Optimization of Elliptical SRF Cavities where $v < c$

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Why $v < c$?

- Acceleration of large subatomic particles
- Accelerator driven systems (ADS)
  - Neutron Spallation
  - Tritium production
  - Nuclear waste transmutation

INFN Milano Cavity, $v/c = 0.5$
Elliptical Cell Geometry

Non-reentrant ($\alpha > 90^\circ$)  
Reentrant ($\alpha < 90^\circ$)

Geometric Constraints
- Half-Cell Length, $L$
- Wall Angle, $\alpha$
- Equatorial Radius, $R_{eq}$
- Aperture Radius, $R_\alpha$

Free Parameters
- Equator Ellipse Axes
  - $A$ and $B$
- Iris Ellipse Axes
  - $a$ and $b$
Geometric Constraints

Half-Cell Length, $L$

- Constrained by mode of operation
  - In-phase mode
  - $\pi$ mode

Wall Angle, $\alpha$

- Constrained by chemical treatment method
  - Non-reentrant
  - Reentrant
Geometric Constraints (cont.)

Aperture Radius, $R_a$
- Propagation of higher-order modes (HOMs)
  \[ f_{cutoff} \propto 1/R_a \]
  - Removed by resistive loads
- Power left in cavity by wakefields
  \[ P \propto 1/R_a \]
- Cell-to-cell coupling in multi-cell cavities

Equatorial Radius, $R_{eq}$
- Tuned to make the frequency of $TM_{01}$ equal to the driving frequency

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Peak Fields

Magnetic Quenching

- Superconductor enters a normal conducting state
  - Magnetic field changes too rapidly
  - Magnetic field is too strong

- Causes heating of the material
  - Spreads the region of normal conductivity

Field Emission

- Electrons are emitted from the superconductor
  - Electric field is too large

- Threshold raised by heat treatment
Numerical Simulation

SUPERLANS

- Simulation for axially symmetric cavities

TunedCell

- Wrapper code for SUPERLANS
  - Adjusts $R_{leq}$ to make the frequency of TM01 equal to the driving frequency
  - Creates geometry file for SUPERLANS
  - Linearly varies free parameters
### Cavity Optimization

#### Goal of Optimization
- **Minimize** $B_{pk}/E_{acc}$ (and equivalently $H_{pk}/E_{acc}$)

- **Optimization constraints**
  - Minimum wall angle, $\alpha$
  - Maximum $E_{pk}/E_{acc}$
  - Minimum radius of curvature of the cell (two times the Niobium sheet thickness $\approx 6$ mm)

#### Cavity Optimizer
- Matlab wrapper code for TunedCell
- Minimizes $B_{pk}/E_{acc}$
- Enforces geometric and electromagnetic constraints

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Multi-Cell Cavity Optimization

Optimization by V. Shemelin

- Reducing wall angle reduces minimum $H_{pk}/E_{acc}$

Optimization when $\beta = \nu/c < 1$

- Same trend for $\beta < 1$
- Increasing $\beta$ increases minimum $H_{pk}/E_{acc}$
Free Parameters

- Equator Ellipse Ratio, $R = B/A$
- Iris Ellipse Ratio, $r = b/a$
- Wall Distance, $d$
- Wall Angle, $\alpha$

- Produces a minimum $E_{plk}$ and $E_{acc}$ for a given $R$, $d$ and $\alpha$
- Increasing wall angle increases optimal iris ellipse ratio

- Increasing wall distance increases optimal iris ellipse ratio
Bhabha Atomic Research Center (BARC)

BARC Optimization

- Single-cell cavity
  - $\beta=0.49$
  - $A=B=20$ mm
  - $a/b=0.7$
  - $R\perp a=39$ mm

Multi-Cell Boundary Conditions

- Qualitatively similar
- Differences attributed to
  - Different levels of free parameter accuracy
  - Different simulation codes (SUPERLANS vs. SUPERFISH)
BARC Verification

**Single-Cell Boundary Conditions**

- Clear minimum in $E_{pk} / E_{acc}$
- Lower values of $E_{pk} / E_{acc}$ and $B_{pk} / E_{acc}$

**Multi-Cell Boundary Conditions**
# BARC Improvement

<table>
<thead>
<tr>
<th>BARC Optimization Results</th>
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<tbody>
<tr>
<td><strong>Free Parameters</strong></td>
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<tr>
<td>$A=20$ mm</td>
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<tr>
<td>$B=20$ mm</td>
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<tr>
<td>$a/b = 0.7$</td>
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<tr>
<td>$\alpha = 96.5^\circ$</td>
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<tr>
<td><strong>Free Parameters</strong></td>
</tr>
<tr>
<td>$A=20.81$ mm</td>
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<tr>
<td>$B=51.3$ mm</td>
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<tr>
<td>$a=10.51$ mm</td>
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<tr>
<td>$b=18.41$ mm</td>
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- Optimized under BARC constraints ($\beta = 0.49$ and $R\perp a = 39$ mm)
- Result for minimum $B_{pk}/E_{acc}$
Single-Cell Cavity Length

Half-Cell Length

- Half-wavelength cell
  - \( L = \frac{v}{4f} \)
  - \( L = \beta \frac{\omega}{c} \frac{c}{4f} \)

Beam Pipe Fields

- Electric field decays exponentially into the beam pipe

\[ E_r = E_0 e^{-\frac{r}{L}} \]
Scaled Cavity Length

- Reducing cavity length decreases $B_{pk}/E_{acc}$
- Reduction from BARC design
  - $B_{pk}/E_{acc}$ by 8%
  - $E_{pk}/E_{acc}$ by 17.8%
Future Work

- Continue optimization of cavities with $\beta < 1$
  - Prove reentrant shape is ineffective

- Optimize the shape and length of single-cell cavity with record setting accelerating gradient
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