Roadblock:

If the aperture is limited by loss you do not win.

For most refractive optics, absorption limits aperture, and hence resolution



Is there a way around this?

Can we beat the loss limitations?

- Yes, but we have to give up something.
- First lets learn something about zone plates

A very brief review of Zone Plates



Equation for fresnel boundaries

$$y_m = \sqrt{(2mf\lambda + m^2\lambda^2)}$$

Table 2.1

Type of zone plate	I ₁ /I ₀ , %	$I_2/I_0, I_3/I_0, \ldots, $ %	Undiffracted portion, %	Absorption, %
Fresnel (amplitude)	10.1	0, 1.1, 0, 0.4,	25	50
Rayleigh-Wood (phase)	40.4	0, 4.5, 0, 1.6,	0	0
Gabor (amplitude)	6.25	0	25	68.75
Gabor (phase)	34	10, 1, 0.1, 0.06,	0	0
Kinoform (phase)	100	0	0	0

(Aristov)

Instead of solid refractive optic:





Deleted sections reduce loss but constrain the bandwidth of optic (There is no free lunch!)

Fresnel lens (kinoform) ; main point

- If you are willing to work at a fixed wavelength, you can reduce loss.
- Remove sections such that at a fixed wavelength the phase shifts by multiples of 2π . Original Fresnel lighthouse lens had large phase shifts (>> 2π).
- Steps are $\frac{\lambda}{\delta}$ thick corresponding to 2π phase shift, or multiples







Consequence of fixed phase shift choice



Is it diffractive or refractive?

One can view the kinoform equivalently as

- a) A blazed zone plate
- b) An array of coherently interfering micro-lenses.

Not really a valid question; refractive limit is $\lambda \rightarrow 0$

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Intuition for the kinoform from the simpler Fresnel prism

$$\rho(\mathbf{k}) = \mathbf{prism}(\mathbf{k}) \times \sum_{\mathbf{n}} \delta(\mathbf{k} - \mathbf{n}(\frac{2\pi}{\mathbf{a}}))$$

Features:

The Fresnel lens has a loss that is almost independent of aperture.

•Transmission
$$T = e^{-(2\pi \frac{\beta}{\delta})}$$

•Implies lens resolution is no longer limited by loss ⇒Back to elliptical shape limit

There is a minimum loss; you need at least enough thickness to give 2π phase shift.
For silicon the best transmission is around 40kV; compton limited

<u>Features:</u> High heat load capacity

- In the best case, the optic is designed not to absorb much heat. Should have a high heat load capability.
- Complicated to calculate
- First pointed out by Lengeler, Snigirev.

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Etching Diamond for high heat loads!

Features:

Favourable phase error comparison with mirrors.

- Consider a mirror with a bump on it
- The path length difference caused by bump is $2dsin\theta$
- To get a 0.5π phase shift bump must be $0.25(\lambda/\sqrt{\delta}) \approx 25$ nm

- Consider a refractive lens
- To get a 0.5π phase shift bump must be $0.5(\lambda/\delta) \approx 15$ microns
- The precision of an e-beam writer is ≈ 1 nm (*). Possible errors very small

Potential road block for zone plate

- The spot size is of order the smallest zone Work at harmonics, reduces efficiency
- As photon energy increases, the zone plate thickness T increases

To get smallest spot sizes at hard x-ray energies requires => Large aspect ratios that are difficult to manufacture

Going beyond the manufacturing tolerance

As in the zone plate, the smallest feature is proportional to the focus spot. Does this limit the spot size?

Answer: Instead of 2π phase shifts, use 4π , or more. The features get bigger and easier to manufacture. We already do this. The limit here is the loss. A single lens should probably not be bigger than exp(-2). Under investigation.

By the way, there is a small factor of 2 improvement in resolution relative to binary zone plate.

Important point: I don't have to have features as small as the spot!*

The fresnel lenses are line focus elements; is this a problem?

Answer:

- 1. Not a problem*. K.B. optics are also line focus optics
- 2. Is an advantage if you have an asymmetric source shape
- 3. Digital processing takes care of this in imaging mode.

