Characterization of Equilibrium Emittance with Monte Carlo

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At the CesrTA run in April 2012, the zero-current bunch width for a single electron bunch was measured to be approximately $247\mu m$ which corresponds to a horizontal emittance of 7 nm while analytic calculations predicted 2.6 nm. In order to better understand this discrepancy Monte Carlo Simulations were used to simulate a bunch traveling through the lattice. A simplified lattice was developed and used for majority of tests.The contributions to the emittance from the wigglers in Monte Carlo were studied and other tests such as placing the radio frequency (RF) cavities in dispersive regions were considered as well. It was found that adding periodic noise to the magnet strength of a simple lattice does not significantly affect the outcome of the equilibrium emittance in Monte Carlo Simulations.

I. INTRODUCTION

The Cornell Electron Storage Ring Test Accelerator (CesrTA) is a testbed for the damping ring needed for the proposed International Linear Collider (ILC). CesrTA has a 768 meter circumference and typically operates at 2 GeV when running in low-emittance conditions. During the April 2012 run the horizontal size of a single electron bunch circulating in the ring was measured with an interferometer. The horizontal emittance, which is analogous to the width of the bunch, for a well-corrected machine was found to be approximately 7 nm[1]. However these measurements disagreed with analytic computations done by Tao, a program developed using Bmad. Bmad is a subroutine library for relativistic charged particle simulations in high energy accelerators and storage rings[2]. Tao is a general purpose program that reads in lattice files and tracks particles according to analytic calculations; it reports basic machine parameters and other commonly sought information about a lattice[3]. Tao reported an ideal horizontal emittance of 2.6 nm which is less than half the emittance measured.

Monte Carlo simulations were introduced to better understand the behavior of the CesrTA lattice. After weeks of testing the CesrTA ideal lattice, Monte Carlo and Tao were not found to be in an acceptable range of agreement. A candidate for the cause of this disagreement was the wiggler tracking model and so investigations into the different types of models was done. To eliminate complexities due to wiggler modeling, a simple lattice was introduced to minimize complications from non-linear elements. The simple lattice is a stripped down structure and only includes dipoles, quadrupoles, sextupoles, RF cavities, and drifts which are needed to keep the beam stored in the ring. Using the simple lattice allowed elements to be added or moved relatively easily. Once it was determined that Monte Carlo agreed within 5% of Tao for the simple lattice, further effects were studied including how adding periodic noise to either the quadrupoles or dipoles could affect the measured emittance for a simple lattice.

II. METHODS

A. Simulation Methods

Monte Carlo Simulations are done by generating an electron bunch with a gaussian distribution of macro particles, each of which represent millions of electrons. Those particles are tracked as they go through the magnets, RF cavities, drift spaces, and other elements which comprise the lattice structure of the ring. When the particle enters a bend, it radiates energy in the transverse direction due to random emission of photons. This energy is restored when the particle enters the RF cavities but only in the longitudinal direction. This combination of radiating transverse energy and restoring only longitudinal energy causes horizontal and vertical damping to occur. The emittance is computed based on the evolved distribution of particles using the sigma matrix of the particles. The sigma matrix is used to calculate a new emittance every 500 turns. It is done by computing the average horizontal, vertical, and azimuthal position and momentum of a particle in the bunch and generating a matrix from these. To calculate, for example, the horizontal emittance of the beam the equation [4]

$$\varepsilon_x^2 = < xx >< x'x' > - < xx' >^2 \tag{1}$$

is used, where x is the position and x' is the momentum. The end coordinates of the bunch distribution are then used as the start coordintates of the next turn, therefore tracking one bunch for many turns. This is done for about 100,000 turns, or 5 damping times, and the beam is greater than 99% damped by the end.

In order to verify the analytic calculations done by Tao, a program was developed that would compute the emittance separately from Tao. Bmad subroutines similar to those in Tao were used but the calculation of the emittance was done manually by the program developed in Fortran90. This provided the flexibility to change computations and better understand certain parts of the process. For example, the contribution to the emittance due to the wigglers and how to slice and step through a wiggler was heavily considered in this program. This Bmad program used an element summation to calculate the radiation integrals and horizontal emittance. For each element in the lattice it calculates the contribution to the radiation integrals and uses the sum of these contributions to comput the emittance.

