Electron Cloud Generation

Dave and Jim,

I managed to develop an analytic expression for the maximum electrons density in a *bend* due to *photo-electrons* alone. The result is:

$$\overline{n}_{\rm ec,pe} \le \overline{n}_{\rm ec,pe}^{(\rm max)} \cong 10.113 \gamma^2 \sqrt{\frac{N_e N_b}{R^3 [cm] \rho^3 [m]}} \frac{\overline{E_{\rm pe}^{1/2}}[eV]}{\overline{E}_{\rm pe}[eV]}$$

where

- N_e number of electrons in a single bunch
- N_b number of bunches in the ring ($N_e N_b$ is the total number of electrons in the ring)
- $\pi R^2 = \pi ab$ is the effective area of the ellipse ($R = \sqrt{ab}$)
- ρ radius of curvature of the bend
- $\overline{E}_{pe} \equiv \int_{0}^{\infty} dE E f_{pe}(E)$ is the average energy of the emitted photoelectrons in eV's

$$\overline{E_{\rm pe}^{1/2}} = \int_{\rm pe}^{\infty} dE \, E^{1/2} \, f_{\rm pe} \left(E \right)$$

• $f_{pe}(E)$ is the probability density of emitting a photo-electron in the range $E \rightarrow E + dE$.

Assuming $R = \sqrt{\frac{9}{2} \frac{4}{2}} = 3[cm], \rho = 80[m], N_e = 5 \times 10^{10}, N_b = 100, \gamma = 10,000$, and

 $\overline{E_{pe}^{1/2}}[eV] \approx 30, \overline{E}_{pe}[eV] \approx 600$ we get $\overline{n}_{ec,pe}^{(max)} \approx 1.36 \times 10^{11} [m^{-3}]$ which seems fairly close to what people report as the cloud density – including secondary emission. The values of the last two parameters $\left(\overline{E_{pe}^{1/2}}, \overline{E}_{pe}\right)$ where chosen fairly arbitrarily but they are not expected to change the order of magnitude. Regardless, I'll try to investigate more thoroughly the parameters of photo-electrons emission.

Please, let me know how this fits to your earlier estimates? The main conclusion is that the density is proportional to the square root of the total charge in the ring!! If you have the patience, the details of the calculations are brought in the following pages and at the end of April I am planning to come to further discuss this result. The effect of secondary electrons is yet to be determined.

Dave, as follow up of your last note regarding e-cloud in the vicinity of the wiggler, I will try to repeat this exercise for that case. Regards, Levi

FLOW OF ELECTRONS

The purpose of this note is to highlight the main steps of a model describing the build-up of the electron cloud and develop some preliminary scaling laws for the electrons' density. Let us denote by $J_+(t,E)$ the flux of electrons of energy (E) impinging upon the wall (r = R) at an instant t. Due to secondary emission (subscript SE), the emerging flux of electrons is denoted by $J_-(t,E)$ the two being related by a "reflection" operator $\mathcal{R}_{SE}(t,E|t',E')$

$$J_{-}(t,E) = \int dt' \int dE' \,\mathcal{R}_{SE}(t,E|t',E') J_{+}(t',E') + J^{(p)}(t,E)$$
(1)

 $J^{(p)}$ representing the flux of *primary* electrons which for the moment it is assumed to be known. Naturally, emerging electrons occur with a delay τ_{SE} according to the collisions they encounter in the metal. This delay depends on both the energy of the impinging electron (E') as well as the energy of the emerging electron (E), or explicitly,

$$\mathcal{R}_{SE}(t, E|t', E') \simeq \delta_{SE}(E|E')\delta[t'-t+\tau_{SE}(E|E')]; \qquad (2)$$

the expression $\delta_{SE}(E|E')$ representing the yield of the secondary emission. Similar to delay, this quantity depends both on the energy of the incoming electron and the emitted one. Consequently,

$$J_{-}(t,E) = \int dE' \,\delta_{SE}(E|E') J_{+}[t - \tau_{SE}(E|E'),E'] + J^{(p)}(t,E).$$
(3)

