Abstract

For ERL injector, superconducting cavities need to deliver high power to the beam. Couplers must provide low $Q_{\text{ext}}$ but small kick. Results of simulation for two low-kick high-power fundamental coupler designs are presented. Possibilities to meet the requirements for the coupler in the range of parameters: $Q_{\text{ext}} = 4.6 \cdot 10^4 - 4.1 \cdot 10^5$, RF power $P = 100 - 150$ kWCW at frequency $f = 1300$ MHz are discussed.

1 INTRODUCTION

A multi-GeV Energy Recovery Linac (ERL), proposed by Cornell University in collaboration with Jefferson Lab, is a low emittance, high average beam current CW accelerator for X-ray science [1]. An injector and a main linac of the ERL are based on the superconducting RF technology. The driving idea behind this machine is to create a low emittance beam using a high-brightness photoemission electron gun and then to preserve the emittance while the beam is being accelerated in the injector and in the main linac. The goal is to have the beam with the normalized emittance of 2 mm-mrad in undulators. An extension of present state-of-the-art technology in several directions is required to achieve this ultimate goal. To address the challenges, development of a 100 MeV, 100 mA average current ERL prototype is in progress at Cornell University [2].

One of the possible sources of emittance dilution is a kick caused by non-zero on-axis transverse electromagnetic fields of fundamental power couplers in superconducting cavities. This effect is especially strong in the injector cavities, where a high average RF power per cavity must be coupled to a low-energy beam. The requirements here are far more demanding than in any existing system. Our design goal is to allow a maximum emittance growth of no more than 10% total for five injector cavities out of the initial emittance of 1 mm-mrad [3].

The five injector cavities are superconducting 2-cell niobium structures. They provide 500 kW (limit is set by the input power specifications) of RF power to the beam. Consequently, the permitted beam current depends on the injector energy and varies from 100 mA at 5 MeV to 33 mA at 15 MeV. The injector cavity coupler has to deliver 100 kW of RF power to the beam and provide matching conditions for a cavity gap voltage of 1 through 3 MV and corresponding beam current of 100 through 33 mA. Thus the coupler must be adjustable with the external $Q$-factor range from $4.6 \cdot 10^4$ to $4.1 \cdot 10^5$. Some ERL injector parameters relevant to the coupler kick and emittance growth calculations are listed in the table below.

<table>
<thead>
<tr>
<th>$\varepsilon_a$</th>
<th>$\sigma_t$</th>
<th>$E$</th>
<th>$V_{\text{acc}}$</th>
<th>$f_{\text{RF}}$</th>
<th>$\lambda_{\text{RF}}$</th>
<th>$\sigma_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mm-mrad</td>
<td>2 mm</td>
<td>0.5 to 5 (15) MeV</td>
<td>1 (3) MV per cavity</td>
<td>1300 MHz</td>
<td>231 mm</td>
<td>0.6 mm</td>
</tr>
</tbody>
</table>

There are several possibilities to completely or partially cure transverse kick from the fundamental RF power coupler and associated with it emittance growth. Among them are:

- An azimuthally symmetric coupler
- Two identical couplers opposite each other (a “twin-coupler”)
- Symmetrizing stub opposite to an input coupler
- Alternate input power couplers in adjacent cavities
- Using larger beam pipe
- Non-protruding antenna

In this paper we describe two possible realizations of a low-kick strongly coupled fundamental power coupler for superconducting cavities of the Cornell ERL injector, namely the twin-coupler and the waveguide-coaxial coupler.

2 CALCULATING RF COUPLER KICK AND ESTIMATING ASSOCIATED WITH IT EMITTANCE GROWTH [3]

Asymmetry of the fundamental power coupler geometry leads to non-zero transverse electric and magnetic fields on the cavity axis and results in the transverse kick to the bunch passing the coupler. A comprehensive study of this effect can be found in [4]. Because of the finite bunch length different parts of the bunch experience different kicks which in turn generates emittance growth. Let us first explain how one can calculate the transverse kick. We will follow M. Dohlus’ approach as it is explained in [5, 6], but will use somewhat different notation. The integrated along the
beam trajectory normalized transverse field strength (kick) can be written as

\[ v_t = \frac{V}{V_{\text{acc}}} = M_1 \cdot v_{\text{in}} + M_2 \cdot v_{\text{out}}, \]

