Evaluation of a New Cavity Focusing Theory

Robin Bjorkquist

Reed College

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Mentor: Georg Hoffstaetter

Cornell University

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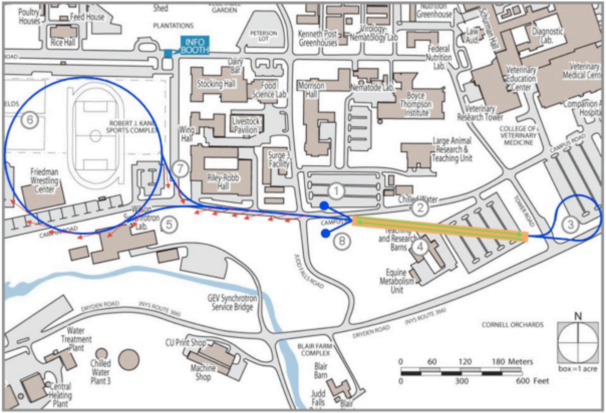
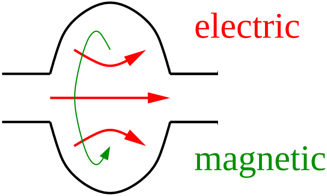
Outline

1. How accelerating cavities work
2. Beam size, and why it's important
3. Cavity focusing
4. My project



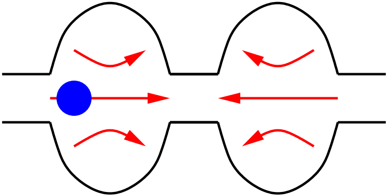
Accelerating Cavities



standing electromagnetic wave in cavity

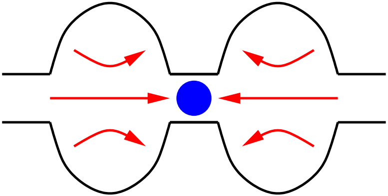
- many modes are possible
- selectively excite the most useful

Accelerating Cavities



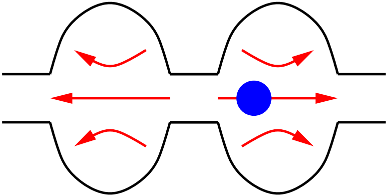
electric field
accelerates particle

Accelerating Cavities



oh no...

Accelerating Cavities

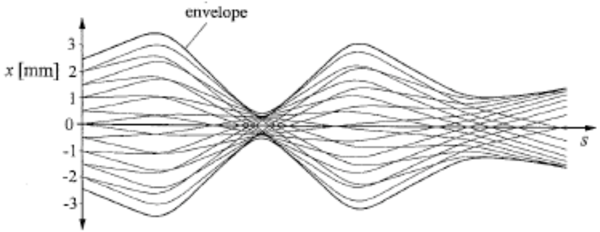


the fields oscillate
(perfect timing)

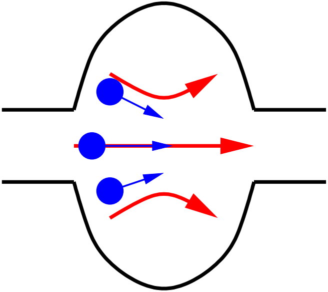
The Beam

Every accelerator has a design path for particles, the orbit.

Deviations from the orbit determine the size of the beam.



Cavity Focusing



What happens to the beam?

In principle, it is easy to find out:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

solve the differential equation

Numerical integration

$$\begin{pmatrix} x_0 \\ v_0 \end{pmatrix} \longrightarrow \begin{pmatrix} x_1 \\ v_1 \end{pmatrix} = \begin{pmatrix} x_0 + v_0 \Delta t \\ v_0 + a(x_0, t_0)\Delta t \end{pmatrix} \longrightarrow \cdots$$

Analytic Solution

Assumptions and approximations

- linearization
- high energy
- small energy change per cavity

Resulting equations

$$\begin{pmatrix} r \\ r' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \epsilon \frac{p'}{p} \end{pmatrix} \begin{pmatrix} \cos\left(\epsilon \ln \frac{p}{p_i}\right) & \sin\left(\epsilon \ln \frac{p}{p_i}\right) \\ -\sin\left(\epsilon \ln \frac{p}{p_i}\right) & \cos\left(\epsilon \ln \frac{p}{p_i}\right) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{\epsilon} \frac{p_i}{p'} \end{pmatrix} \begin{pmatrix} r_i \\ r_i' \end{pmatrix}$$

- p is the design momentum

$$\epsilon^2 = \frac{1}{4p^2} \left(\frac{\langle p'^2 \rangle}{\langle p' \rangle^2} - 1 \right)$$

Project Goals

1. Use Mathematica and actual cavity fields to find particle trajectories for a variety of initial conditions
2. Compare to Georg's matrix theory
3. Compare to an earlier, somewhat unsatisfactory theory