# Influence of undulators on nonlinear ERL optics

Charalambos Koupparis (Wayne State University) and Georg H. Hoffstaetter \* (Cornell University)

#### Abstract

Cornell University is currently investigating the feasibility and benefits of building an Energy Recovery Linear Accelerator (ERL) to produce high quality x-ray beams. Due to the small emittances and short bunch length that linear accelerators can produce, an ERL has the potential to provide significantly better x-ray beam parameters than a storage ring. Part of this investigation is optimizing the electron optics to make sure that the required beam parameters can be achieved at the location of the undulators and after the energy recovery path. In order to manipulate the beams and to achieve the proposed results, quadrupole and sextupole strengths within the ERL are adjusted. Here it is studied how the undulators' effect the nonlinear optics and therefore the quadrupole and sextupole strength in the energy recovery path of an ERL that would be located in Cornell's Wilson tunnel. We will show that the linear optics is hardly perturbed by the undulators and that the change in sextupole strength is no more than a factor of 2. Nonlinear optimization features and nonlinear wiggler descriptions of LEPP's BMAD codes were used to perform this study.

## **INTRODUCTION**

The benefit of building an ERL at Cornell [1] is it's ability to produce lower electron emittances, thus higher brilliance x-rays, and shorter pulse length than the state of the art. As opposed to storage rings, which let the bunches circle over and over around the ring until the stochastic emission of radiation leads to an equilibrium beam size, the ERL will only let them circle once so that the emittance is largely determined by the gun and the injector [2]. Currently Cornell operates the  $e^+, e^-$  storage ring CESR, which produces x-rays for the CHESS laboratory, but their parameters are far inferior to what an ERL, or a 3rd generation light source for that matter, could provide. An ERL, like a storage ring, achieves the intense x-ray beams by means of undulators. However in the case of an ERL the electrons only travel through the device once so that small emittances coming from a DC photo-cathode electron gun can be preserved, while in a storage ring the electrons travel through each undulator for millions of turns. In spite of this, as in a storage ring, the beam sizes and bunch-length in each undulator are determined by linear and nonlinear optics of the used magnet structure. We have optimized the linear and nonlinear optics for one possible layout for an ERL at Cornell, as shown in Fig. 1 [3, 4]. In this paper we compare the optimization results with and without undulators in order to point out that the perturbations due to undulator fields can easily be incorporated and do not significantly change the magnet requirements.



Figure 1: One of the proposed construction sites for the ERL, using existing infrastructure of CESR.

# ENERGY RECOVERY LINEAR ACCELERATOR (ERL)

As the name implies, the energy recovery scheme is able to recover the energy from used electron beams [5]. In doing so it saves electricity and allows for higher currents. Electrons aren't dumped with high energies thus re-using this energy saves power and money. Furthermore, the problem of finding a sufficient and safe method of dumping very high beam powers is reduced.

The anticipated method for retrieving the energy from the used bunches involves sending these bunches around a return loop once, extracting their energy on a second turn through the linac, and then dumping the beams. While the particle bunches are sent around the ERL they need to have the desired properties in each undulator. Once they have been used, they are sent through the linac a second time. However, this time the electron beams are 180 degrees out of phase with the accelerating fields. The fields will then decelerate the beam which transfers its energy to the ERL cavities where it is used to accelerate the following bunches that pass through them.

In order to optimize the beam size in the undulators, the magnetic optics was adjusted to the following constraints:

a)  $\beta_x$  in the center of each undulator was required to be equal to half the undulator length.

b)  $\beta_y = \beta_x$  was required in the center of each undulator.

<sup>\*</sup> Georg.Hoffstaetter@cornell.edu

- c)  $\alpha_x = 0$  and  $\alpha_y = 0$  were required in the center of each undulator.
- d) For the dispersion, D = 0 and D' = 0 were required in each undulator.
- e) The first and second order time of flight terms R56 and T566 were required to lead to 100fs bunch length in the central undulator and to minimal energy spread at the end of the energy recovery pass [6].
- f) For the second order dispersion,  $D_2 = 0$  and  $D'_2 = 0$ were required in the central undulator and in the linac.

