CALCULATION OF COHERENT SYNCHROTRON RADIATION IN GENERAL PARTICLE TRACER

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Abstract

General Particle Tracer (GPT) is a particle tracking code, which includes 3D space charge effect based on a non-equidistant multigrid Poisson solver or a point-to-point method. It is used to investigate beam dynamics in ERL and FEL injectors. We have developed a new routine to simulate coherent synchrotron radiation (CSR) in GPT based on the formalism of Sagan [1]. The routine can calculate 1D-wake functions for arbitrary beam trajectories as well as CSR shielding effect. In particular, the CSR routine does not assume ultrarelativistic electron beam and is therefore applicable at low beam energies in the injector. Energy loss and energy spread caused by CSR effect were checked for a simple circular orbit, and compared with analytic formulas. In addition, we enhanced the 3D space charge routine in GPT to obtain more accurate results in bending magnets.

INTRODUCTION

Both ERLs and FELs require electron bunches of low emittance and short duration for production of high quality synchrotron radiation. The optimization of various parameters to suppress emittance growth due to the space charge effects at low energy in a DC gun based photoinjector has been carried out using a space charge code ASTRA [2, 3]. Additionally, the effects of space charge and CSR on beam dynamics in an ERL merger must be carefully considered. The usual 1D treatment of CSR [4], however, assumes ultrarelativistic electron beam energies, and therefore cannot be applied to low energy transport lines typical for ERL injection system. Full accounting of space charge and CSR forces requires self-consistent solution to Lienard-Wiechert potentials [5, 6], in turn leading to CPU-time costly calculations, which are of limited use in extensive optimizations necessary in obtaining ultimate performance of an injector. Alternatively, in order to investigate the beam dynamics in a full injector, we have developed and implemented a new CSR routine, GPT/CSR, for a particle tracking code, GPT [7], which includes efficient 3D space charge effect treatment based on nonequidistant multigrid Poisson solver. GPT/CSR does not assume ultrarelativistic electron beam [1], and is therefore effective for the merger or chicane simulations at low energies where both CSR and space charge may be important at the same time.

Additionally, the original mesh-based 3D space charge routine in GPT is found to contain a serious error when applied to bunches in a bending magnet. It is caused by inappropriate disposition of the mesh in the rest frame when the electron bunch does not move along the z-axis. To treat the space charge effect accurately in the merger, we have corrected the 3D space charge routine in GPT.

We summarize changes to GPT code that extend its applicability to treatment of space charge and CSR effects in the bends at low energy.

CSR CALCULATION IN GPT

The GPT/CSR routine calculates 1D wake using the formalism which does not assume ultrarelativistic electron beam energies, which makes it effective at low energies [1]. Furthermore, the routine correctly calculates CSR transient wake-fields for arbitrary beam trajectories, which are stored in parametric form for the purpose of retarded time calculations. Besides, the routine can calculate the effects of vacuum chamber shielding using image charges.

Energy Loss and Spread

The steady-state energy loss and spread for various beam energies are compared as calculated by GPT/CSR, `elegant`, and analytical expression for a circular orbit with a bending radius of $\rho = 1.0$ m. In the calculation, the bunch length is $\sigma_s = 0.6$ mm, the initial distribution is Gaussian, and the bunch charge is 80 pC. Figures 1 and 2 show the energy loss $d\varepsilon/dt$ and incremental energy spread $d\sigma_\delta/dt$ respectively for the steady-state case. The red line in Fig. 1 is the analytical result derived by C. Mayes [8],

$$\frac{d\varepsilon}{dt} = -\frac{2}{3}\frac{(r_e m_e c^2)c\beta^4\gamma^4}{\rho^2} N\left(1+(N-1)T(a)\right), \quad (1)$$

where $a = 3/2 \cdot \gamma\, 3\sigma_s/(\beta\rho)$,

$$T(a) = \frac{9}{32\pi}\frac{1}{a^3}\left(\mathrm{e}^{\frac{1}{(8a^2)}}\sqrt{\pi}K_{5/6}\left(\frac{1}{8a^2}\right) - 2\pi a\right), \quad (2)$$

$K_{5/6}(x)$ is the modified Bessel function, N is the number of electrons in the bunch, m_e is the electron mass, r_e is the classical electron radius, c is the speed of light, γ is the Lorentz energy factor and $\beta = (1 - 1/\gamma^2)^{1/2}$. The agreement between `elegant` and the theory is good only for higher beam energy, $E_0 > 40$ MeV, because the CSR routine in `elegant` includes the assumption of ultrarelativistic beam [4]. On the other hand, GPT/CSR reproduces the analytical result accurately, as seen in Fig. 1.