B. Wiggler Tracking Models

There were struggles to match the emittances reported by Monte Carlo simulations with those reported by Tao for the ideal CesrTA lattice. As shown in Table 1, different methods of computation for the wigglers led to different equilibrium emittances. The wigglers are a series of close alternating dipoles, whose purpose is to bend the beam back and forth fairly quickly so that the electrons emit photons and release copious amounts of radiation. 90% of the energy radiated is lost in the wigglers which is then restored by the RF cavities. Because most of the energy lost is from the wigglers, it is very important that the wiggler model is accurate when computing the emittance of the beam. There are various ways to model a wiggler, including Taylor mapping, symplectic lie tracking, and bend drift bend modeling. In each method, radiation emission is handled differently. When using a Taylor map, half the radiation is emitted at the beginning and end of each wiggler. A more accurate version, symplectic lie tracking, splits the wiggler into 10s of slices per wiggler period and releases 50% of the energy at the beginning and end of each slice. This makes it a closer model of the continuous radiation emission in the actual wigglers. The bend drift bend (BDB) model is the most straightforward because it only includes the bends and drifts in the wigglers but also lacks other important field features and is the least accurate of the three methods. While none of these methods gave Monte Carlo agreeable results with Tao for the ideal CesrTA lattice, the symplectic lie tracking gave emittances consistent with Tao for the simple lattice.

TABLE I: Different types of Wiggler models for an ideal CesrTA lattice

Type of model	Horizontal Emittance	% error from Tao
BDB	5.13 nm	93.7%
Taylor Map	4.28 nm	64.6%
Symplectic Lie Tracking	3.28 nm	26.2%
Tao	2.6 nm	N/A

C. Simple Lattice

Due to the disagreement of Monte Carlo with Tao for an ideal CesrTA lattice, the simple lattice was introduced to minimize the difficulties of using a complicated lattice like Cesr. The advantage of using the simple lattice is that it does not have any of the non-linear elements (besides sextupoles) which can complicate the dispersion and beta functions, both of which are important when computing the emittance of the beam. As shown in 1, the dispersion and beta functions for the simple lattice with one wiggler is periodic and easily examined whereas for the ideal CesrTA lattice they are complicated and non-uniform. It was also easier to include additional effects to study, for example misalignments and corrections.

III. RESULTS

As shown in Table 2, Monte Carlo and Tao were found to agree well for the simple lattices. The element summation program, developed with Bmad agreed well for machines run at 5.3 GeV however when the 2 GeV machine was introduced they did not agree. The simple lattice was found to have an equilibrium emittance of 14.51 nm by Tao. The average equilibrium emittance for Monte Carlo was found to be 15.1 +/- 0.2 nm. An example of typical data collected for a simple lattice is shown in 2. The average emittance from 15 initial random seeds was used as the basis for comparison when periodic noise was added to the quadrupoles and dipoles in the simple lattice.

A. Dispersion in RF cavities

Dispersion, η, means that the energy a particle has affects how it will bend in a magnet. In regions with dispersion, if a particle changes energy (either through photon emission or passing through an RF cavity), its trajectory will change. Because the RF cavities in CesrTA are located in a high dispersion area, it was important to look into their contribution

