f_1-exchange effects in vector-meson production #5

Asymmetries in decay angular distribution

In this note, we review the formalism of vector-meson decay distribution in vector-meson photoproduction from the nucleon targets in order to verify the different roles of the Pomeron- and f_1-trajectories.

1 Coordinate Systems

Here we start with the formula of Ref. [1]. In this formalism, we assume that the target nucleon is unpolarized and the polarization of the recoiling nucleon is not observed so that we are interested in the vector-meson production by polarized photons.

In this Section, we start with our coordinate systems and definitions. First, we use q and q_V for the momenta of the incoming photon and outgoing vector-meson, respectively, in CM frame. The three-vectors $\mathbf{q}$ and $\mathbf{q}_V$ define a right-handed coordinate system;

$$\hat{Z} = \frac{\mathbf{q}}{|\mathbf{q}|}, \qquad \hat{Y} = \frac{\mathbf{q} \times \mathbf{q}_V}{|\mathbf{q} \times \mathbf{q}_V|}, \qquad \hat{X} = \frac{(\mathbf{q} \times \mathbf{q}_V) \times \mathbf{q}}{|(\mathbf{q} \times \mathbf{q}_V) \times \mathbf{q}|}. \tag{5.1}$$

The scattering angle in CM frame is defined to be Θ. We should be careful since there are many angles in our systems.

The density matrix of the produced vector-meson reads

$$\rho_{\lambda_V \lambda'_V} = \frac{1}{N} \sum_{\lambda'_N, \lambda_\gamma, \lambda_N, \lambda'_\gamma} T_{\lambda_V \lambda'_N, \lambda_\gamma \lambda_N} \rho(\gamma)_{\lambda_\gamma \lambda'_\gamma} T^*_{\lambda'_V \lambda'_N, \lambda'_\gamma \lambda_N}, \tag{5.2}$$

where N is the normalization factor defined as

$$N = \frac{1}{2} \sum_{\lambda\text{'s}} |T_{\lambda_V \lambda'_N, \lambda_\gamma \lambda_N}|^2. \tag{5.3}$$

The normalization of the T matrix can be chosen, e.g., such as

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol.}} = \left(\frac{2\pi}{q}\right)^2 \frac{1}{4} \sum_{\lambda\text{'s}} |T_{\lambda_V \lambda'_N, \lambda_\gamma \lambda_N}|^2. \tag{5.4}$$

Note that the helicity amplitude T is a function of Θ as well as W.

Now we define the frame for the vector-meson decay distribution. Here we consider the two-particle decay of the vector-meson such as $\rho \to \pi\pi$ and $\phi \to K\bar{K}$. In order to compute the vector-meson decay distribution, we first have to fix our system since it is convenient to use the

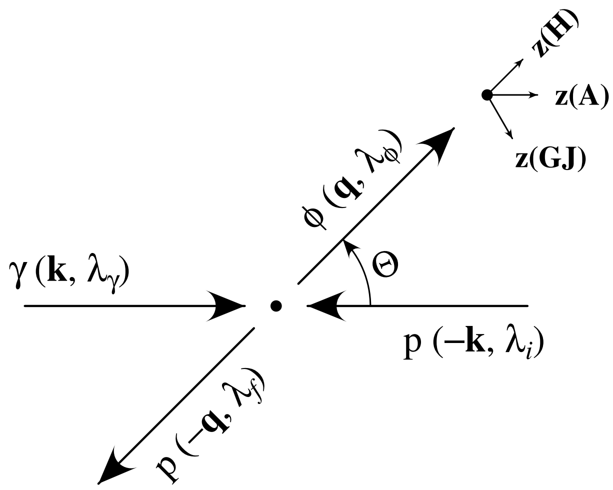

Figure 1: The z-axis of the three reference frames in the vector-meson rest frame.

vector-meson rest frame for this purpose. Usually, there are three systems in the literature for describing the vector-meson decay, which was produced by photons from the nucleon targets. (See Refs. [1, 2].) They are Gottfried-Jackson system, helicity frame, and Adair system. These three reference systems differ in the choice of the spin-quantization axis (i.e., z axis). See Fig. 1.

Gottfried-Jackson (GJ) system: The z axis is the direction of the incident photon in the vector-meson rest frame.

Helicity (H) system: The z axis is the direction of the vector-meson in the γp c.m. system, i.e., opposite to the direction of the outgoing proton in the vector-meson rest frame.

Adair (A) system: The z axis is along the direction of the incident photon in the γp c.m. system.

