TUNING ELLIPTICAL NIOBIUM CAVITIES BY PLASTIC DEFORMATIONS *

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Abstract

The niobium superconducting accelerating cavities used in many contemporary accelerators require tuning to a working frequency after fabrication. This tuning is necessary because small variations between cells can substantially alter the field profile and decrease the accelerating rate. ANSYS simulations are done to understand changing of the initial shape due to plastic deformations at tuning. Simulations are presented for inner cells of a multicell cavity and for the end group including the end cell and the beam pipe. The tuning device has different contact areas on the cell surface where the force is applied, so stretching and squeezing the cavity cell are studied separately. The influence of the stiffening rings is also studied. The cavity tuned to the working frequency with the tuning device has a shape different from the designed shape. So, parameters of the higher order modes (HOMs) excited by the beam will also differ from the parameters calculated for the designed shape. Knowledge of these disturbed parameters is necessary for calculations of the beam breakup caused by HOMs.

INTRODUCTION

If the accelerating cavity has more than one cell, the cells need to be tuned relative to each other so that the accelerating field is the same in each cell [1]. Each cell is tuned by squeezing or stretching it in a special device. The place of the contact between this device and the cell is different for squeezing and stretching, so the deformations can occur in the different places along the cell. These tuning deformations are of necessity plastic, contrary to elastic deformations used for cavities in the cryomodule when a fine tuning is needed. Deformations discussed in [2] were calculated for the elastic case. A more general approach with the plastic deformations is needed.

A multicell cavity for high \( \beta \) values has different inner and end cells, Fig. 1. The inner cell is usually symmetric relative to the left and right side, so each halfcell can be described by 7 parameters related to the inner surface that define the RF properties of the cell: equatorial radius \( R_{eq} \), radius of aperture \( R_a \), length \( L \), half-axis of the equatorial and iris elliptic arcs: \( A \), \( B \), \( a \), and \( b \), respectively, Fig.2. There is a straight segment \( l \) between the arcs, the wall slope angle \( \alpha \) is usually chosen > 90° for easier drainage of chemical liquids at the preparation of the surface.

First of all, we will analyze deformations of the inner cell on the example of the Cornell ERL accelerating cavity with inner dimensions presented in Fig. 2. Wall thickness can be different in different points of a cell due to the procedure of stamping from a flat sheet niobium. For simplicity we will take a uniform thickness of 3 mm. Actual thickness varies from approximately 2.8, at the iris, to 3.2 mm, at the equator [3].

Then we will discuss the mechanical properties of the cavity with stiffening rings and of the end group that consists of the cell and a beam pipe.

![Multicell elliptic cavity.](image)

Figure 1: Multicell elliptic cavity.

As can be found from the data published in [2], the fundamental frequency change in the case of elastic deformation is \( df/dL = 5.9 \text{ MHz/mm} \), whereas the above mentioned simplified model from [4] gives \( df/dL = 3.9 \text{ MHz/mm} \). This means that the parameters of HOMs will also change differently in the process of elastic tuning than in the simplified model. We can be sure that the plastic de-

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formations will also be quite different from the elastic ones, moreover from the simplified model.

We will suggest, like in [2] and [4], that the deformed cavity is still the elliptic cavity, i.e. can be described by two elliptic arcs connected with a straight segment tangent to the arcs at the points of connection. However, we will evaluate the deviation from the elliptic model because for large deformations this model can become unsatisfactory.

The design of the tuning device for plastic deformations can have a decisive effect on the values of deformation. The plates which contact the cell walls are different for the cavities with and without stiffening rings.

Calculations were performed with the ANSYS structural mechanics software [5].

MECHANICAL PROPERTIES OF NIOBIUM

Summary of mechanical properties of niobium was compiled in [6]. The data presented here from various sources are sometimes contradicting and depend on many different conditions: RRR, heat treatment – temperature and duration of annealing, grain size, mechanical processing. The electron welded iris area can also have different mechanical properties compared to the rest part of the cavity cell. These details can be taken into account in future research. To the best of our knowledge this research is the first on simulating the process of tuning the acceleration cavity, so we will chose a definite set of properties to find the most general features of the cell behaviour in the process of plastic deformation.

For our simulations we used the model with the bilinear isotropic hardening and the following parameters of the material were taken: Yield Strength = $5 \times 10^7$ Pa, Tangent Modulus = $2 \times 10^9$ Pa, Young Modulus = $5 \times 10^{10}$ Pa, Fig. 3.

Figure 3: Reactor grade niobium with RRR = 70; 1200 C 6 hour with RRR = 120; 1250 C 6 hour with RRR = 210; crystal grain size unknown [6]. The bold red line is the approximation for our simulations.

Figure 4: Deformation of the dumbbell stretched by a force of 7 kN.

Figure 5: Elasto-plastic behavior of the halfcell with a free end at the iris (1) and as part of a dumbbell for maximal stretching force from 5 to 8 kN (2 – 5).