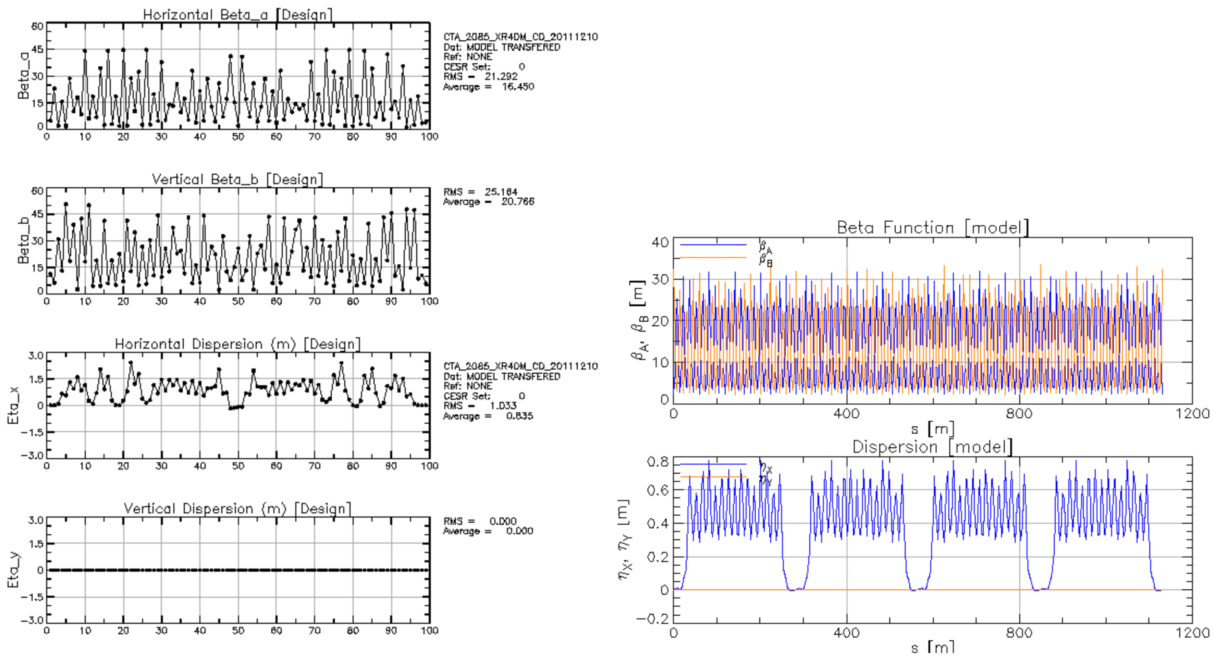

FIG. 1: x and y Dispersion and Beta functions computed by Tao. Left plot is for an ideal CesrTA lattice. Right plot is for a simple lattice with 1 wiggler.

TABLE II: Horizontal Equilibrium Emittance for Simple Lattices

Energy(GeV), # wigglers	Tao	Monte Carlo	Element Sum	MC % err from Tao	ES % err from Tao
(5.3 GeV)	14.51 nm	15.1 nm	14.57 nm	4.1%	0.41%
(5.3) 1 wigg	7.422	7.09	7.3728	4.5%	0.66%
(5.3) 8 wiggs	3.99	3.98	3.8727	0.15%	3.0%
(2 GeV)	2.255	2.27	2.264	0.84%	0.4%
(2) 1 wigg	1.510	1.48	1.1817	1.8%	20.1%
(2) 8 wiggs	1.609	1.68	.962	4.9%	40%

to the emittance. Since dispersion is a transverse-longitudinal coupling, it is possible that the effect of having dispersion cavities corresponds to the emittance observed in the April runs. The simple lattice was altered slightly to introduce the RF cavities into a dispersive region, and was run with Monte Carlo and Tao, as shown in 3. The overlapping data sets are x and y modes and a and b modes, and basically refer to different coordinate definitions. The normal mode (a and b) were included because that is what Tao reports in. In this case, Tao reported 20.6 nm, more than twice what Monte Carlo did.

B. Periodic Noise

The idea of testing periodic noise is to see if introducing a time dependent signal to the lattice will affect the emittance. This is done by taking a Fast Fourier Transform (FFT) of the time dependent signal to produce its frequency spectrum, whose frequencies are chosen in the parameter input for the lattice. The amplitude for the frequencies can be chosen