We will use the GJ system which is favored by the literature. In this system, the polarization vectors of the photon and vector-meson have their simplest form, i.e., no angle dependence. So the $\hat{z}$-axis of the VM rest frame is fixed by working with the GJ system. The y axis is always normal to the production plane, defined by $\hat{q} \times \hat{q}_V$, and the x axis is given by $\hat{x} = \hat{y} \times \hat{z}$. Since we will use linearly polarized photon beams, we need to specify the angle Φ between the photon polarization vector $\hat{\epsilon}$ and the production plane in the total c.m. system. It is defined by

$$\sin\Phi = \hat{y} \cdot \hat{\epsilon} \tag{5.5}$$

with the sign ambiguity resolved by

$$\cos\Phi = \hat{\epsilon} \cdot (\hat{y} \times \hat{q}). \tag{5.6}$$

Now we define the decay angles: θ and φ are the polar and azimuthal angles of the direction of flight $\hat{\pi}$ of the K-meson (or π meson) in the ϕ (ρ) rest frame:

$$\begin{aligned} \cos\theta &= \hat{\pi} \cdot \hat{z}, \\ \cos\varphi &= \hat{y} \cdot (\hat{z} \times \hat{\pi})/|\hat{z} \times \hat{\pi}|, \\ \sin\varphi &= -\hat{x} \cdot (\hat{z} \times \hat{\pi})/|\hat{z} \times \hat{\pi}|. \end{aligned} \tag{5.7}$$

2 Decay Angular Distribution

The decay angular distribution of the vector-meson in its rest frame is defined as

$$\begin{aligned}\frac{d\mathcal{N}}{d\cos\theta d\varphi} &\equiv W(\cos\theta,\varphi) = M\rho M^\dagger \\ &= \sum_{\lambda_V,\lambda'_V} \langle\theta,\varphi|M|\lambda_V\rangle \rho_{\lambda_V\,\lambda'_V} \langle\lambda'_V|M^\dagger|\theta,\varphi\rangle,\end{aligned} \tag{5.8}$$

where λ_V is the helicity of the vector-meson, ρ is the density matrix of the vector-meson and M is the decay amplitude which can be written as

$$\langle\theta,\varphi|M|\lambda_V\rangle = C\sqrt{\frac{3}{4\pi}} D^{1*}_{\lambda_V\,0}(\varphi,\theta,-\varphi), \tag{5.9}$$

when the vector-meson decays into spinless particles. (See Appendix A for details.) W is also a function of t although not written explicitly. Here $|C|^2$ is proportional to the vector-meson decay width independent of λ_V because of rotation invariance. Since we are considering a normalized decay angular distribution we can set $C = 1$ without loss of generality. The Wigner rotation functions are

$$\begin{aligned}D^1_{10}(\varphi,\theta,-\varphi) &= -\frac{1}{\sqrt{2}}\sin\theta e^{-i\varphi}, \\ D^1_{00}(\varphi,\theta,-\varphi) &= \cos\theta, \\ D^1_{-10}(\varphi,\theta,-\varphi) &= \frac{1}{\sqrt{2}}\sin\theta e^{i\varphi}.\end{aligned} \tag{5.10}$$

Thus we have

$$W(\cos\theta,\varphi) = \frac{3}{4\pi}\sum_{\lambda_V,\lambda'_V} D^{1*}_{\lambda_V\,0}(\varphi,\theta,-\varphi)\rho_{\lambda_V\lambda'_V} D^1_{\lambda'_V\,0}(\varphi,\theta,-\varphi). \tag{5.11}$$

Since the ρ is hermitian, $\rho_{\lambda_V\lambda'_V} = \rho^*_{\lambda'_V\lambda_V}$, we have

$$\begin{aligned}W(\cos\theta,\varphi) = \frac{3}{4\pi}\Big\{&\frac{1}{2}(\rho_{11}+\rho_{-1-1})\sin^2\theta + \rho_{00}\cos^2\theta \\ &-\frac{1}{\sqrt{2}}\,\mathrm{Re}\,(\rho_{10}-\rho_{-10})\sin 2\theta\cos\varphi + \frac{1}{\sqrt{2}}\,\mathrm{Im}\,(\rho_{10}+\rho_{-10})\sin 2\theta\sin\varphi \\ &-\mathrm{Re}\,(\rho_{1-1})\sin^2\theta\cos 2\varphi + \mathrm{Im}\,(\rho_{1-1})\sin^2\theta\sin 2\varphi\Big\},\end{aligned} \tag{5.12}$$

which is the decay angular distribution of the vector-meson.

Next we consider the case with polarized photon beams. We start with the photon density matrix,

$$\rho(\gamma) = \frac{1}{2}(1 + \mathbf{P}_\gamma\cdot\boldsymbol{\sigma}), \tag{5.13}$$

where $\boldsymbol{\sigma}$ is the Pauli matrix and $\mathbf{P}_\gamma$ specifies the photon polarization, e.g.,

$$\mathbf{P}_\gamma = p_\gamma(0, 0, \pm 1), \tag{5.14}$$

for circular polarization with $\lambda_\gamma = \pm 1$, and

$$\mathbf{P}_\gamma = p_\gamma(-\cos 2\Phi, -\sin 2\Phi, 0), \tag{5.15}$$

for linearly polarized photons where Φ denotes the angle between the photon polarization vector and the vector-meson production plane (i.e., xz plane), which leads to $\boldsymbol{\varepsilon} = (\cos\Phi, \sin\Phi, 0)$. In the above expressions, p_γ denotes the strength of the polarization ($0 \le p_\gamma \le 1$).