STRETCHING AN INNER CELL WITHOUT STIFFENING RINGS

For the simplest case of stretching a cell without stiffening rings we have to take a dumbbell – two halfcells having a common iris. The model used for calculations is shown in Fig. 4. Because of axial symmetry one quarter only of the whole dumbbell was considered in most cases. The dumbbell is fixed at the plane passing through its equator on the left side but can move in the radial direction. The force is applied uniformly and parallel to the axis of rotation to the cross-section passing through the right equatorial plane. If the model consists of a halfcell only (left half of the model in Fig. 4) and the force is applied to the iris cross-section, the iris has no constrains in the radial direction, and the iris part of the halfcell starts to bend out of the axis of rotation leading to results very different from the results with a dumbbell model.

In Fig. 5, the values of the halfcell elongation are shown.
for different maximal forces applied to the halfcell or to the dumbbell. Deformations of different parts of the dumbbell are shown in Fig. 4, and the residual stresses are presented in Fig. 6.

Figure 6: Residual stress of the dumbbell stretched by a force of 7 kN.

Figure 7: Residual deformation of the unstiffened halfcell vs applied stretching force.

One can see that the deformation occurs mainly in the iris region, the equatorial region is practically not deformed and has no residual stress. This is also seen in Fig. 7 where the increments of the geometric parameters of the halfcells are presented after application of different maximal forces. These dependences on force are very strong starting from some point, but if we plot these increments for different increments of the halfcell length $\Delta L$, we can see that they are practically linear, Fig. 8. On the same Fig. 8 the values of the rms deviation $\sigma$ from the elliptic shape are presented. One can see that $\sigma$ is more than 3 orders of magnitude less than any parameters: $R_{eq}$, $A$, $B$, and so on, so the deformed halfcell can be considered as an elliptic one. It is difficult to explain physically the increase of $R_{eq}$ by stretching the cell. It is however seen that this increase of $R_{eq}$ is less than the value of $\sigma$. This means that actually $R_{eq}$ can not increase by stretching the cell but this apparent increase is due to rms fitting of the equatorial elliptic arc, see also Fig. 9. Let’s note that the increment $\Delta b$ in Fig. 8 is decreased 10 times to make the graph more compact. The bisector $\Delta L$ vs $\Delta L$ is also shown to highlight that the elongation of the halfcell happens mainly due to the increase of the half-axis $a$.

Deviation from the “ideal” elliptic cavity shape, consisting of 2 elliptic arcs connected with a straight segment, is shown in Fig. 9 for the stretching force of 8 kN. One can see that the maximal deformation occurs near the transition from the equatorial arc to the straight segment.

The change of the working frequency of the halfcell also
Table 1: Coefficients of proportionality for the dimensions and frequency changes with elongation and compression of the halfcell as a part of the dumbbell and as a part of the end group.

<table>
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<th>∆A/∆L</th>
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<th>∆b/∆L</th>
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*For compression ∆L is negative.

Figure 10: Shift of the working frequency of the unstiffened halfcell vs elongation and compression. Points are simulated by ANSYS, the thin black line is a linear approximation.

The frequency change is defined for the individual halfcell limited by the electric wall in the equatorial plane and by the magnetic wall in the iris plane. The measured frequency shift when the halfcell is incorporated into the multicell cavity will be of course different from this value. The actual influence on the multicell cavity frequency can be calculated with SLANS or another specialized software.

In reality, for stretching the special dies are used as will be described in the next section. So, not only the stretched cell is deformed but the adjacent cells on both external sides of the die also. More detailed calculations are needed to analyze deformation of these cells.

**COMPRESSING AN INNER CELL WITHOUT STIFFENING RINGS**

For compressing the cell with the goal of decreasing its frequency, the cell was squeezed between two special dies. Because of symmetry only one die and one half of the compressed cell was used in the simulation as it is shown in Fig. 11. The die moves parallel to the cavity axis to the left, the left halfcell is fixed in the equatorial plane but can expand or shrink radially in this plane. To secure the rigidity of the iris area, the model includes the whole dumbbell as in the above section.

As can be seen in Fig. 12, the force needed for the same
deformation as in the case of stretching is about 4 times larger. This is because now the equator area having larger cross-section is deformed rather than the iris area.

Figure 12: Elasto-plastic behavior of the halfcell as part of a dumbbell for maximal compressing force 24, 30, and 34 kN (1, 2, 3).

The increments of the geometric parameters of the halfcell under compression are shown in Fig. 13. Coefficients of proportionality for linear approximations versus halfcell length increment $\Delta L$ are in the Table 1.

Deviations from the elliptic halfcell shape after application of 34 kN compressing force is shown in Fig. 14. Now deformation of the equatorial arc is comparable with the deformation of the transition between the straight segment and the arcs, cf. Fig. 9.

Frequency change under compression is shown together with the change under stretching, in Fig. 9. Let us note that though for stretching the frequency change is practically linear, there is a smooth transition to a steeper slope of the curve for compression. The linear approximation with $\Delta f/\Delta L = 6.62$ MHz/mm (see the Table 1) is not very accurate near $\Delta L = 0$.