The red line in Fig. 2 is the analytical result for energy spread [9, 10], which includes the assumption of $\gamma \gg (\rho/\sigma_s)^{1/3}$,

$$\frac{d\sigma_\delta}{dt} \approx 0.22\frac{r_e N c\beta}{\gamma\rho^{2/3}\sigma_s^{4/3}}. \quad (3)$$

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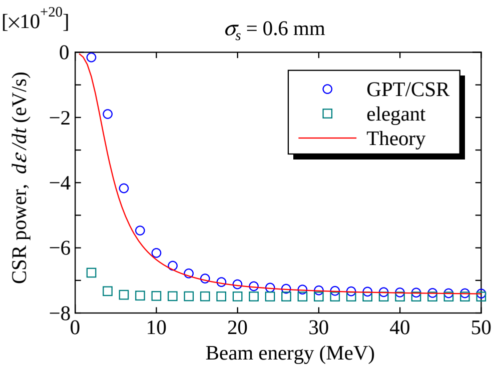

Figure 1: Steady-state energy loss for a circular orbit with $\rho = 1$ m.

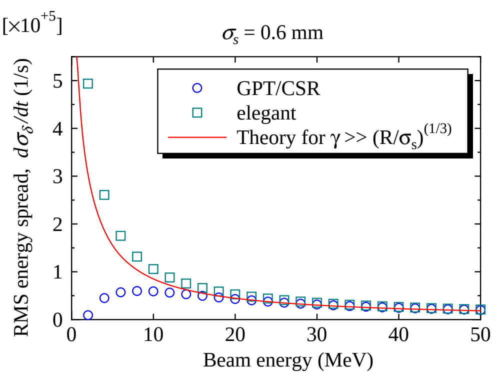

Figure 2: Steady-state incremental rms energy spread for a circular orbit with $\rho = 1$ m.

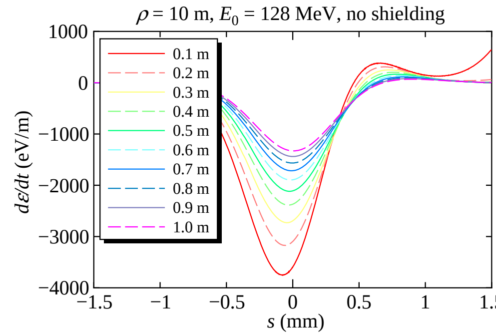

Figure 3: CSR wake without shielding from the end of bending magnet. The bunch length is 0.3 mm.

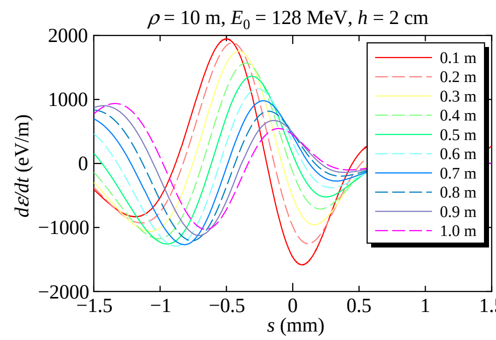

Figure 4: CSR wake with shielding from the end of bending magnet. The shielding chamber height is 2 cm.

Fig. 2 shows that the results of GPT/CSR and `elegant` both reproduce well the analytical result for higher beam energy, $E_0 > 40$MeV. However, the results of `elegant` and Eq. 3 diverge to infinity for $E_0 \rightarrow 0$ unlike GPT/CSR, which approaches zero as expected.

These results show that the GPT/CSR is effective for wide range of beam energies, and can be used to investigate beam dynamics in ERL and FEL photoinjectors.

CSR in Transient State

As an example of CSR effect in a transient state, the CSR wake form is calculated by GPT/CSR after the exit of a bending magnet for the following parameters: the beam energy 128 MeV, Gaussian particle distribution, the bunch length 0.3 mm, and the bunch charge 80 pC. Figures 3 and 4 show the transient CSR wake with and without the effect of shielding. In Fig. 4, the shielding chamber height and the number of image charge layers are 2 cm and 32, respectively. Before the starting point, $\Delta s = 0$ m, the bunch moves in a bending magnet with the bending radius $\rho = 10$ m having reached a steady-state CSR condition. After the starting point, the bunch moves in a drift space. The figures show that the CSR wake reduces as the distance from the exit of the bending magnet increases as expected.

CSR Shielding Effect

The effect of CSR shielding is calculated by GPT/CSR for a circular orbit with the bending radius $\rho = 10$ m. Figure 5 shows the energy loss for various shield heights in a steady-state case. In the calculation, the initial particle distribution is Gaussian, the bunch length is 1.0 mm, the bunch charge is 80 pC, and the number of image charges is 32. The solid lines in Fig. 5 are the analytical values without the shielding as calculated by Eq. (1). As the shielding height increases, the energy loss approaches to the analytical value.