where \( M_1 \) and \( M_2 \) are the coupling constants for the normalized amplitudes of incoming and outgoing waves in the coupler, \( v_{\text{in}} \) and \( v_{\text{out}} \), \( V_{\text{acc}} \) is the accelerating voltage. For a perfectly matched coupler one has pure traveling wave. To reconstitute this wave we can use results from a computer code like MAFIA or Microwave Studio. The calculations have to be performed for two different boundary conditions at the end of the coupler: perfect electric wall and perfect magnetic wall. Then one can reconstitute the incoming traveling wave by combining the two standing wave solutions in quadrature with appropriate normalization and calculate the transverse kick as it is illustrated in Fig. 1 and determine first coupling constant from

\[ M_1 = \frac{v_t}{v_{\text{in}}}. \]

Similarly one can reconstitute outgoing traveling wave to calculate the second coupling constant. Finally, in the stationary case the incoming and outgoing waves are fully determined by the running conditions (beam current \( I_b \), beam phase relative to maximum acceleration \( \phi_0 \), \( V_{\text{acc}} \), operating frequency \( \omega \) and the cavity parameters (\( Q_{\text{ext}} \), \( Q_0 \), \( R/Q \), coupling coefficient \( \beta_c \), and the cavity detuning \( \delta \omega ):

\[ v_{\text{in}} = \frac{I_b}{2V_{\text{acc}}} \left( \frac{R/Q}{Q_{\text{ext}}} \cos \phi_0 + i \sin \phi_0 \right) + \frac{\beta_c + 1}{2} \left( 1 + i \tan \psi \right), \]

\[ v_{\text{out}} = -\frac{I_b}{2V_{\text{acc}}} \left( \frac{R/Q}{Q_{\text{ext}}} \cos \phi_0 + i \sin \phi_0 \right) + \frac{\beta_c + 1}{2} \left( \frac{\beta_c - 1}{\beta_c + 1} - i \tan \psi \right), \]

\[ \tan \psi = 2Q_L \frac{\delta \omega}{\omega}, \quad \frac{1}{Q_{\text{ext}}} = \frac{1}{Q_0} + \frac{1}{\beta_c}, \quad \beta_c = \frac{Q_0}{Q_{\text{ext}}}. \]

Fig. 1. Illustration to the coupler kick calculation.
Knowing the coupling constants and the formulae for the traveling waves, one can easily calculate the transverse kick for any combination of running conditions and the cavity detuning. For a reflection free operation \((v_{\text{out}} = 0)\) it is necessary that
\[
\tan \psi = -I_b \frac{R/Q \cdot Q_{\text{acc}}}{V_{\text{acc}}} \sin \phi_0 ,
\]
and
\[
I_b = \frac{V_{\text{acc}}}{R/Q \cdot Q_{\text{ext}} \cos \phi_0} \cdot \frac{\beta_c - 1}{\beta_c}.
\]

The kick received by the center of a passing bunch depends on the relative phase \(\phi_0\) between the bunch and the RF voltage:
\[
\alpha = \frac{p}{p_t} = \frac{eV_{\text{acc}}}{\gamma c} \cdot \text{Re} \left( e^{i\phi_0} \right),
\]
where \(p_t\) is the longitudinal momentum, \(p_t\) is the transverse momentum due to the RF coupler. This kick can be easily compensated. We are interested here in a kick change along the bunch, from head to tail, which leads to the transverse emittance growth. The normalized transverse emittance growth can be estimated, assuming that before kick \(\alpha = 0\) and \(d\sigma = 0\), from [7]
\[
\frac{d\xi_{\alpha}}{d\sigma} = \frac{d\sigma}{\sigma} \cdot \frac{2\pi \sigma}{\lambda_{\text{RF}}} \cdot \frac{1}{p}.
\]

Here \(\sigma_t\) and \(\sigma_r\) are the bunch sizes in the transverse plane \(t\), \(\sigma_z\) is the bunch length, \(\lambda_{\text{RF}}\) is the RF wavelength. The derivative of the transverse momentum with respect to phase is
\[
\frac{d\sigma}{\sigma} = \frac{eV_{\text{acc}}}{\gamma c} \cdot \text{Re} \left( \frac{dV_{\text{acc}}^*}{d\phi} \right) = \frac{eV_{\text{acc}}}{\gamma c} \left[ -\text{Re}(v_i) \cdot \sin \varphi - \text{Im}(v_i) \cdot \cos \varphi \right].
\]