Electrons emerging from the surface are eventually reflected back to the surface under the influence of: (i) the image-charges, (ii) the space-charge accumulated in the cloud and (iii) the bunch. This process is described by the electrostatic (subscript ES) reflection operator $\mathcal{R}_{ES}(t, E|t', E')$ or explicitly by

$$J_{+}(t,E) = \int dt' \int dE' \mathcal{R}_{\rm ES}(t,E|t',E') J_{-}(t',E').$$
(4)

For simplicity sake, we assume that the effect of the bunch on the cloud may be included in an effective way in the term describing the primary electrons. Consequently, the operator \mathcal{R}_{ES} represents only the *elastic scattering*. Subject to this assumption, it takes an electron (of an initial energy E') from the instant it has left the metallic surface until it hits it again, an average time $\tau_{ES}(E')$ depending primarily on its initial energy and the density of electrons in the cloud. With this conclusion in mind we may write for the electrostatic reflection operator in the "vacuum" region the following expression

$$\mathcal{R}_{\rm ES}(t,E|t',E') \simeq \delta(E'-E)\delta[t'-t+\tau_{\rm ES}(E')]$$
⁽⁵⁾

Combining equations (3) and (4) in conjunction with (5) we conclude that the equation for the for the flux of electrons emerging from the surface is

$$J_{-}(t,E) = \int dE' \,\delta_{SE}(E|E') J_{-}[t - \tau(E|E'),E'] + J^{(p)}(t,E);$$
(6)

wherein the "round-trip" time was conveniently defined as $\tau(E|E') \equiv \tau_{SE}(E|E') + \tau_{ES}(E')$. One should not be mislead by the form of this equation and conclude that this is a linear equation. The electrostatic delay time (τ_{ES}) depends on the total charge in the volume of interest and the latter depends in turn on both fluxes (J_-, J_+) .

Although the secondary yield depends primarily on the energy of impinging and emitted electrons, in principle it depends on the incoming and outgoing angles of the corresponding electrons. In this note we shall limit the discussion to a bend or a quadropole therefore the electrons may be assumed to move (and gyrate) along the magnetic fields. Consequently, both the impinging and the emitted electron are assumed to be parallel. Without significant loss of generality, based on the experimental data, it is reasonable to further assume that the energy of the secondary electrons is uniformly distributed between zero and the incident electron $0 \le E \le E'$ i.e.,

$$\delta_{SE}(E|E') \simeq \tilde{\delta}_{SE}(E') \left[h(E) - h(E - E')\right] / E'$$
(7)

and as a result,

$$J_{-}(t,E) = J^{(p)}(t,E) + \int_{E}^{\infty} dE' \frac{1}{E'} \tilde{\delta}_{SE}(E') J_{-}[t - \tau(E|E'),E'] .$$
(8)

Before proceeding it is instructive to examine the two components of $\tau(E|E')$. Assuming an effective decelerating electric field \mathcal{E}_{dec} , then the average time it takes an electron of energy E to return back to the surface due to electrostatic processes in the "vacuum" region is given by

$$\tau_{ES}(E) \simeq \frac{r_e}{c} \sqrt{\frac{E}{E_{\text{eff},v}}}$$
(9)

wherein $E_{\text{eff},v} = (\varepsilon_0 \mathcal{E}_{\text{dec}}^2 / 2) \pi r_e^3$ is the effective energy associated with the decelerating processes and r_e is the classical radius of the electron. In order to establish the order of magnitude of τ_{ES} , let us consider a cloud of density $n \approx 10^{12} [\text{m}^{-3}]$ uniformly¹ distributed in a waveguide of radius $R \sim 5[\text{cm}]$. The average decelerating field is $\mathcal{E}_{\text{dec}} \approx enR/2\varepsilon_0 \sim 500[\text{V/m}]$ and consequently for an electron of initial energy $E \sim 300[\text{eV}]$ we get $\tau_{ES} \sim 200[n \sec]$ which is five orders of magnitude slower than the duration of a single bunch $\sim 1[p \sec]$ which is the time-scale of the photons' pulse that in turn generate the primary electrons.

¹ This is a very rough approximation that aims to provide us with an estimate of the decelerating field – the latter is expected to be proportional to this density and the radius of the pipe.