Calculational Approach

Consider first a rectangular aperture with an amplitude transmittance given by

$$t_A(\xi, \eta) = \operatorname{rect}\left(\frac{\xi}{2w_X}\right)\operatorname{rect}\left(\frac{\eta}{2w_Y}\right).$$

The result is

$$I(x, y) = \frac{A^2}{\lambda^2 z^2} \operatorname{sinc}^2 \left(\frac{2w_X x}{\lambda z} \right) \operatorname{sinc}^2 \left(\frac{2w_Y y}{\lambda z} \right).$$

Some reasons we resort to numerical simulation

The function U(ξ) contains phase and amplitude of lens For material of thickness t, the phase shift is $\frac{2 \pi t \delta}{\lambda}$

For the familiar, solid, lossless refractive lens U(ξ)=exp($i\phi$)

$$\exp(-i\frac{2\pi t\delta}{\lambda}) = \exp(-i\frac{2\pi\delta(\frac{\xi^2}{2R})}{\lambda}) = \exp(-\frac{i\pi\xi^2}{\lambda f})$$

Both λ and δ are energy dependent. $\delta \propto \rho E^{-2}$ Transverse scan is difficult analytically

Phase profile comparison between "full" lens and kinoform

In the calculation the 2pi phase shifts make no difference

Another connection between the zone plate and kinoform

Zone plate boundaries
$$y_m = \sqrt{(2mf\lambda + m^2\lambda^2)}$$

Very similar but not identical. What is the connection?

All is well!

1) Let $f \to (f - \frac{\lambda}{\delta})$ then the two expressions become identical $y_m = \sqrt{(2mf\lambda + m^2\lambda^2)}$ $y_1 = \sqrt{(2f\lambda + \lambda^2(1 - (\frac{2}{\delta})))}$

2) The zone plate can be thought of as a lens with an infinite refractive index (Sweatt, 199x). Works here also.

A simplified analytical model to explain kinoforms with feature sizes M times the resolution

Start with Fresnel Kirchhoff: $A(z) \propto \iint T(\xi, \eta) e^{\frac{i\pi}{\lambda z} (\xi^2 + \eta^2)} d\xi d\eta$ Switch to radial coordinates: $2\pi \int_{0}^{\rho_N} T(\rho) e^{\frac{i\pi\rho^2}{\lambda z}} \rho d\rho$ Use t(ρ) which is lens
thickness, is discontinuous in
M multiples of Fresnel zones $T(\rho) = e^{\frac{i2\pi\delta t(\rho)}{\lambda}} = e^{i2\pi(\frac{\rho^2}{2\lambda f} - nM)}$

$$2\pi \int_{v_0}^{v_N} T(v) e^{\frac{i\pi v}{\lambda z}} dv$$

Finally we get a sum over all the M sized Fresnel zones up to the full size of the lens

$$\int_{v_0}^{v_L} e^{\frac{i\pi v}{\lambda} \left(\frac{1}{f} - \frac{1}{z}\right)} dv = \int_{v_0}^{v_L} e^{i\pi\alpha v} dv = \int_{v_0}^{v_1} e^{i\pi\alpha v} dv + \int_{v_1}^{v_2} e^{i\pi\alpha v} dv + \dots + \int_{v_{L-1}}^{v_L} e^{i\pi\alpha v} dv$$
$$\alpha = \frac{1}{\lambda} \left(\frac{1}{f} - \frac{1}{z}\right)$$

Focal length f

In the top panel are all the allowed foci at 1/n in normalized units. In b the middle panel the is shown the "form factor" with zeroes at most of these allowed foci and in c we show the product, leaving a single focus.

How to use larger feature sizes than the desired optics resolution. Here we use zones twice the size, and so the normalized foci are 2/n, but the "form factor" only allows the desired focus F. So for this lens, the lens resolution is half the size of the smallest kinoform feature size.

- This is why the kinoform is 100% (*?) efficient
- When one works at the harmonics one can get all the light into one of the higher orders!
- But it is not free.
- We pay with bandwidth.

