Evaluation of a New Cavity Focusing Theory

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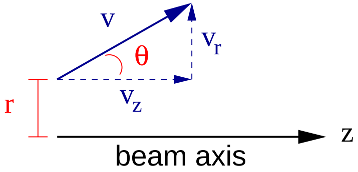
Outline

1. Transport matrices
2. Cavity focusing
3. Initial project goals
4. What happened?
5. Plans for the future

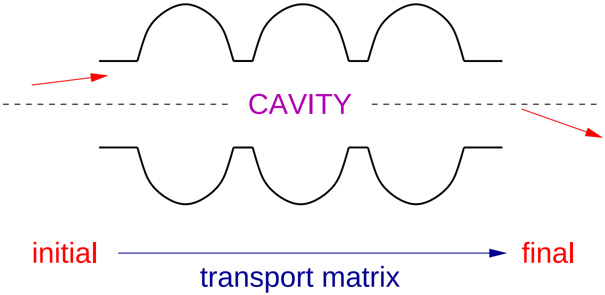
Transport Matrices

Introduction

$$\begin{pmatrix} r \\ r' \end{pmatrix}_f = \begin{pmatrix} \text{transport} \\ \text{matrix} \end{pmatrix} \begin{pmatrix} r \\ r' \end{pmatrix}_i$$

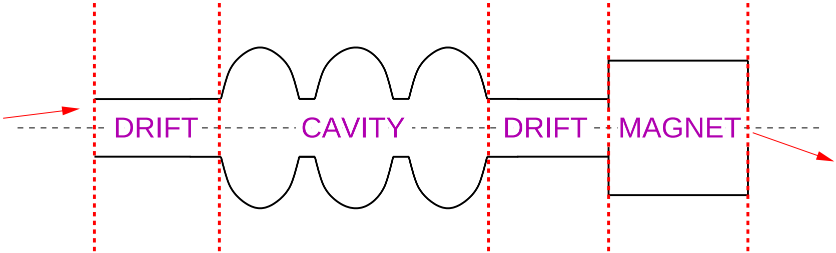


$$r' = \frac{dr}{dz} = \frac{v_r}{v_z} \approx \theta$$



Transport Matrices

Accelerator elements in series



$$\begin{pmatrix} r \\ r' \end{pmatrix}_f = \begin{pmatrix} \text{magnet} \\ \text{matrix} \end{pmatrix} \begin{pmatrix} \text{drift} \\ \text{matrix} \end{pmatrix} \begin{pmatrix} \text{cavity} \\ \text{matrix} \end{pmatrix} \begin{pmatrix} \text{drift} \\ \text{matrix} \end{pmatrix} \begin{pmatrix} r \\ r' \end{pmatrix}_i$$

Cavity Focusing

What does the radial force look like?

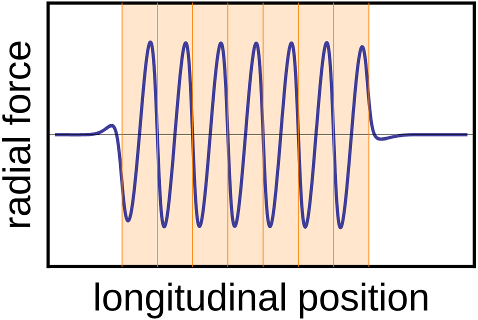
The particle experiences an oscillatory radial force inside the cavity.

$$F_r = q(E_r - v_z B_\phi)$$

radial force
on particle with $\longrightarrow$

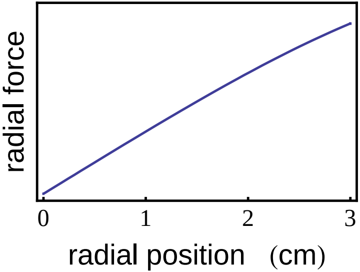
- constant r
- $v_z = c$

Net force $= 0$



But actual particles have changing r...

The radial force is stronger farther from the axis.



How does this affect the particle motion?