Finally,
\[
\frac{d\xi_{\alpha}}{d\sigma} = \frac{2\pi \sigma}{\lambda_{\text{RF}}} \cdot \frac{eV_{\text{acc}}}{\gamma c} \cdot \text{Re}(v_i) \cdot \sin \varphi + \text{Im}(v_i) \cdot \cos \varphi.
\]

(here \(E_0 = 0.511\) MeV is the rest mass energy of electron). This formula, of course, gives only a rough estimate of the emittance growth for a bunch on axis and does not take into account transverse dependence of electromagnetic fields. Computer simulations are necessary for a more precise evaluation of the emittance growth.

3 SINGLE AND TWIN-COAXIAL COUPLERS

We plan to use TESLA-like cavities in the main linac of ERL, so it was natural for us to take one of the TTF coaxial coupler designs as a starting point. The TTF2 and TTF3 couplers use coaxial line with outer diameter of 40 mm. As we know, the TTF2 coupler were tested up to 1.8 MW in a pulse of 1.3 ms with average power of 4.68 kW [8]. One of the newer designs, TTF5, utilizes bigger (60 mm) coaxial line. Because the ERL injector cavity coupler has to transfer much higher average power of 100-150 kW, we have decided to use 60 mm diameter outer conductor. To minimize losses in the inner conductor we chose an impedance of 60 Ohm. The coaxial line of these dimensions is multipactor-free up to 200 kW even in a standing wave regime according to [9].

The coupler can be made adjustable in a fashion similar to TTF couplers by adjusting the position of the antenna. The left-hand beam-pipe (see Fig. 2) has a radius of 39 mm and the internal iris has a 35 mm radius like the TESLA cavity.

![Fig. 2. Single coaxial coupler with a two-cell cavity.](image)

The right-hand part of the cavity was optimized with a goal to increase the coupling and to decrease the kick. This involves an increase in the tube radius. At the same time we wanted to keep the length of the tube in reasonable limits. So, we needed to get necessary coupling and the kick with as small radius as possible. The right-hand tube radius was taken to be 48.7 mm. A funnel-shaped tube and a disk-shaped antenna tip serve to increase coupling without the need to penetrate into the beam pipe. For the traveling wave in the coupler maximal electric field on the antenna disk is about 20 % of the maximal field on the cavity iris. The minimal distance between the right cell and the coupler is 45 mm that is defined by the position of the cryostat helium vessel wall.
Because of a high coupling, the single coaxial coupler (Fig. 2) would cause a big kick to the bunch. The fields on the axis of the cavity with such a coupler are shown in Fig. 3. The transverse fields \( E_y \) and \( H_z \) are high in part due to the big diameter of the outer conductor that in turn is caused by high power to be transferred. Indices \( e \) and \( m \) designate fields with an electric and a magnetic wall at the end of the coaxial line respectively. The term “electric” or “magnetic wall” means that the tangential component of the corresponding field is zero on this “wall”. Different amplitudes of the axial fields \( E_\text{ze} \) and \( E_\text{zm} \) on Fig. 3 are related with normalization: we should multiply fields of one of the solution (e.g. with the magnetic wall) by such a factor that amplitudes of the real and imaginary fields were equal. Example of this normalization is shown in Fig. 1 (formula for \( E \) as a complex sum of \( E_1 \) and \( E_2 \)). Magnetic fields are also multiplied by \( Z_0 \), the free space impedance, to have all fields in the same units. Integration performed in accordance to expression presented in Fig. 1 gives the value of \( v_\text{i} = (0.7 - 3.0i) \cdot 10^{-3} \) for the kick of the single coupler. Here we consider the case of a matched coupler, so that \( v_{\text{out}} = 0 \). This value of the kick, in accordance to (1), leads to emittance growth \( \text{se} = 2 \cdot 10^{-7} \), i.e. 20% for one 2-cell cavity that is unacceptable as stated in the Introduction.