*i.e. no light goes into any alternate foci like binary zone plate

- •Use Fresnel limit diffraction calculation*
- $\bullet\Delta E/E \sim 10\%$
- •Now take each profile, fit to a gaussian shape*

"Comparison" of experimental data with simulation

How compound kinoform lenses can improve resolution

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Focusing Hard X Rays to Nanometer Dimensions by Adiabatically Focusing Lenses

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We address the question of what is the smallest spot size that hard x rays can be focused to using refractive optics. A thick refractive x-ray lens is considered, whose aperture is gradually (adiabatically) adapted to the size of the beam as it converges to the focus. These adiabatically focusing lenses are shown to have a relatively large numerical aperture, focusing hard x rays down to a lateral size of 2 nm (FWHM), well below the theoretical limit for focusing with waveguides [C. Bergemann *et al.*, Phys. Rev. Lett. **91**, 204801 (2003)].

*K. Evans-Lutterodt et al., "Single-element elliptical hard x-ray micro-optics", Optics Express 11 (8) 919-926, 21 April 2003.

.....One implication of the elliptical shape is that for a given focal length and refractive index, the diffraction-limited resolution given by the Rayleigh criterion

is dependent only on the choice of material and the wavelength, even for lossless material and in the refractive limit. For $\delta = -10^3$, one gets a resolution of $-10^3 \lambda$. This is not a fundamental limit; by using more than one element i.e. a compound lens, one can exceed this limit.

Optimize Gain

- Each lens gives some loss
- Each lens increases gain (flux into spot)
- $I \propto Nx^N$
- N is the number of lenses, x is the loss of a lens
- Note: you do not have to optimize gain; you can choose to get a smaller spot, and increase the background.

A dummy lens calculation for NSLS2

•We consider a compound lens fabricated out of a stack of Fresnel lenses. For Beryllium at 10keV $\delta \approx 3.1 \times 10^{-6}$, and $\beta \approx 7.5 \times 10^{-10}$, and so the transmission T of a single Beryllium Fresnel lens T ≈ 0.9985 . For 200 lenses, corresponding to 100 lenses for each axis, the total transmission is 75% of the incident light.

• For the paraxial limit we make the standard approximation that the focal length of the stack is (f_0/N) where f_0 is the focal length of an individual lens and N is the number of lenses. If we conservatively stay within the paraxial limit, we estimate a focal length of $f_0 \approx 2.2y/\sqrt{\delta}$ where y is the required aperture and δ is the refractive index.

•The aperture y is of order 5x10-4 m, ($\approx 3 \text{ x}$ (distance from source) x $\sigma_y' = 3 \text{ x} 40 \text{m} \text{ x} 4x10^{-6}$). The estimated f_0 is 0.64 m. The net focal length for 100 lenses is 6.5mm, and the resulting resolution is λ /(Numerical Aperture) is 1.6nm.

Consider breakdown of linear approximation Incident un-focused light

Line Focus?

$$U(P_0) = \frac{1}{i\lambda} \iint_{(\xi,\eta)} U(P_1) \frac{\exp(ikr_{01})}{r_{01}} \cos\theta ds$$
$$r_{01} = \sqrt{z^2 + (x - \xi)^2 + (y - \eta)^2} \qquad r_{01} \approx z[(1 + \frac{1}{2}(\frac{x - \xi}{z})^2 + \frac{1}{2}(\frac{y - \eta}{z})^2]$$

showing the replacement of the spherical wave by a pair of orthogonal parabolic terms.

$$z^{3} \geq \frac{\pi}{4\lambda} \left[(x - \xi)^{2} + (y - \eta)^{2} \right]^{2}$$

For 100micron aperture, focal length 1cm, we can use crossed lenses down to at least 10nm, but how far can we go?

Please fabricate the structure below to obtain the best resolution

Radially symmetric kinoform structures. Material: Single crystal material Arrays of these, as we have seen

What are we doing today?

- Improve depth and fidelity of etch, currently 80microns deep
- Figure out how to create cylindrically symmetric self-supporting structures (Complicated micro-fabrication, not planar)
- 3. Figure out how to use materials other than Silicon

Summary

- •If you are willing to accept the fixed wavelength limitation, kinoform lenses have some useful features.
- •For some applications even the bandwidth issues are not a problem
- •Clearly they work, and are improving.