Adopting a similar approach for the secondary emission we find

$$\tau_{SE}(E|E') \simeq \frac{r_e}{2c} \left(\sqrt{\frac{E}{E_{\text{eff,acc}}}} + \sqrt{\frac{E'}{E_{\text{eff,dec}}}} \right)$$
(10)

wherein $E_{\rm eff,dec}(E_{\rm eff,acc})$ is the typical energy associated with the decelerating (accelerating) process of the incident (reflected) electrons in the solid material. By analogy to the process in vacuum, the number of charges is the same but it is anticipated to be concentrated in a layer which is orders of magnitude thinner therefore, the bouncing time including both the accelerating and decelerating processes in the solid are negligible in comparison to the electrostatic bouncing time i.e.,

$$\tau_{SE}(E|E') \ll \tau_{ES}(E') \tag{11}$$

implying that $\tau(E|E') \simeq \tau_{ES}(E')$ and as a result,

$$J_{-}(t,E) = J^{(p)}(t,E) + \int_{E}^{\infty} dE' \frac{1}{E'} \tilde{\delta}_{SE}(E') J_{-}[t - \tau_{ES}(E'),E'] .$$
(12)

Taking advantage of the fact that the integrand is independent on the emerging energy (E), it is possible to replace the integral equation with a differential one

$$\frac{\partial}{\partial E}J_{-}(t,E) + \frac{1}{E}\tilde{\delta}(E)J_{-}[t - \tau_{ES}(E),E] = \frac{\partial}{\partial E}J^{(p)}(t,E).$$
(13)

Before any further simplification is possible we need to discuss in more detail the source term namely, the contribution of the photo-electrons.

PHOTO-ELECTRONS GENERATION

Let us evaluate the number of photo-electrons generated in an energy range $E \rightarrow E + dE$. For this purpose let us assume an arc of radius of curvature ρ , angle α_0 and the vacuum pipe having a radius R. Bending the trajectory of the electron/positron leads to synchrotron radiation of intensity [7]

$$\frac{dI}{d\omega} \simeq \sqrt{3\pi} N_e \frac{e^2}{4\pi\varepsilon_0} \frac{\gamma}{c} \sqrt{\frac{\omega}{\omega_{\rm cr}}} \exp\left(-2\frac{\omega}{\omega_{\rm cr}}\right)$$
(14)

wherein $\omega_{cr} = 3\gamma^3 c / \rho$ and N_e representing the number of electrons in the bunch². With this expression in mind we may calculate the *total energy* emitted

$$\mathcal{E}_{\rm ph} \simeq \sqrt{3\pi} N_e \frac{e^2}{4\pi\varepsilon_0} \frac{\gamma}{c} \int_0^\infty d\omega \sqrt{\frac{\omega}{\omega_{\rm cr}}} \exp\left(-2\frac{\omega}{\omega_{\rm cr}}\right)$$

$$\simeq \sqrt{3\pi} N_e \frac{e^2}{4\pi\varepsilon_0} \frac{\gamma}{c} \omega_{\rm cr} \int_0^\infty dx \sqrt{x} \exp\left(-2x\right) \simeq \frac{\sqrt{6\pi}}{8} N_e \frac{e^2}{4\pi\varepsilon_0} \frac{\gamma}{c} \omega_{\rm cr}$$
(15)

² Only the incoherent process is considered

as well as the total number of photons emitted

$$N_{\rm ph} \approx \sqrt{3\pi} N_e \frac{e^2}{4\pi\varepsilon_0} \frac{\gamma}{c} \int_0^\infty d\omega \frac{1}{\hbar\omega} \sqrt{\frac{\omega}{\omega_{\rm cr}}} \exp\left(-2\frac{\omega}{\omega_{\rm cr}}\right)$$
$$\approx \sqrt{3\pi} N_e \frac{e^2}{4\pi\varepsilon_0} \frac{\gamma}{c\hbar} \int_0^\infty dx \frac{1}{\sqrt{x}} \exp\left(-2x\right)$$
$$\approx \frac{\sqrt{6\pi}}{2} N_e \frac{e^2}{4\pi\varepsilon_0} \frac{\gamma}{c\hbar} \approx \frac{\sqrt{6\pi}}{2} \alpha \gamma N_e \approx 2.8 \times 10^{-3} \gamma N_e.$$
 (16)