We will also make use of the symmetry property of the helicity amplitudes,

$$T(\Theta)_{-\lambda_V -\lambda'_N, -\lambda_\gamma -\lambda_N} = (-1)^{(\lambda_V - \lambda'_N) - (\lambda_\gamma - \lambda_N)} T(\Theta)_{\lambda_V \lambda'_N, \lambda_\gamma \lambda_N}, \tag{5.16}$$

where Θ is the c.m. production (scattering) angle.

From Eq. (5.13), we can decompose the vector-meson density matrix ρ as

$$\rho = \rho^0 + \sum_{\alpha=1}^{3} P_\gamma^\alpha \rho^\alpha, \tag{5.17}$$

where

$$\rho^0 = \frac{1}{2} T T^\dagger, \qquad \rho^\alpha = \frac{1}{2} T \sigma^\alpha T^\dagger, \tag{5.18}$$

with $\alpha = (1, 2, 3)$. Explicitly, they are written as

$$\begin{aligned}
\rho^0_{\lambda_V \lambda'_V} &= \frac{1}{2N} \sum_{\lambda_\gamma, \lambda_N, \lambda'_N} T_{\lambda_V \lambda'_N, \lambda_\gamma \lambda_N} T^*_{\lambda'_V \lambda'_N, \lambda_\gamma \lambda_N}, \\
\rho^1_{\lambda_V \lambda'_V} &= \frac{1}{2N} \sum_{\lambda_\gamma, \lambda_N, \lambda'_N} T_{\lambda_V \lambda'_N, -\lambda_\gamma \lambda_N} T^*_{\lambda'_V \lambda'_N, \lambda_\gamma \lambda_N}, \\
\rho^2_{\lambda_V \lambda'_V} &= \frac{i}{2N} \sum_{\lambda_\gamma, \lambda_N, \lambda'_N} \lambda_\gamma T_{\lambda_V \lambda'_N, -\lambda_\gamma \lambda_N} T^*_{\lambda'_V \lambda'_N, \lambda_\gamma \lambda_N}, \\
\rho^3_{\lambda_V \lambda'_V} &= \frac{1}{2N} \sum_{\lambda_\gamma, \lambda_N, \lambda'_N} \lambda_\gamma T_{\lambda_V \lambda'_N, \lambda_\gamma \lambda_N} T^*_{\lambda'_V \lambda'_N, \lambda_\gamma \lambda_N}.
\end{aligned} \tag{5.19}$$

Here N is the normalization factor which gives a constraint $\operatorname{Tr} \rho^0 = 1$,

$$N = \frac{1}{2} \sum_{\lambda_V, \lambda_\gamma, \lambda_N, \lambda'_N} T_{\lambda_V \lambda'_N, \lambda_\gamma \lambda_N} T^*_{\lambda_V \lambda'_N, \lambda_\gamma \lambda_N}. \tag{5.20}$$

Then the parity conservation (5.16) leads to

$$\begin{aligned}
\rho^\alpha_{\lambda\lambda'} &= (-1)^{\lambda - \lambda'} \rho^\alpha_{-\lambda -\lambda'} \qquad \text{for } (\alpha = 0, 1), \\
\rho^\alpha_{\lambda\lambda'} &= -(-1)^{\lambda - \lambda'} \rho^\alpha_{-\lambda -\lambda'} \qquad \text{for } (\alpha = 2, 3),
\end{aligned} \tag{5.21}$$

which shows that ρ^0_{1-1} and ρ^1_{1-1} are real and ρ^2_{1-1} and ρ^3_{1-1} are purely imaginary.

Using Eq. (5.17) we can also decompose the decay distribution as

$$W(\cos\theta,\varphi) = W^0(\cos\theta,\varphi) + \sum_{\alpha=1}^{3} P^\alpha_\gamma \rho^\alpha(\cos\theta,\varphi), \tag{5.22}$$

where W^α is defined as Eq. (5.12) by using ρ^α. To write down the explicit forms of W^α, we use that ρ^0_{1-1} is purely real, $\rho^0_{-1-1} = \rho^0_{11}$ and $\rho^0_{-10} = -\rho^0_{10}$ together with $\mathrm{Tr}\,\rho^0 = 1$. This holds also for ρ^1 except the normalization condition. This procedure leads us to

$$\begin{aligned}
W^0(\cos\theta,\varphi) &= \frac{3}{4\pi}\left\{\frac{1}{2}(1-\rho^0_{00}) + \frac{1}{2}(3\rho^0_{00}-1)\cos^2\theta\right. \\
&\qquad \left. - \sqrt{2}\,\mathrm{Re}\rho^0_{10}\sin 2\theta\cos\varphi - \mathrm{Re}\rho^0_{1-1}\sin^2\theta\cos 2\varphi\right\}, \\
W^1(\cos\theta,\varphi) &= \frac{3}{4\pi}\left\{\rho^1_{11}\sin^2\theta + \rho^1_{00}\cos^2\theta\right. \\
&\qquad \left. - \sqrt{2}\,\mathrm{Re}\rho^1_{10}\sin 2\theta\cos\varphi - \mathrm{Re}\rho^1_{1-1}\sin^2\theta\cos 2\varphi\right\}.
\end{aligned} \tag{5.23}$$