From the data presented in Table 1 we can conclude that frequency increase under stretching is due mainly to the increase of the distance between the consecutive irises and increase of $R_e$. Both changes decrease the capacitive part of the resonator. The value of $R_{eq}$ keeps practically constant. Under compression, both the gap decrease and the $R_{eq}$ increase (increase of the capacitive and inductive components) are responsible for the decrease of the frequency. This observations give more detail to the results presented in [7] where the frequency decrease was related to the increase of the equatorial radius.

As it was mentioned in the previous section, the adjacent cells can be also deformed when a cell is squeezed between two dies, mainly in the iris area. This is also a subject of further research.

STRETCHING AN INNER CELL WITH STIFFENING RINGS

The model for simulation of the dumbbell with a stiffening ring is shown in Fig. 15.
Simulation results of stretching of the stiffened halfcell are presented in Figs. 16, 17 and in Table 1. One can see that dependences of $\Delta B$ and $\Delta b$ on $\Delta L$ are not linear anymore but are close to linear for other increments.

**COMPRESSING AN INNER CELL WITH STIFFENING RINGS**

Residual deformation of the stiffened dumbbell after compressing by 40 kN is shown in Fig. 18. Coefficients for linearization of increments versus change of the halfcell length are presented in the Table 1. Graphs for increments of the geometric parameters versus applied compressing force are shown in Fig. 19. A distinctive feature of this graph is a lengthening (not shortening under compressing force) of the halfcell when the force is below 30 kN. The cause of this behavior is not fully understood, possibly this is because the contact between the die and the cell is point-like at the first moment when the force is small, possibly the mesh is too large (1 mm). Anyway, this anomalous behavior happens at the level less than 10 micrometers, and if the level of tuning is of the order of 100 kHz, the dependencies of the geometrical increments and $\Delta f$ on $\Delta L$ are quite linear, see Figures 20 and 21.

**STRETCHING OF THE UNSTIFFENED END GROUP**

Results of stretching simulations of the unstiffened end group are shown in Figs. 22 - 26 and in the Table 1. In Fig. 26 $\Delta a$ and $\Delta b$ seem nonlinear with $\Delta L$. This can be an effect of the short arc when the curvature is the same for two different arcs, radius of curvature $r_c = a^2/b = const$. A small deviation of the external points of the dumbbell surface can cause a big deviation in $a$ and $b$ but simultaneously. The curvature keeps for a given elongation if these
errors in $a$ and $b$ are small. It varies linearly with the half-cell elongation, see the lower line in Fig 26.

Dependence of the halfcell frequency on the elongation is shown in Fig. 27. The linear dependence of frequency on the elongation is not kept for high values of $\Delta L$. However, we do not expect the deformations as big as more than 2 mm with frequency change about 10 MHz. So, the coefficient $\Delta f/\Delta L = 4.49$ MHz/mm in the Table 1 is calculated for this initial part of the plot. Here, as it was done above for the dumbbell, we define the frequency of the halfcell when it is limited with the electric boundary in the equatorial plane and with the magnetic boundary in the iris plane.

**COMPRESSING OF THE UNSTIFFENED END GROUP**

This work was not finished because of lack of financing.

**STRETCHING OF THE STIFFENED END GROUP**

Residual deformation of the stiffened end group after stretching by 30 kN are shown in Fig. 28. Elasto-plastic
Figure 25: Residual deformation of the unstiffened end group halfcell vs applied stretching force.

Figure 26: Residual deformation of the unstiffened end group halfcell vs elongation.

behavior of the stiffened end group for maximal stretching force from 10 to 30 kN is presented in Fig. 29. One can see that no residual deformation occurs at 10 kN and up to approximately 14 kN.

COMPRESSING OF THE STIFFENED END GROUP

This work was not finished because of lack of financing.

CONCLUSION

Tuning of a multicell superconducting cavity with the goal to compensate errors of manufacturing and to obtain necessary field flatness between the cells is used in any laboratory working with these cavities as a part of the cavity preparation. The present work, to the best of our knowledge, is the first attempt to connect forces for this tuning with changes of dimensions and frequency, to find limits of elastic and plastic deformations. Simulations were done with the ANSYS software. Properties of niobium used for the simulations were chosen from a variety of data, actually they are different depending on the preparation of the material, but we hope that the main dependences will be qualitatively the same for any quality of the metal. Usage of stiffening rings needed for limitation of the Lorentz forces when a cavity works with strong fields, changes significantly its mechanical properties. These simulations are done for the inner cells but were not finished for the end group.

It is shown that the shape of the cavity cell, inspite of its deformations, can be treated with a good accuracy as an elliptic shape. In this case, it is easy to calculate frequency change versus the cell elongation or shortening, using a special software, e.g. SLANS [8]. The correct account of the shape change gives a way for calculation of higher order modes. It is shown that a simplified approach to the deformations used earlier leads to unreliable results. It is found that the change of the geometrical parameters of the cavity cell, half-axes of the elliptic arcs, equatorial
and aperture radii, its working frequency, are in most cases linear function of the increment of the cavity length.

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REFERENCES