Consequently, the number of photons in the energy range $E \rightarrow E + dE$ is

$$\tilde{N}_{\rm ph}(E) \simeq 2.8 \times 10^{-3} \gamma N_e \left[\frac{2}{\sqrt{2\pi} E_{\rm cr}} \frac{1}{\sqrt{E/E_{\rm cr}}} \exp(-2E/E_{\rm cr}) \right]$$
(17)

satisfying $\int_{0}^{\infty} dE \,\tilde{N}_{\rm ph}(E) \simeq 2.8 \times 10^{-3} \gamma N_e$ thus $f_{\rm rh}(E) = \frac{2}{\sqrt{1-2}} \frac{1}{\sqrt{1-2}} \exp(-2E/E_{\rm er})$

$$f_{\rm ph}(E) = \frac{2}{\sqrt{2\pi}E_{\rm cr}} \frac{1}{\sqrt{E/E_{\rm cr}}} \exp\left(-2E/E_{\rm cr}\right)$$
(18)
probability of emitting a photon in the energy range $E \to E + dE$.

represents the probability of emitting a photon in the energy range $E \rightarrow E + dE$. Denoting by $\delta_{pe}(E|E')$ the yield of generating a photo-electron of energy E by a photon of energy E' we conclude that the spectrum *photo-electrons* charge in this energy range is

$$Q_{\rm pe}(E) \simeq e \int_{0}^{\infty} dE' \delta_{\rm pe}(E|E') \tilde{N}_{\rm ph}(E').$$
⁽¹⁹⁾

This yield is dependent only on the properties of the material and is independent of the properties of the bunch or the cloud. It tacitly includes information about incident angle of incidence of the photons as well as the reflection process.

At this point we may evaluate the density of photo-electrons by first defining the average yield

$$\overline{\delta}_{\rm pe} \equiv \int_{0}^{\infty} dE \int_{0}^{\infty} dE' \delta_{\rm pe} \left(E | E' \right) f_{\rm ph}(E') \tag{20}$$

implying that the total charge of photo-electrons is

$$\overline{Q}_{\rm pe} \simeq 2.8 \times 10^{-3} e \gamma N_e \overline{\delta}_{\rm pe} \,. \tag{21}$$

The volume of the ring is $(\pi R^2)(2\pi\rho)$ therefore, the average density is

$$\overline{n}_{\rm pe} \simeq 2.8 \times 10^{-3} \gamma \frac{N_e}{\pi R^2 (2\pi\rho)} \overline{\delta}_{\rm pe}$$
⁽²²⁾

For a bend of an angle $\alpha_0 \ll 2\pi$ the number of photons is reduced proportionally to the angle and correspondingly the number of photo-electrons is reduced

$$\bar{Q}_{\rm pe} \simeq 2.8 \times 10^{-3} e \gamma N_e \bar{\delta}_{\rm pe} \frac{\alpha_0}{2\pi}$$
⁽²³⁾

Before concluding this section, let us develop an upper estimate for the average photoelectrons yield relying on energy conservation. Specifically, the expression in Eq.(15) provides the total energy emitted by the bunch in one revolution. Based on Eq.(20) we may define the probability density for generating a photo-electron in the energy range $E \rightarrow E + dE$ as

$$f_{\rm pe}(E) \equiv \frac{1}{\overline{\delta}_{\rm pe}} \int_{0}^{\infty} dE' \delta_{\rm pe}(E|E') f_{\rm ph}(E')$$
⁽²⁴⁾