For $\alpha = 2,3$, we have $\rho^\alpha_{-1-1} = -\rho^\alpha_{11}$, $\rho^\alpha_{-10} = \rho^\alpha_{10}$, and $\rho^\alpha_{00} = 0$. Thus we have

$$\begin{aligned}
W^2(\cos\theta,\varphi) &= \frac{3}{4\pi}\left\{\sqrt{2}\,\mathrm{Im}\rho^2_{10}\sin 2\theta\sin\varphi + \mathrm{Im}\rho^2_{1-1}\sin^2\theta\sin 2\varphi\right\}, \\
W^3(\cos\theta,\varphi) &= \frac{3}{4\pi}\left\{\sqrt{2}\,\mathrm{Im}\rho^3_{10}\sin 2\theta\sin\varphi + \mathrm{Im}\rho^3_{1-1}\sin^2\theta\sin 2\varphi\right\}.
\end{aligned} \tag{5.24}$$

This is what we want, i.e., the decay distribution with polarized photon beams.

The next job is to insert various forms of $\mathbf{P}_\gamma$ so that we can obtain the decay distributions according to the photon polarization. All dynamics is in the density matrix and by measuring the decay distribution experimentally, we can obtain the expt. data of the density matrices. Depending on the photon polarization, we get

- unpolarized photons, $\mathbf{P}_\gamma = \mathbf{0}$.
$$W_{\text{unpol.}} = W^0. \tag{5.25}$$

- circulary polarized photons of helicity $\lambda_\gamma = \pm 1$.
$$W^\pm = W^0 \pm p_\gamma W^3(\cos\theta,\varphi). \tag{5.26}$$

- linearly polarized photons.
$$W^L(\cos\theta,\varphi,\Phi) = W^0 - p_\gamma\cos 2\Phi\, W^1(\cos\theta,\varphi) - p_\gamma\sin 2\Phi\, W^2(\cos\theta,\varphi). \tag{5.27}$$

3 Physical Quantities

Since VM production is observed by analyzing the two-mesons, it would be possible to measure the decay distributions. An interesting case is when we use the linearly polarized photon beams at $(\theta = \frac{\pi}{2}, \varphi = \frac{\pi}{2}, \Phi = 0, \frac{\pi}{2})$. Then we can define $\sigma_\parallel$ and $\sigma_\perp$ as the cross sections for symmetric decay particle pairs produced parallel and normal to the photon polarization plane respectively, so $\sigma_\parallel$ $(\sigma_\perp)$ corresponds to the case with $\Phi = \pi/2$ (0). Then the photon polarization asymmetry Σ is defined as

$$\Sigma \equiv \frac{\sigma_\parallel - \sigma_\perp}{\sigma_\parallel + \sigma_\perp} = \frac{1}{p_\gamma} \frac{W^L(0, \frac{\pi}{2}, \frac{\pi}{2}) - W^L(0, \frac{\pi}{2}, 0)}{W^L(0, \frac{\pi}{2}, \frac{\pi}{2}) + W^L(0, \frac{\pi}{2}, 0)}. \tag{5.28}$$

Then using the explicit forms of W^L which we obtained in the previous Section, we have

$$\Sigma = \frac{\rho^1_{11} + \rho^1_{1-1}}{\rho^0_{11} + \rho^0_{1-1}}. \tag{5.29}$$

Note that this is defined with $\cos\theta = 0$ and $\varphi = \frac{\pi}{2}$. An important point here is that one should not take average over the angles since doing so will erase the informations on some density matrix elements.

Another interesting quantity is the parity asymmetry P_σ. This can be defined by decomposing the helicity amplitudes as

$$T = T^N + T^U, \tag{5.30}$$

or

$$T^{N/U}(\Theta)_{\lambda_V \lambda'_N, \lambda_\gamma \lambda_N} = \frac{1}{2} \left\{ T^(\Theta)_{\lambda_V \lambda'_N, \lambda_\gamma \lambda_N} \mp (-1)^\lambda_V T^(\Theta)_{-\lambda_V \lambda'_N, -\lambda_\gamma \lambda_N} \right\}. \tag{5.31}$$

This decomposition is from the observation that if only natural ($P = (-1)^j$) or only unnatural parity ($P = -(-1)^j$) exchanges in the t-channel contributes, one has an additional symmetry to leading order in the incoming photon energy [3],

$$\begin{aligned} T(\Theta)_{-\lambda_V \lambda'_N, -\lambda_\gamma \lambda_N} &= \pm(-1)^{\lambda_V - \lambda_\gamma} T(\Theta)_{\lambda_V \lambda'_N, \lambda_\gamma \lambda_N} \\ &= \mp(-1)^{\lambda_V} T(\Theta)_{\lambda_V \lambda'_N, \lambda_\gamma \lambda_N}, \end{aligned} \tag{5.32}$$

where the upper (lower) sign applies to natural (unnatural) parity exchanges. Then one can write

$$\rho^{0(N/U)}_{\lambda\lambda'} = \frac{1}{2} \left[\rho^0_{\lambda\lambda'} \mp (-1)^\lambda \rho^1_{-\lambda\lambda'} \right]. \tag{5.33}$$

