DESIGNING MULTIPLE CA VITY CLASSES FOR THE MAIN LINAC OF CORNELL’S ERL

N. Valles† and M. Liepe, Cornell University, CLASSE, Ithaca, NY 14853, USA

Abstract
Cornell is currently developing a high current Energy Recovery Linac. The baseline 7-cell cavity design for the main linac has already been completed, and prototyping has begun, as of Fall 2010. Previous work showed that increasing the relative cavity-to-cavity frequency spread increases the beam break-up current through the linac. Simulations show that expected machining variations will introduce a relative HOM frequency spread of $0.5 \times 10^{-3}$, corresponding to 150 mA of threshold current. The key idea of this work is to increase the relative cavity-to-cavity frequency spread by designing several classes of 7-cell cavities obtained by making small changes to the baseline center cell shape. This allows a threshold current in excess of 450 mA, which is well above the 100 mA goal for the Cornell Energy Recovery Linac.

INTRODUCTION
Central to the intended operation of an Energy Recovery Linac (ERL) is the proper design and functioning of the superconducting RF cavities comprising its main accelerating structure. Cornell has chosen to implement superconducting niobium seven-cell accelerating structures into the main linac design enabling a high current (100 mA), very low emittance (30 pm-rad at 77 pC bunch charge) 5 GeV beam capable of producing short pulses ($\sigma_z/c = 2$ ps) of hard x-rays with a high repetition rate (1.3 GHz)[1].

Because of the large number of free parameters available in a 7-cell cavity, the design was broken down into several steps allowing for a robust cavity design within a reasonable time frame. Staring from a geometry which minimized the cryogenic load due to the fundamental accelerating mode[2], the center cells were designed to increase cell-to-cell coupling and be stable under unavoidable machining perturbations[3]. Then the end cells were tuned independently to minimize the effect of higher order modes (HOMs) that most strongly limit the beam break-up (BBU) current through the linac[4].

The threshold current that can be sent through a linear accelerator before coherent effects cause the beam to be lost is called the beam break-up current. This limit is due to excitation of transverse HOMs by the beam, and can be amplified strongly by coherent excitation of a given HOM present in multiple cavities, which will occur if the mode has similar frequencies in the individual cavities (if $\Delta f < \text{few mode bandwidths})[4].$ The effect of a given HOM, denoted by $\lambda$, on the BBU current is quantified by $\zeta_{\lambda}$, a function of the $R/Q$, $Q$ and frequency, $f$ of the mode given by

$$\zeta_{\lambda} \equiv \left(\frac{R}{Q}\right)_\lambda \cdot \sqrt{Q} \cdot f_{\lambda}^{-1}.$$  

Minimizing $\zeta \equiv \max(\zeta_{\lambda})$ maximizes $I_{th}$ in an ERL with large number of cavities. In general, BBU current must be determined through particle tracking simulations, but using $\zeta$ as the goal function in cavity optimization significantly reduces the computational load, and yields a solution more quickly.

Since the BBU current can be lowered strongly by coherent excitation of a HOM in multiple cavities in an ERL main linac, coherent mode excitation can be reduced and the BBU current increased. Study has also shown that further loosening of the machining tolerances is not sufficient to guarantee an increase the beam break-up current because certain cavity shapes lead to trapped modes that will strongly degrade performance and in fact suppress BBU current.

To get the benefits of large relative cavity-to-cavity frequency spread and avoid the drawbacks of cavity shapes with trapped modes, this study investigates the design of multiple cavity shapes to be used in the same linac as a means to increase the HOM frequency spread in a controlled way without causing trapped modes.

For a HOM pass band with nominal frequency $f_{\lambda}$ assuming machining variations yield uniformly distributed spread in HOM frequency with a width $\Delta f_{\lambda}$, the relative cavity-to-cavity spread, $\sigma_f$, is given by

$$\sigma_f = \frac{\Delta f_{\lambda}}{2\sqrt{3}f_{\lambda}}.$$  

Machining tolerances of a single class of cavities will be controlled tightly, yielding small relative frequency spreads, but with multiple cavity classes, the total relative frequency spread can be made large, allowing for a large threshold current before HOM driven instabilities cause beam degradation and loss.

SIMULATIONS
Beam tracking was used to compute the threshold current through the ERL as a function of frequency spread, and presented in Fig. 1, utilizing the properties of a HOM at 1.7 GHz. For each point, hundreds of linacs were simulated, using relative cavity-to-cavity frequency spread as a parameter. In the actual linac, relative frequency spread comes from slight differences between cavities.

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† nr5@cornell.edu
Previous work demonstrated that allowing ±1/16 mm machining tolerances result in cavity designs with a relative frequency spread of $0.5 \times 10^{-3}$[5]. This corresponds to 150 mA of BBU current, or 1.5 times the design value of Cornell’s ERL. To have a safety factor that is better than this value, there needs to be a way to reliably increase the threshold current through the linac.