A twin-coaxial coupler was proposed to eliminate the kick. In this case power for each half of the double coupler would go down to 50-75 kW and the value of \( Q_{\text{ext}} \) for each coupler would be 2 times bigger than for a single coupler.

Cross-sections along the axis of the twin-coaxial coupler are shown in Fig. 4. Dimensions of this coupler are the same as for the single coupler above.
This is a preliminary design; some dimensions of the coupler and the cavity shape will be further optimized. Here we want only to show advantage of the symmetric coupler over the asymmetric one for a case of dimensions and figures of merit close to the specified values. In this case the external quality factor obtained for the double coupler is \( Q_{\text{ext}} = 4.4 \cdot 10^4 \). The technique of calculating the external \( Q \) factor is discussed in another paper presented at this workshop [10].

The kick received by an on-axis beam is zero for a symmetric twin-coupler. However, asymmetry can be caused by the shift of the top of the inner conductor or/and by the phase shift between two coaxial lines. For the shift \( \delta_t \) of the antenna tip the kick is

\[ \kappa_t = (4.5 - 7.3i) \cdot 10^{-5} \cdot \delta_t / \text{mm}. \]

The phase shift was modeled by a small difference in lengths of the coaxial lines. For the shift \( \delta_w \) of the shortened end of the coaxial line with electric or magnetic wall the kick is

\[ \kappa_w = (2.5 - 0.8i) \cdot 10^{-5} \cdot \delta_w / \text{mm}, \]

\[ \kappa_w = (1.6 - 0.5i) \cdot 10^{-5} \cdot \phi / \text{degrees}. \]

In both cases the emittance growth is two orders of magnitude less than for the asymmetric coupler.

4 WAVEGUIDE-COAXIAL COUPLER

The other design that we had briefly explored is a hybrid waveguide-coaxial coupler (Fig. 5). Bellows assist to adjust the coupling in a wide range. This way to couple RF power to a cavity supposedly should not create any transverse kick and there should not be any wake fields harmful to beam (if the beam is traveling from left to right on the picture). However, our calculations showed that the presence of the waveguide created a field asymmetry and consequently a transverse kick to the beam.

The value of the kick modulus for this geometry is shown in Fig. 6a. It falls exponentially with the length \( B \) (see Fig. 5) and is an order of magnitude less than for a single coaxial coupler. The distance \( D \) determines the coupling and is 47.2 mm for \( Q_{\text{ext}} = 4.0 \cdot 10^4 \).

\[ v_t = (4.5 - 7.3i) \cdot 10^{-5} \cdot \delta_t / \text{mm}. \]

The waveguide-coaxial transition can be matched separately of the whole cavity. For the presented case the matching is obtained by adjusting the dimensions \( A \) and \( C \), see Fig. 5 and 6b.

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In both cases the emittance growth is two orders of magnitude less than for the asymmetric coupler.
The traveling wave electric field in the transition is shown in Fig. 7. One can see that the wave front is slightly tilted near the rectangular waveguide and becomes as flat with the coaxial output at its end. When the waveguide-coaxial transition is connected to the beam pipe, this tilted field causes a kick. We believe that a simple change of geometry: the inclination of the flat waveguide top or/and of the flat bottom of the coaxial and the following matching should eliminate the distortion of the field and secure a zero kick on the axis. However, it is necessary to check that the kick will not change when the coupling changes, as desired, from $Q_{\text{ext}} = 4.6 \cdot 10^4$ to $4.1 \cdot 10^5$. The field symmetry can be also improved by elongating the coaxial part to allow higher-order modes (HOMs) generated by the transition to attenuate better. There are also other uncertainties in this design that require extensive studies: higher-order mode excitation by the beam, coupling strength tuning, waveguide connection to the cryostat, etc.

![Fig. 7. Traveling wave in the WCC.](image)

### 4 CONCLUSIONS

Two different designs were considered for the ERL injector cavity coupler: the twin-coaxial and the waveguide-coaxial couplers. Both designs look as feasible high-power low-kick couplers. However, we settled down on the twin-coaxial coupler as a more practical approach. Presented electromagnetic designs can serve as a basis for developing a strongly-coupled low-kick high average power couplers.

CST Microwave Studio™ [11] code was used for calculations and to produce pictures for this paper.

### 5 REFERENCES