Then the parity asymmetry is defined by σ^N and σ^U which are the contributions of natural and unnatural parity exchanges to the total cross section:

$$P_\sigma \equiv \frac{\sigma^N - \sigma^U}{\sigma^N + \sigma^U} = 2\rho^1_{1-1} - \rho^1_{00}, \tag{5.34}$$

where we have used Eq.(5.33) with $\sigma \propto \mathrm{Tr}\,\rho^0$. So when we have natural parity exchange only, then we get $P_\sigma = +1$ and $P_\sigma = -1$ for unnatural parity exchange only. Note also that this quantity can be defined only in the large energy limit where the decomposition (5.32) is possible.

However, note that Σ is not related unambiguously to natural and unnatural parity exchange. Only if the helicity-flip amplitudes are zero, Σ is equivalent to P_σ [2, 4]. This can be seen as follows. First, using the parity conservation (5.16), we can write

$$\sum_{\lambda'_N,\lambda_N} T_{\lambda_V\lambda'_N,\lambda_\gamma\lambda_N} T^*_{\lambda'_V\lambda'_N,\lambda'_\gamma\lambda_N} = (-1)^{(\lambda_V-\lambda'_V)-(\lambda_\gamma-\lambda'_\gamma)} T_{-\lambda_V\lambda'_N,-\lambda_\gamma\lambda_N} T^*_{-\lambda'_V\lambda'_N,-\lambda'_\gamma\lambda_N}. \tag{5.35}$$

To leading order of the energy of the incoming photon, there arises the additional symmetry (5.32) if only natural or only unnatural parity exchanges contribute in the t channel. From the definition of $\rho^\alpha_{\lambda\lambda'}$ we can obtain the elements of density matrices which appear in W^α as[#1]

$$\begin{aligned}
\rho^0_{00} &= \frac{1}{2N}\left(T_{0+}T^*_{0+} + T_{0-}T^*_{0-}\right), \\
\rho^0_{11} &= \frac{1}{2N}\left(T_{++}T^*_{++} + T_{+-}T^*_{+-}\right), \\
\rho^0_{10} &= \frac{1}{2N}\left(T_{++}T^*_{0+} + T_{+-}T^*_{0-}\right), \\
\rho^0_{1-1} &= \frac{1}{2N}\left(T_{++}T^*_{-+} + T_{+-}T^*_{--}\right), \\
\rho^1_{00} &= \frac{1}{2N}\left(T_{0+}T^*_{0-} + T_{0-}T^*_{0+}\right), \\
\rho^1_{11} &= \frac{1}{2N}\left(T_{++}T^*_{+-} + T_{+-}T^*_{++}\right), \\
\rho^1_{10} &= \frac{1}{2N}\left(T_{++}T^*_{0-} + T_{+-}T^*_{0+}\right), \\
\rho^1_{1-1} &= \frac{1}{2N}\left(T_{++}T^*_{--} + T_{+-}T^*_{-+}\right), \\
\rho^2_{10} &= \frac{i}{2N}\left(T_{+-}T^*_{0+} - T_{++}T^*_{0-}\right), \\
\rho^2_{1-1} &= \frac{i}{2N}\left(T_{+-}T^*_{-+} - T_{++}T^*_{--}\right), \\
\rho^3_{10} &= \frac{1}{2N}\left(T_{++}T^*_{0+} - T_{+-}T^*_{0-}\right), \\
\rho^3_{1-1} &= \frac{1}{2N}\left(T_{++}T^*_{-+} - T_{+-}T^*_{--}\right).
\end{aligned} \tag{5.36}$$

Now if the parity asymmetry $P_\sigma = \pm 1$, we have a condition that

$$2\rho^1_{1-1} - \rho^1_{00} = \pm 1, \tag{5.37}$$

which leads to

$$|T_{++} \mp T_{--}|^2 + |T_{+-} \mp T_{-+}|^2 + |T_{0+} \pm T_{0-}|^2 = 0. \tag{5.38}$$

Thus we find

$$T_{++} = \pm T_{--}, \qquad T_{+-} = \pm T_{-+}, \qquad T_{0+} = \mp T_{0-}, \tag{5.39}$$

[#1] Here our notation is $T_{\lambda_V,\lambda_\gamma}$ and the sum over nucleon helicities are understood.