The key idea of this work is to increase the relative frequency spread between the cavities without risking creating dangerous HOMs by creating several cavity classes. These different cavity shapes must preserve the fundamental mode figures of merit, $R/Q$, $G$ and frequency, but have HOM passbands that are shifted in frequency relative to one another. Starting from the baseline center cell shape[4], a parameter sweep was performed by varying 4 center cell cavity parameters, shown in Fig. 2.

The center cell parameters were varied in 1 mm steps, 5 mm around the original values. Each center cell shape that was generated was tuned to 1.3 GHz, and a 2D field solver, CLANS[6], was used to calculate the $Q$, $R/Q$, $E_{pk}/E_{acc}$, $H_{pk}/E_{acc}$, and geometry factor of the fundamental mode. The first 6 dipole passbands were also calculated, and the boundary conditions were varied to obtain the 0 and $\pi$-mode. By choosing multiple cell shapes whose HOM dispersion curves cover large regions, the relative frequency spread can be maximized. Dispersion curves for the cavities calculated in the parameter sweep is shown in Fig. 3.

As shown in Fig 3, the HOM dispersion curves are fairly wide, suggesting that properly selected, a large cavity-to-cavity relative frequency spread can be obtained. For instance, the 1700 MHz passband has a width of about 30 MHz giving a maximal possible relative frequency spread of about $5.1 \times 10^{-3}$, if all cavity shapes are considered.

While all cavities computed had a fundamental mode frequency of 1300 MHz, the other figures of merit for the fundamental mode and higher-order modes varied widely. Possible center cell cavity shapes were required to meet five conditions: $E_{peak}/E_{acc}$ had to remain below 2.1, $R/Q$-$G$ and $H_{pk}/E_{acc}$ of the fundamental mode had to stay within ±5% of the base line value and the wall angle of the cavity had to be less than 85°,1 and the radius of curvature everywhere on the cell could not be smaller than 6 mm.2

Out of the set of cavities satisfying the above requirements, the objective is then to select $N$ cavities such that they cover the entire acceptable passband region. Filtering

1The wall angle constraint is a necessity for proper chemical treatment and cleaning by high-pressure rinsing.

2This consideration is necessary for making reproducible bends in the fabrication process.
out cavities that don’t satisfy the requirements, one finds that there is a relative frequency spread of about \(3.9 \times 10^{-3}\) is achievable. Assuming that machining variations will account for a frequency spread of \(0.5 \times 10^{-3}\), 8 cavity classes are necessary to get full coverage of the passband.

To select the \(N\) ‘best’ center cell shapes, one must consider how to choose cell geometries that have dispersion curves covering the largest region of a given HOM passband. Let the dispersion curves, which are functions of the phase \(\phi\), of the \(m\) cell shapes generated in the parameter sweep, and filtered to have acceptable fundamental mode properties be denoted by \(k_m(\phi)\). Furthermore, let \(h_n(\phi)\) be an ‘ideal’ dispersion curve, in the sense that for a given phase, \(\phi'\), if the possible cavity shapes allow for the dispersion curve to take on frequency values within \([f_{\lambda}(\phi'), f_{\lambda}(\phi') + \Delta f_{\lambda}(\phi')]\), then \(h_1(\phi'), h_2(\phi'), \ldots, h_N(\phi')\) are evenly distributed within this range of width \(\Delta f_{\lambda}\).

Each of the \(N\) classes of cavities were chosen by minimizing the residual sum of squares between the ideal dispersion curve and the cavity dispersion curve

\[
RSS_n = \min_m \left[ \sum_i (h_n(\phi_i) - k_m(\phi_i))^2 \right],
\]

for \(n \in 1, \ldots, N\), where \(\phi_i\) is sampled at discrete points within \([0, \pi]\).

This procedure was carried out taking the functions \(h_n(\phi)\) from the first dipole passband, which has most of the strongest dipole modes. The dispersion curves of the selected cell shapes are presented in Fig. 4 and the corresponding cell geometries are plotted in Fig. 5.

![Figure 5: Plot comparing the geometry of the selected center cell shapes. Note that despite the significant geometry variation among them, though they all possess very similar fundamental mode properties.](image1)

**CONCLUSIONS**

The key development this work presents is that instead of relying on machining variation to increase relative frequency spread, multiple cavity classes can be designed, having shifted HOMs relative to one another, which reduces coherent effects and increases the threshold current through the accelerator.

Eight center cell classes with a relative frequency shift of \(>11.5\) MHz were obtained. These designs still maintain sufficiently wide passbands to mitigate the introduction of dangerous HOMs under machining variation. The minimal frequency splitting is for the 3.0 GHz passband, and the splitting are much larger for the other passbands. The baseline design has the strongest HOM in the 1.7 GHz band, where the frequency splitting is 23 MHz, which gives a relative frequency spread of approximately \(3.9 \times 10^{-3}\), corresponding to a threshold current of 450 mA. This result is more than double the threshold current obtained previously, and yields a safety factor for the Cornell ERL of 4.5 for maximal current through the accelerator.

**REFERENCES**