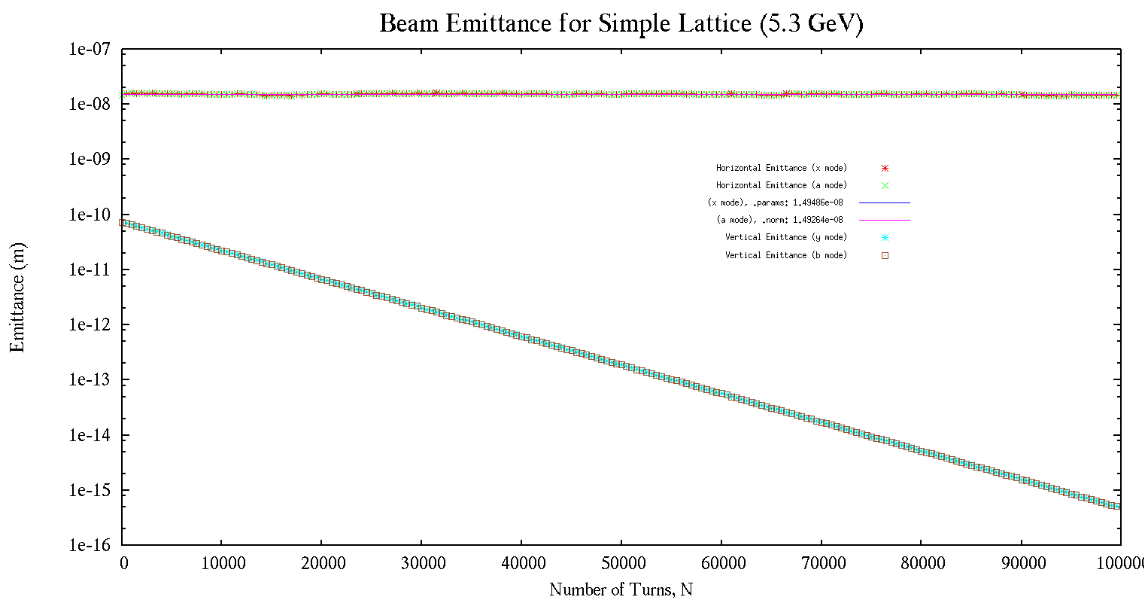

FIG. 2: Log plot for equilibrium emittances for a simple lattice. Monte Carlo reports 14.95 nm for this run while Tao reports 14.51 nm for a simple lattice.

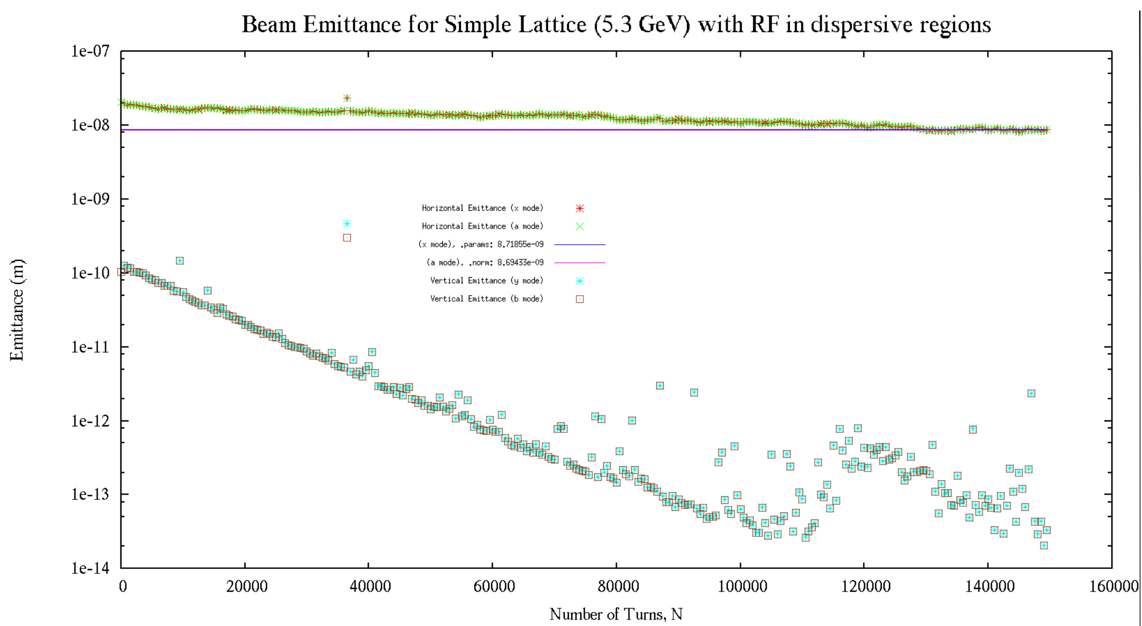

FIG. 3: Log plot for equilibrium emittances for a Simple lattice with dispersion in RF cavities. The equilibrium emittance from Monte Carlo was 8.7 nm while Tao reported 20.6 nm.

and harmonics can be added if desired. Frequencies of 60 Hz with amplitudes ranging from 0.0-2.0% in the quadrupole and dipole strength separately and tested for both the simple lattice and the simple lattice with misalignments and corrections. A 360 Hz signal in the quadrupoles and dipoles at similar amplitude ranges was done for the simple lattice as well.

Adding periodic noise in the quadrupoles of a simple lattice, for example, did change the equilibrium emittance more than one σ, however, when considering the difference between

Monte Carlo and Tao it seems that these changes might be due to random initial distributions in Monte Carlo, rather than an effect from the noise. In Figure 4, a 60 Hz noise signal was added to the dipole strength in a misaligned lattice. While it does seem that adding noise affects the horizontal equilibrium emittance, an average for the misaligned lattice was not computed so the scatter in the points could be explained by the error in Monte Carlo itself.