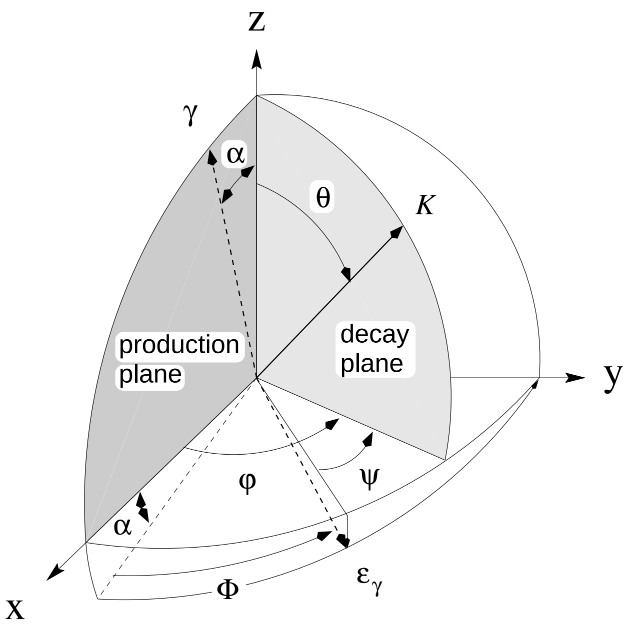

Figure 2: Angles used in the study of vector-meson decay [2]. The angle α is zero in the GJ system. The angle ψ is defined as $\psi = \varphi - \Phi$.

which are identical to the relation (5.32). Then we consider the case that $\Sigma = \pm 1$. Using its definition, we can find that $\Sigma = \pm 1$ gives

$$(T_{++} \mp T_{+-})(T^*_{-+} \mp T^*_{+-}) + (T_{++} \mp T_{+-})(T^*_{++} \mp T^*_{--}) = 0. \tag{5.40}$$

So $P_\sigma = \pm 1$ gives $\Sigma = \pm 1$. But $\Sigma = \pm 1$ can happen when $T_{++} = \pm T_{+-}$. Therefore, Σ can be related to the parity of the t channel exchange only when T_{+-} is zero. Otherwise, even in the case that we have $\Sigma = \pm 1$, we cannot say that the parity of the t channel exchange is natural or unnatural.

4 Helicity Conserving Process

For experimental study, it is more convenient to use the angle $\psi \equiv \varphi - \Phi$ which is the angle between the photon polarization and the vector-meson decay plane in the forward direction. (See Fig. 2.)

The following discussion is from Refs. [2, 5]. If the vector-meson production mechanism conserves s-channel helicity, i.e., the vector-meson is transverse and linearly polarized like the photon,, then in the helicity system

$$\rho^1_{1-1} = -\mathrm{Im}\,\rho^2_{1-1} = \frac{1}{2} \tag{5.41}$$

and all other ρ^α_{ij} appearing in the expression of $W(\cos\theta, \varphi, \Phi)$. If we assume the s-channel helicity conservation,

$$T_{\lambda_V \lambda_\gamma} \propto \delta_{\lambda_V \lambda_\gamma}, \tag{5.42}$$

we obtain the normalization constant as

$$N = \frac{1}{2}\left(T_{++}T_{++}^{*} + T_{--}T_{--}^{*}\right) = T_{++}T_{++}^{*}, \tag{5.43}$$

thanks to the parity relation. The summation over the target and recoil nucleon helicities are suppressed. The explicit expression for the ρ_{ij}^{α} then gives

$$\rho_{11}^{0} = \rho_{1-1}^{1} = -\mathrm{Im}\,\rho_{1-1}^{2} = \frac{1}{2}. \tag{5.44}$$

Thus there is one additional nonvanishing ρ_{ij}^{α}, i.e., $\rho_{11}^{0} = \frac{1}{2}$. But ρ_{11}^{0} does not appear in the expression of $W(\cos\theta, \varphi, \Phi)$. Then from Eq. (5.44), one can easily show that

$$W(\cos\theta, \psi) = \frac{3}{8\pi}\sin^2\theta\,(1 + p_\gamma \cos 2\psi)\,, \tag{5.45}$$

where we have used $\psi = \varphi - \Phi$. So for 100% linearly polarized photons, the decay distribution can be further simplified. As before, if we assume natural parity exchange, then $T_{++} = T_{--}$ and we get $\rho_{1-1}^{1} = -\mathrm{Im}\,\rho_{1-1}^{2} = \frac{1}{2}$. This leads to

$$P_\sigma = +1 \qquad \text{and} \qquad W(\cos\theta, \psi) = \frac{3}{4\pi}\sin^2\theta\cos^2\psi. \tag{5.46}$$

Or if we assume unnatural parity exchange, then $T_{++} = -T_{--}$ and we get $\rho_{1-1}^{1} = -\mathrm{Im}\,\rho_{1-1}^{2} = -\frac{1}{2}$. Then we have

$$P_\sigma = -1 \qquad \text{and} \qquad W(\cos\theta, \psi) = \frac{3}{4\pi}\sin^2\theta\sin^2\psi. \tag{5.47}$$

So one can see that the asymmetry P_σ is directly related to the W distribution.