With this quantity we may determine the average energy of the photo-electrons

$$\mathcal{E}_{\rm pe} = 2.8 \times 10^{-3} \gamma N_e \overline{\delta}_{\rm pe} \int_0^\infty dE \, E \, \frac{1}{\overline{\delta}_{\rm pe}} \int_0^\infty dE' \delta_{\rm pe} \left(E | E' \right) f_{\rm ph} \left(E' \right)$$

$$= 2.8 \times 10^{-3} \gamma N_e \overline{\delta}_{\rm pe} \int_0^\infty dE \, E \, f_{\rm pe} \left(E \right) = 2.8 \times 10^{-3} \gamma N_e \overline{\delta}_{\rm pe} \overline{E}_{\rm pe}$$

$$(25)$$

 \overline{E}_{pe} being the average energy of the photo-electrons. Now, since energy conservation implies $\mathcal{E}_{ph} \ge \mathcal{E}_{pe}$ we conclude that

$$\overline{\delta}_{pe} \le \overline{\delta}_{pe}^{(max)} \equiv 1.44 \times 10^{-8} \frac{1}{\rho[m]} \frac{\gamma^3}{\overline{E}_{pe}[eV]}$$
(26)

For example, in case of a 5[GeV] bunch, 100[m] radius of curvature of the bend and assuming that $\overline{E}_{pe} = 1[keV]$, the maximum average yield of the photo-electrons is $\overline{\delta}_{pe}^{(max)} \sim 0.15$. Moreover, the electron cloud density due to photo-electrons alone is $\overline{n}_{pe} \simeq 4 \times 10^{10} [m^{-3}]$ assuming $N_e = 5 \times 10^{10}$ electrons in the bunch.

So far we assumed that the ring consists of a continuous bend. In practice there are straight sections ("straights") that have no contribution to synchrotron radiation. Ignoring the straights inflates the estimate of photo-electrons density but since their overall length compared with the circumference of the ring is relatively small, the error is estimated to be small [???].

Finally, in case of a straight of length L between two bends, it is important to realize that there are photo-electrons in the straight because of the bend but in the first region of the second bend there are no photo-electrons except if the straight is sufficiently short.

DYNAMICS OF CLOUD BUILDUP

Relying on the time-scales indicated earlier (pico-seconds long bunch and many nanoseconds separation) the primary electrons (ignoring "snow-plow") are assumed to be virtually delta-function occurring at prescribed intervals T_n and their energy distribution may be derived from the synchrotron radiation photons namely,

$$J_{p}(t,E) = Q_{pe}(E) \sum_{n=1}^{N} \delta(t-T_{n})$$
(27)

wherein $Q_{pe}(E)$ is the spectrum of photo-electrons and it was defined in Eq.(19). Further defining $\psi(\omega, E) = \int_{0}^{E} dE' \delta_{SE}(E') (1/E') \exp[-j\omega\tau_{ES}(E')]$ the formal solution of Eq.(13) is

$$J_{-}(t,E) = \sum_{n=1}^{N} \int_{0}^{E} dE' \frac{dQ_{pe}(E')}{dE'}$$

$$\times \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \exp\left[j\omega(t-T_{n}) - \psi(\omega,E) + \psi(\omega,E')\right]$$

$$= J^{(p)}(t,E) - \sum_{n=1}^{N} \int_{0}^{E} dE' Q_{pe}(E') \delta_{SE}(E') \frac{1}{E'}$$

$$\times \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \exp\left\{j\omega\left[t-T_{n} - \tau_{ES}(E')\right] - \psi(\omega,E) + \psi(\omega,E')\right\}$$
(28)

Since τ_{ES} is orders of magnitude longer than the characteristic time of the triggering pulses, we consider a zero order (quasi-static) approximation for the phase term i.e.,

$$\psi(\omega, E) \simeq \psi(0, E) = \int_{0}^{E} dE' \delta_{SE}(E')(1/E').$$
⁽²⁹⁾

In the framework of this approximation the current is given by

$$J_{-}(t,E) = J^{(p)}(t,E) - \sum_{n=1}^{N} \int_{0}^{E} dE' \frac{1}{E'} G(E|E') Q_{pe}(E') \delta[t - T_n - \tau_{ES}(E')]$$

$$G(E|E') \equiv \delta_{SE}(E') \exp\left[-\int_{E'}^{E} dE'' \frac{\delta_{SE}(E'')}{E''}\right]$$
(30)