The here investigated version of an ERL at Cornell University incorporates 7 undulators. They are arranged in a mirror symmetric fashion around the current location of CESR's south interaction point. Coming form the east there is a 2m undulator, then two 5m undulators. Then, close to the current  $e^+/e^-$  interaction point, there is a undulator of 25m length. Finally, there are two 5m undulators followed by a 2m undulator, all shown in Fig. 2. The undulators had a period of  $\lambda = 17$ mm, so that the 2m undulator had 118, the 5m undulator had 294, and the 25m undulator had 1470 poles. A field strength of 1T, which provides a bending radius of  $\rho = 17$ m, and the harmonic approximation of infinitely wide poles was used [7]. While the horizontal focusing of such undulators averages to zero over many cells, they provide vertical focusing with an average focal strength of approximately  $\frac{L}{2a^2}$ , which is only 0.04/m for the longest undulator. Since this is about 10% of a typical quadrupole strength, the undulators are not expected to influence the linear optics strongly. While the undulators produce and average octupole strength of about  $\frac{L}{12}(\frac{2\pi}{\rho\lambda})^2$ , they do not produce a sextupole strength. Nevertheless, the undulators influence the second order optics since they produce first and second order dispersion.



Figure 2: Green arrows indicate positions of the undulators in the ERL. Red arrows indicate fit sections.

For optimization purposes, the arc was split into the 8 pieces indicated in Fig. 2. These 8 sections are

- 1) Start to center of 2m undulator
- 2) center of 2m undulator to center of 5m undulator
- 3) center of 5m undulator to center of 5m undulator

- 4) center of 5m undulator to center of 25m undulator
- 5) center of 25m undulator to center of 5m undulator
- 6) center of 5m undulator to center of 5m undulator
- 7) center of 5m undulator to center of 2m undulator
- 8) center of 2m undulator to the end of the arc.

Within each piece, except the last, the conditions of the linear optics (a-d in the above list) were matched by an appropriate choice of quadrupole strengths. All except the first and last sections have exactly as many quadrupoles as constraints. The second order constraints are taken care of by using six sextupoles which are located at high dispersion regions to reduce their strength. Sequentially in order, these fits are

- A) Fit first order constraints in sections 1 to 4.
- B) Use section 1 to fit the first order time of flight term R56 in the middle of the 25m undulator
- C) Fit first order constraints in sections 5 to 8.
- D) Use section 8 to fit the first order time of flight term R56 at the end of the arc.
- E) Fit second order constraints e-f at the center of the 25m undulator, using 3 the sextupoles in section 1.
- F) Fit second order constraints e-f at the end of the arc, using the 3 sextupoles in section 8.

We are concerned with the first order time of flight constraint only in the middle of the arc and at the end. Therefore, after completing the fits A) and B), section 1 was used to adjust R56 at the end of section 4 while additionally requiring all first order constraints a-d to remain satisfied at the end of section 1. Only section 1 can be used for adjusting the time of flight since all other sections have exactly as many magnets as associated constraints a-d. Similarly the first order time of flight at the end of the arc was adjusted by the extra magnets in section 8. Finally, the second order optics requirements e-f were satisfied by the six sextupoles. The first three in section 1 are used to fix the three second order constraints at the center of the 25m undulator, the last three sextupole magnets in section 8 fix these constraints at the end of the arc.

## **OPTICS CALCULATIONS**

Once the above procedure had been established, initially only taking into account the first order optics and no effect of the undulators, satisfactorily small quadrupole strength are obtained and the beta functions are acceptable. After that the second order optics constraints were added. Once this procedure was completed the undulators and their linear and non-linear effects were added. In order to arrive at an optics with undulators that is as close as possible to the optics without undulators, the optimization process with and without the undulators was performed in the same order as described above. Furthermore, the described fits were computed after only two undulators were added symmetrically at a time. Finally after adding the long 25m undulator the final magnet strengths were obtained.

The particle optics code BMAD [8] was used for all required computations. The undulators were split up into little sections, since the undulators are made up of 100 to 300 alternating polarity dipoles. For each section the magnetic field is calculated and used to determine the nonlinear beam transport map. The concatenation of all these nonlinear maps determines the total nonlinear effect of the undulators.