Cavity Focusing

r-dependence of the radial force

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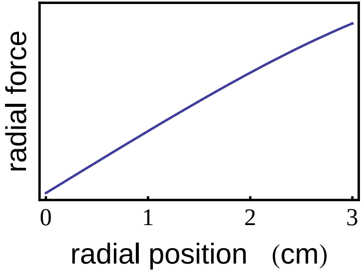

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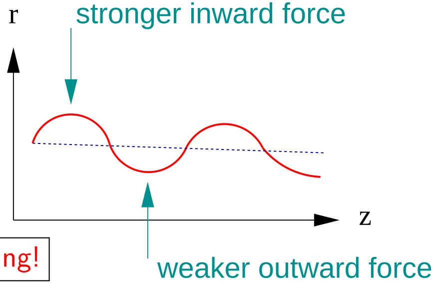
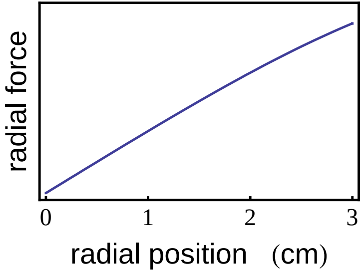


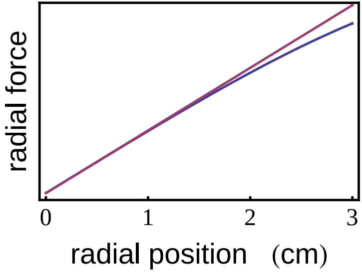
Average force is inward $\Longrightarrow$ Focusing!

The radial force is stronger farther from the axis.



Linear?

The radial force is stronger farther from the axis.



Linear?

Yes, for small enough r

The Beginning

Initial project goals

I started with

- a paper by J. Rosenzweig and L. Serafini
  [Phys. Rev. E **49**, 1599 (1994)]
  in which they derive a transport matrix
- an alternate matrix theory developed by G. Hoffstaetter

I set out to

- compare the two matrices to each other
- compare the results of both matrix theories to actual particle motion

The Middle

How I spent my time

For the past several weeks, I have

- read the paper by Rosenzweig and Serafini
- learned how both derivations go
- written programs in *Mathematica* to
  - read in data about the cavity fields
  - numerically solve equations of motion
  - plot actual and matrix trajectories
- discovered needed adjustments and additions to Georg's theory

The End

What I found

The Rosenzweig and Serafini matrix:

$$\begin{pmatrix} \cos\left(\frac{\sqrt{\eta/8}}{\cos(\Delta\phi)}\ln\frac{\gamma_f}{\gamma_i}\right) & \frac{\cos(\Delta\phi)}{\sqrt{\eta/8}}\frac{\gamma_i}{\gamma'}\sin\left(\frac{\sqrt{\eta/8}}{\cos(\Delta\phi)}\ln\frac{\gamma_f}{\gamma_i}\right) \\ -\frac{\sqrt{\eta/8}}{\cos(\Delta\phi)}\frac{\gamma'}{\gamma_f}\sin\left(\frac{\sqrt{\eta/8}}{\cos(\Delta\phi)}\ln\frac{\gamma_f}{\gamma_i}\right) & \frac{\gamma_i}{\gamma_f}\cos\left(\frac{\sqrt{\eta/8}}{\cos(\Delta\phi)}\ln\frac{\gamma_f}{\gamma_i}\right) \end{pmatrix}$$

The Hoffstaetter matrix:

$$\begin{pmatrix} \cos\left(\epsilon\ln\frac{p_f}{p_i}\right) & \frac{1}{\epsilon}\frac{p_i}{p'}\sin\left(\epsilon\ln\frac{p_f}{p_i}\right) \\ -\epsilon\frac{p'}{p_f}\sin\left(\epsilon\ln\frac{p_f}{p_i}\right) & \frac{p_i}{p_f}\cos\left(\epsilon\ln\frac{p_f}{p_i}\right) \end{pmatrix}$$

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Are they the same?

- Does $\epsilon = \frac{\sqrt{\eta/8}}{\cos(\Delta\phi)}$?
- Does $p = \gamma$?

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The End

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Are they the same?

- Does $\epsilon = \frac{\sqrt{\eta/8}}{\cos(\Delta\phi)}$? Yes
- Does $p = \gamma$? No, but they are proportional $p = \gamma m v$

That's not the end of the story!

It is the central matrices that are the same,
but you also need edge matrices for the cavity entrance and exit

- Georg's theory did not originally include this
- we don't have it fully worked out yet

Possibilities for the future

- find better way to deal with the cavity entrance/exit
- see where the high γ approximation breaks down
  - improve by including $\frac{1}{\gamma^2}$ terms?
- check with ERL injector parameters
  - and compare to real-life measurements

Acknowledgments

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