The last term reflects the important role of the dynamics of the electrons in the cloud and in particular their impact on the e-cloud lifetime. Moreover, the energy integral may be further simplified if we consider the relatively simple relation between the delay time and the initial energy E' as it prescribed in Eq.(9). Bearing in mind that the Dirac delta function satisfies

$$\delta[f(x)] = \frac{1}{\left|f'(x_0)\right|} \delta(x - x_0) \tag{31}$$

wherein $f(x_0) = 0$, we observe that the argument of the delta function vanishes if

$$E' = E_{n}(t) \equiv E_{\text{eff},v} \left(\frac{t - T_{n}}{r_{e}/c}\right)^{2} \text{ thus}$$

$$J_{-}(t, E) = J^{(p)}(t, E) - \frac{2\sqrt{E_{\text{eff},v}}}{(r_{e}/c)} \sum_{n=1}^{N} \left[\frac{1}{\sqrt{E'}} G(E|E') Q_{\text{pe}}(E') h(E - E')\right]_{E' = E_{n}(t)} (32)$$

With the solution in Eq.(30) and (32), it is possible to evaluate the amount of charge in the vacuum region at a given time

$$Q(t) = \int_{0}^{t} dt' \int_{0}^{\infty} dE \left[J_{-}(t', E) - J_{+}(t', E) \right]$$

$$= \int_{0}^{t} dt' \int_{0}^{\infty} dE \left\{ J_{-}(t', E) - J_{-}[t' - \tau_{ES}(E), E] \right\}$$
(33)

evidently, the amount of charge at any given instant is directly related to the time it takes each electron of energy E to return to the metallic wall (electrostatic delay - $\tau_{ES}(E)$ or explicitly $Q(t) = Q_1(t) + Q_2(t)$. This expression reflects the direct contribution of the photo-electrons to the electron cloud

$$Q_{1}(t) = \sum_{n} \int_{0}^{\infty} dE Q_{pe}(E) \left\{ h(t - T_{n}) - h[t - T_{n} - \tau_{ES}(E)] \right\}$$
(34)

whereas the second expression

$$Q_{2}(t) = \sum_{n} \int_{0}^{\infty} dE \int_{0}^{E} dE' Q_{pe}(E') \frac{1}{E'} G(E|E') \times \left\{ h \left[t - T_{n} - \tau_{ES}(E') - \tau_{ES}(E) \right] - h \left[t - T_{n} - \tau_{ES}(E') \right] \right\}$$
(35)

represents the contribution of the secondary electrons.

E-CLOUD BUILDUP DUE TO PHOTO-ELECTRONS ALONE

In the framework of the present model the dependence of the bouncing time on the energy of the particle has been established assuming that the average decelerating field due to the cloud is known. In other words, the charge in the cloud was effectively assumed to be known. Obviously this is not the case, but this assumption has facilitated to determine the expression which may provide a good estimate of charge in the cloud Eqs.(20-21). It is therefore natural at this stage to determine a self-consistent solution of the charge in the electron cloud.

For a rough estimate let us assume a repetition rate $T \simeq 15$ [nsec], $\mathcal{E}_{dec} \sim l[kV/m]$ the latter implying $\tau (E_{\rm cr} \simeq 100 [\text{eV}])/T \simeq 4.5$, $\tau (E_{\rm cr} \simeq 300 [{\rm eV}])/T \simeq 7.8$ and $\tau (E_{\rm cr} \simeq 700 [{\rm eV}])/T \simeq 11.9$. The left frame of Figure 7 reveals the impact of the spectrum of the photo-electrons alone. For a 1kV/m average decelerating field it is evident that the time it takes a typical electron to propagate into the pipe and bounce back, is longer than the period of the bunches. Consequently, during the first few bunches we observe accumulation of electrons in the volume of the pipe. Eventually, the net flow of electrons is zero and the cloud reaches equilibrium – assuming that t > NT, typically this happens when after a time duration which is of the order of the electrostatic delay time ($t \simeq \tau_{ES}$). The right frame reveals the impact of the number of bunches on this accumulation. If the train is shorter than τ_{ES} , the cloud does not reach the equilibrium. Moreover, if the pulse separation (T=100nsec) is not very short comparing to τ_{ES} , there is no accumulation of charge as illustrated in Figure 8. In fact, in what follows, we shall develop an analytic expression for the accumulated charge. It should be pointed out that in all these figures we assumed that the yield of generating photo-electrons (defined in Eq.(20)) is unity, $\delta_{pe} \simeq 1$.