The analysis of ρ production of Ref. [2] shows that at low $|t|$, say $|t| \leq 0.4$ GeV^2, $P_\sigma \simeq +1$, $\rho_{1-1}^{1} \simeq -\mathrm{Im}\,\rho_{1-1}^{2} \simeq \frac{1}{2}$, and other ρ_{ij}^{α} close to zero. Diviations from these values are expected to come from the background processes such as Drell terms and rescattering terms. But at large momentum transfers, the s-channel helicity conservation does not hold and helicity-flip amplitudes give non-negligible contributions. By analyzing their data, Ballam *et al.* concluded that $P_\sigma \simeq 1$ and $\Sigma \simeq 1$ up to $|t| = 1$ GeV^2. They also found that the helicity-flip amplitudes are at a minimum in the helicity frame.

5 Numerical Results

Since the Pomeron has natural parity while f_1 has unnatural parity, we can expect that $P\sigma$ is a good quantity to distinguish the both processes. From our earlier study, we learned that the Pomeron dominates in the low $|t|$ region, i.e., $|t| \leq 1$ GeV^2 and f_1 dominates at $|t| \geq 1$ GeV^2 region. So we expect that P_σ is nearly $+1$ at $|t| \leq 1$ GeV^2 and -1 at $|t| \geq 1$ GeV^2 and the change of P_σ occurs near $|t| = 1$ GeV^2.

In Fig. 3, we plot the results of P_σ for ρ and ϕ photoproduction with $W = 94$ GeV.

Given in Fig. 4 is the result for the asymmetry Σ, which has the similar behavior with P_σ. This is because the helicity-flip amplitudes are suppressed. (It should be checked)

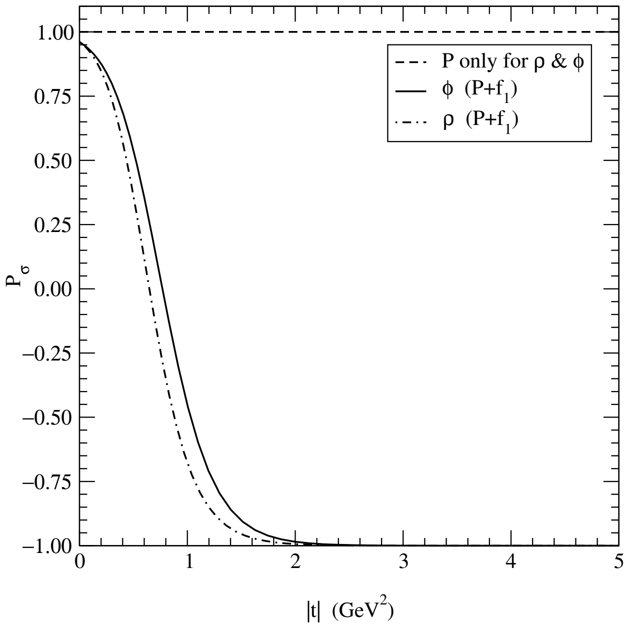

Figure 3: P_σ in vector-meson decays. The dotted line is the Pomeron-model prediction. The solid (dot-dashed) line is the prediction of Pomeron plus f_1 exchange for ϕ (ρ) meson. The initial CM energy is $W = 94$ GeV.

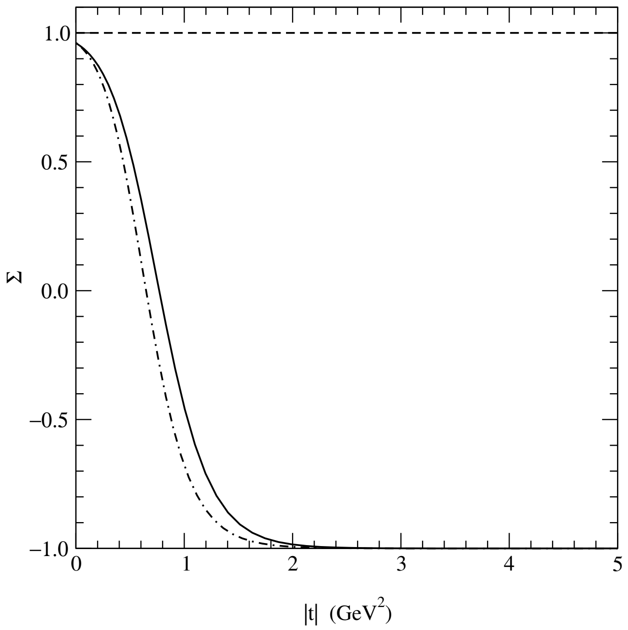

Figure 4: Σ in vector-meson decays. The dotted line is the Pomeron-model prediction. The solid (dot-dashed) line is the prediction of Pomeron plus f_1 exchange for ϕ (ρ) meson. The initial CM energy is $W = 94$ GeV.

A Decay of particles

Here we discuss the formula (5.9) in detail by following the work of Gottfried and Jackson [6]. Studying the decay of resonance is essential since it is the decay of the reaction product which makes $\rho(V)$ accessible to experimental study. Let us consider, therefore, the angular distribution of the decay $d \to \alpha + \beta$, where α and β have spin s_α and s_β, respectively. Let $\mathbf{p}$ and $-\mathbf{p}$ be the momenta of α and β, respectively, in the vector-meson rest frame.