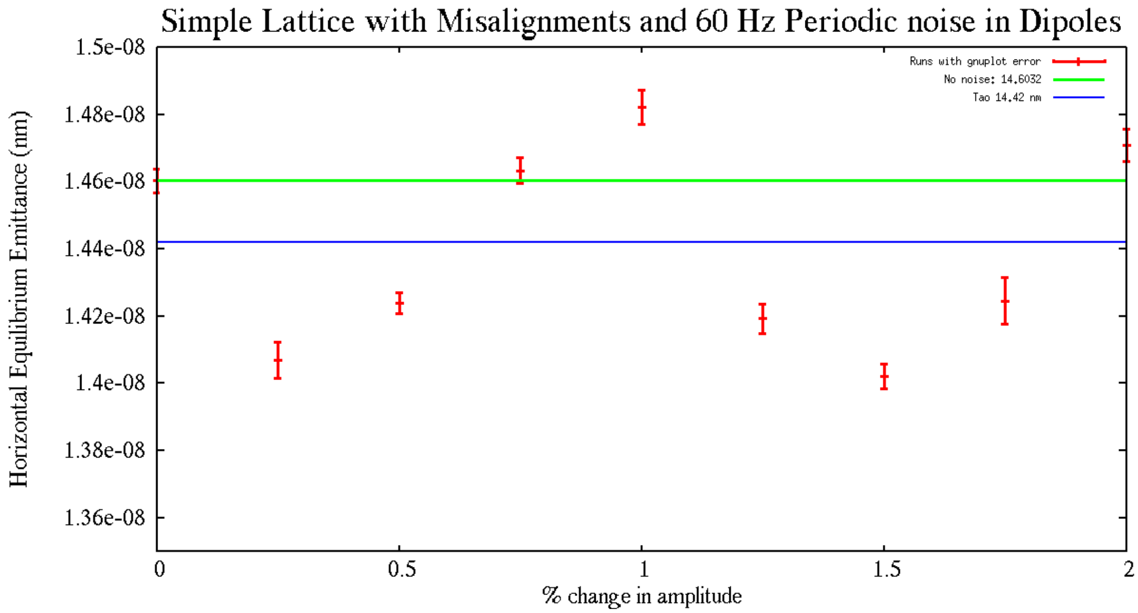

FIG. 4: 60 Hz noise added to dipole strength in a simple lattice with misalignments. Note the scale on the emittance axis is zoomed in to include only a small range.

IV. DISCUSSIONS

For the simple lattice with no wigglers or additional features, Tao and Monte Carlo agreed very well. It was also shown in the simple lattice that the wigglers could not contribute more that 5% error in a simple lattice and by inference, are not a likely candidate for the observed discrepancies between Tao and Monte Carlo in the ideal CesrTA lattice. The leading candidate for a 26.6% discrepancy could be dispersion in the RF cavities. While Monte Carlo produced unexpected lower emittances when RF cavities were placed in regions of dispersion, this suggests that RF cavities in dispersive regions play an important role in computing equilibrium emittances, though more simulations need to be run to make better conclusions.

One way to make more informed conclusions about effects on the equilibrium emittance of the electron beam would be to use an ideal CesrTA lattice like the one that ran in April. Since a simple lattice was used for these studies, only definite conclusions about effects to the simple lattice can be made. Also, in order to better understand the results of adding other effects to a lattice, better understanding of the average equilibrum emittance for that lattice should be computed in order to determine whether observed outcomes are just due to the random initial distribution of particles that changes from run to run. Further investigations into the Bmad element summation program could be pursued in order to better calculate the horizontal emittance.

Suggestions for future research include examining the dispersion in the RF cavities and determining if their location in the lattice is significant. By placing the RF cavities in a low or high dispersion region, effects on the emittance, if any, could be studied. Another task for consideration is to change the tracking method in Monte Carlo. Currently photons that are emitted by particles passing through the bending magnets are only emitted in the horizontal plane, that is the same plane the particles are circulating in. This causes the beam to emit what looks like a fan of radiation as it passes through. However, the photons emitted actually have a vertical component to them so that the beam produces more of a cone of radiation. This could be incorporated into the Monte Carlo to better the tracking simulation. After values reported by Tao and Monte Carlo agree for more lattices, further work can be done into examining the measured emittances from April.

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