CSR in Merger Section

As an example, the transverse emittance in a 3-dipole merger of ERL project at Cornell University is calculated by GPT/CSR and `elegant` for two different conditions: (a) $p_0 = 10$ MeV/c and (b) $p_0 = 500$ MeV/c. CSR calculations are done without shielding. Space charge calculations have been suppressed in GPT in this example. The layout of the merger is shown in Fig. 6, which consists of three bending magnets and two quadrupoles. Magnetic fields have been scaled for the two energy cases so that the optics conditions are identical. Figure 7 shows the evolution of the normalized horizontal emittance as a function of longitudinal position. The bunch length is 0.3 mm, the particle distribution is Gaussian and the charge is 80 pC. For Fig. 7

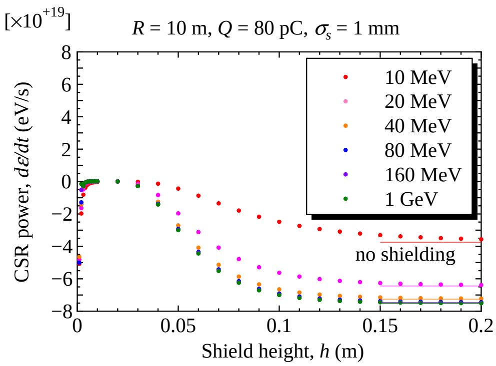

Figure 5: Energy loss for various shield heights in a steady-state case. The number of image charges is 32.

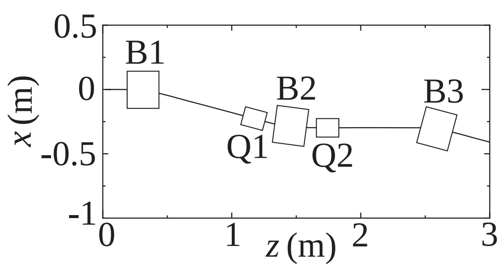

Figure 6: Layout of 3 dipoles merger in Cornell University.

(a) $p_0 = 10$ MeV/c, the GPT/CSR and elegant results disagree. On the other hand, for Fig. 7 (b) $p_0 = 500$ MeV/c, the agreement is good demonstrating that GPT/CSR reproduces `elegant` CSR calculations at higher beam energies as expected.

ENHANCED 3D SPACE CHARGE ROUTINE IN GPT

To calculate the space charge field in the 3D mesh-based routine in GPT, the particle coordinates are transformed from the laboratory frame to the rest frame according to $\boldsymbol{r}'_\perp = \boldsymbol{r}_\perp$ and $\boldsymbol{r}'_\parallel = \gamma \boldsymbol{r}_\parallel$ relative to the direction of motion. See Fig. 8. When the bunch does not move along the z-axis, the bounding box ends up improperly oriented as illustrated in Fig. 8 (a) and (b). In this case, for example, the transverse emittance incorrectly depends on the angle relative to the z-axis in a straight trajectory. To fix this problem, we have added a transformation of rotation in the rest frame in the space charge routine. Using the enhanced space charge routine, we are able to calculate transverse emittance accurately for an orbit with a finite angle to the z-axis.

SUMMARY

We have developed a new CSR routine for GPT in order to investigate beam dynamics in ERL and FEL injectors. To check GPT/CSR, energy loss and energy spread are calculated by GPT/CSR, `elegant` and analytical expression. The results show GPT/CSR to be effective in a wide range of beam energies. In addition, we have corrected 3D space charge routine in GPT so that it is made applicable to calculating the space charge effect in bending magnets.

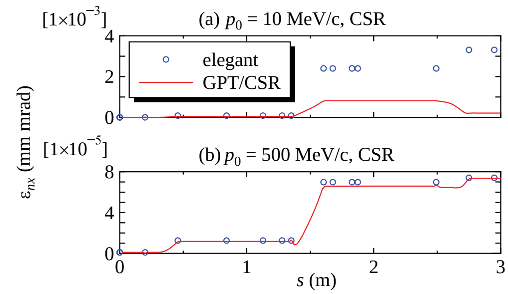

Figure 7: Normalized emittance in 3 dipoles merger of Fig. 6 calculated by elegant and GPT/CSR.

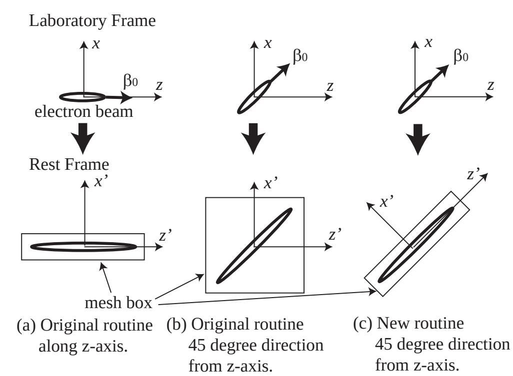

Figure 8: Bounding box sizes in the rest frame for the original GPT space charge mesh routine and the enhanced version.

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