We investigated how many sections were required and found that the fit results did not change when each pole of the undulator was split into five or more section. To minimize the computation time, we therefore chose to split each undulator into 5 times the number of poles it contains.

#### RESULTS

As can be seen from Fig. 3 the quadrupole strength  $K_1$  that are originally obtained without undulators and those finally obtained with undulators differ very little. The quadrupoles that are most affected by the undulators are naturally those closest to the 25m undulator. Including the undulators changed those quadrupole strength by about 3.5 to 8%.



Figure 3: Original and final quadrupole strength  $K_1$ , without and with undulators.

Also the sextupoles' strengths  $K_2$  remain sufficiently small, within a range of -0.00005/m<sup>3</sup> to 0.02/m<sup>3</sup>, as can be seen in Fig. 4, when the undulators are included into the optics computation.

The  $\beta_x$  and  $\beta_y$  function values were within acceptable limits, usually below 100m as shown in Fig. 5. Only toward the end,  $\beta_x$  rises to about 130m. The position of all quadrupoles in the section 1 and 8 have remained where they are currently in CESR. A rearrangement of quadrupole positions can most likely further reduce the beta function in this region. In the Figs. 5, 6, 7, and 8, the 7 undulators are



Figure 4: Original and final sextupole strength  $K_2$ , without and with undulators.

located at 113m, 133m, 155m, 188m, 220m, 242m, and 262m.



Figure 5: Beta function values after optimization.

The  $\alpha_x$ ,  $\alpha_y$ ,  $\eta$  and  $\eta'$  constraints were all satisfied to a high accuracy as shown in Figs. 6 and 7.



Figure 6: Alpha function values after optimization.

Also, the time of flight constraint was satisfied very well, as shown in Fig. 8. To compress the 2ps bunch length in the linac to 100fs, R65 = -0.22437m is required in the center of the arc. For energy recovery, the total arc must have R56 = 0 so that the second half of the arc has R65 =0.22437. This difference is responsible for the asymmetry in the optics between the right and the left side of the ERL return arc.



Figure 7: Dispersion and dispersion prime function values after optimization.



Figure 8: Time of flight function values after optimization.

### CONCLUSIONS

Even though further optimization of magnet positions are possible, even the here presented results show that the perturbations that undulators produce in the first and second order optics can easily be compensated by moderate changes to quadrupole and sextupole strengths for an ERL in the CESR tunnel that were obtained without undulators [4].

#### ACKNOWLEDGMENTS

C.K. performed this work as an REU student and would therefore like to thank the local REU coordinators Giovanni Bonvicini and Rich Galik. David Sagan's support with BMAD is also appreciated. Support came from the NSF REU grant PHY-0101649 and the research cooperative agreement PHY-9809799.

#### REFERENCES

- [1] S.M. Gruner, M. Tigner (eds.), Phase I Energy Recovery Linac (ERL) Synchrotron Light Source at Cornell University, Report CHESS-01-003 at http://erl.chess.cornell.edu/papers/ERL\_Study.pdf (2001)
- [2] G.H. Hoffstaetter, et al., The Cornell ERL Prototype Project, proc. of PAC03 (2003)
- [3] R. Talman, Energy Recovery Linac in the Wilson Tunnel, Report LEPP-ERL 02-06 (2002)
- [4] G.H. Hoffstaetter, et al., A Lattice for a 5GeV ERL in the CESR Tunnel, proc. of PAC03 (2003)

- [5] M. Tigner, A Possible Apparatus for Electron Clashing-Beam Experiments, Nuovo Cimento Vol 37, 1228-1231 (1965)
- [6] I.V. Bazarov, G.H. Hoffstaetter, *Lattice options for a 5GeV Light Source at Cornell*, proc. of PAC03 (2003)
- [7] D. Sagan, J. A. Crittenden, D. Rubin, E. Forest, I. Oho, A magnetic field model for wigglers and undulators, Portland/OR (2003)
- [8] D. Sagan, "BMAD Lattice Language Guide", March 2003, cesrelog.lns.cornell.edu/documents/software/bmad.html, (August 8 2003)