Then the decay amplitudes

$$A_{\lambda_V}(\hat{\mathbf{p}}\lambda_\alpha\lambda_\beta) \equiv \langle\hat{\mathbf{p}}\lambda_\alpha\lambda_\beta|M|s_d\lambda_V\rangle, \tag{5.48}$$

can be written as

$$A_{\lambda_V}(\hat{\mathbf{p}}\lambda_\alpha\lambda_\beta) = \langle\hat{\mathbf{p}}\lambda_\alpha\lambda_\beta|s_d\lambda_V\lambda_\alpha\lambda_\beta\rangle\langle s_d\lambda_V\lambda_\alpha\lambda_\beta|M|s_d\lambda\rangle. \tag{5.49}$$

By making use of the formula [7],

$$\langle\theta,\varphi;\lambda_1\lambda_2|JM;\lambda_1'\lambda_2'\rangle = \sqrt{\frac{(2J+1)}{4\pi}}\delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'}D^{J*}M\Lambda(\varphi,\theta,-\varphi), \tag{5.50}$$

with $\Lambda = \lambda_1 - \lambda_2$, and defining

$$M_d(\lambda_\alpha\lambda_\beta) = \langle s_d\lambda_V\lambda_\alpha\lambda_\beta|M|s_d\lambda\rangle, \tag{5.51}$$

we can write

$$A_{\lambda_V}(\hat{\mathbf{p}}\lambda_\alpha\lambda_\beta) = \sqrt{\frac{2s_d+1}{4\pi}}D^{s_d*}_{\lambda_V\Lambda}(\varphi,\theta,-\varphi)M_d(\lambda_\alpha\lambda_\beta), \tag{5.52}$$

with $\Lambda = \lambda_\alpha - \lambda_\beta$.

There is one more restriction on the amplitude M_d, reflection invariance [7], which reads

$$M_d(\lambda_\alpha\lambda_\beta) = \eta_\alpha\eta_\beta\eta_d(-1)^{s_d-s_\alpha-s_\beta}M_d(-\lambda_\alpha-\lambda_\beta), \tag{5.53}$$

where η's are the intrinsic parity of each particle. Then if α and β are both spinless, or if $s_\alpha = \frac{1}{2}$ and $s_\beta = 0$, there is only one constant M_d characterizing the decay. So in our case with $s_\alpha = s_\beta = 0$, $s_d = 1$ and $\eta_\alpha = \eta_\beta = \eta_d = -1$, we have only one M_d for the decay. In this case, Eq. (5.52) becomes

$$A_m(\hat{\mathbf{p}}) = \sqrt{\frac{3}{4\pi}}D^{1*}_{\lambda_V 0}(\varphi,\theta,-\varphi)M_d, \tag{5.54}$$

which we have used in this note.

From the invariances under reflection, we can have constraints on the decay distribution W. First, the reflection invariance (5.53) leads to

$$W(\theta,\varphi) = W(\pi-\theta,\pi+\varphi). \tag{5.55}$$

Second, the reflection invariance of the production mechanism,

$$\langle m|\rho|m'\rangle = (-1)^{m-m'}\langle -m|\rho|-m'\rangle, \tag{5.56}$$

gives

$$W(\theta,\varphi) = W(\theta,-\varphi). \tag{5.57}$$

B The Söding Model

In dimeson photoproduction, there can be other processes which are not related to the vector-meson production. This is called Söding model [8] and serves as background of dimeson production from vector-meson photoproduction. The typical Söding model is given in Fig. 5. So the matrix element used to describe $\gamma p \to pK\bar{K}$ is the sum of ϕ production, Drell diagrams, and rescattering terms. (We do not consider Δ production piece.) We are interested in the process of Fig. 5(a) and assume that the background effects are calculable and can be distinguished in experiemental study. Thus we neglect the contributions from Drell diagrams and its rescattering corrections.

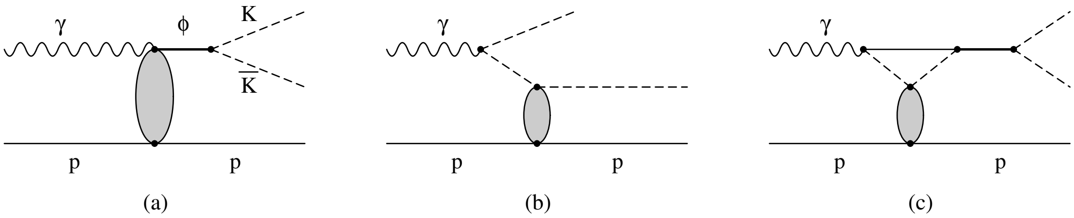

Figure 5: Diagrams for dimeon production corresponding to the Söding model: (a) through vector-meson production, (b) Drell diagrams, (c) rescattering correction to the Drell diagrams.

Note that in the case of $\pi\pi$ production, the vector-meson becomes the ρ meson. And the solid line between the photon line and the vector-meson line in Fig. 5(c) can be either π or ω. However, in the case of $K\bar{K}$ production, there is no contact term of ppK and the blob in Fig. 5(b,c) should contain intermediate baryon (Λ and Σ etc.) states.

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