Figure 7: The left frame reveals the impact of the spectrum of the photo-electrons. For an average decelerating field of 1kV/m the time it takes a typical electron to propagate into the pipe and bounce back, is longer than the period of the bunches. During the first stages we observe accumulation of electrons in the volume of the pipe. Eventually, the average net flow of electrons is zero and the cloud reaches equilibrium – assuming that t > NT, typically this happens when $t \approx \tau$. The right frame reveals the impact of the number of bunches on this accumulation; $E_{\rm cr} = 300[{\rm eV}]$. In both frames the charge is normalized to the charge of photo-electrons $\overline{Q}_{\rm pe}$ defined in Eq.(21).

It is evident that in certain circumstances, the photo-electrons alone may have a significant contribution to the density of the electron cloud.



Figure 8: With the exception of the pulses separation T which is more than six times longer, all the parameters are as in Figure 7. It is evident that the electrons accumulation in this case is negligible. In both frames the charge is normalized to the charge of photo-electrons \overline{Q}_{pe} defined in Eq.(B.7).

Based on these results we may conceive a simple model for the build-up of the electron cloud due to photo-electrons alone. For this purpose let us assume that the reduction in the number of photo-electrons in the cloud in time is proportional to their number and the decay time is just the average time it takes a photo-electron to return to the metallic wall

$$\frac{d}{dt}Q_1 + \frac{1}{\overline{\tau}_{ES}}Q_1 = \overline{Q}_{pe}\sum_n \delta(t - T_n)$$
(36)

where the typical decay time is $\overline{\tau}_{ES} = \frac{1}{\overline{Q}_{pe}} \int_{0}^{\infty} dE Q_{pe}(E) \tau_{ES}(E)$. The solution of this

equation may be readily found

$$Q_{1}(t) = \overline{Q}_{pe} \sum_{n} \exp\left[-(t - T_{n})/\overline{\tau}_{SE}\right] h(t - T_{n})$$
(37)

h(t) being the step function. Assuming that the round trip in the ring is $\tau_C = 2\pi\rho/c$ we may evaluate the amount of charge in the ring as

$$\overline{Q}_{ec,pe} = \frac{1}{\tau_C} \int_0^{\tau_C} dt Q_1(t) \Rightarrow \boxed{\overline{Q}_{ec,pe} \simeq \overline{Q}_{pe} \frac{\overline{\tau}_{ES}}{\tau_C} N_b}$$
(38)

where N_b represents the number of the bunches in the ring. The result in Eq.(38) clearly reflects the fact that the average charge which accumulates is a function of the average bouncing time $\overline{\tau}_{ES}$. The latter, according to Eq.(xx), depends on the charge in the cloud and therefore, in what follows we shall establish a self-consistent and analytic estimate of the e-cloud density.

Relying on Eq.(38) we substitute the definition of $\overline{\tau}_{ES}$ hence

$$\overline{Q}_{\text{ec,pe}} \simeq \frac{N_{\text{b}}}{\tau_{C}} \int_{0}^{\infty} dE Q_{\text{pe}}(E) \tau_{ES}(E)$$
(39)

further using the previous definitions $Q_{\rm pe}(E) \simeq 2.8 \times 10^{-3} e \gamma N_e \overline{\delta}_{\rm pe} f_{\rm pe}(E)$ and

$$\tau_{ES}(E) \simeq \frac{r_e}{c} \sqrt{\frac{E}{E_{\text{eff},v}}} \text{ we get}$$

$$\overline{Q}_{\text{ec,pe}} \simeq 2.8 \times 10^{-3} e \gamma N_e \overline{\delta}_{\text{pe}} \frac{r_e}{c} \frac{N_b}{\tau_C} \frac{1}{\sqrt{E_{\text{eff},v}}} \int_0^\infty dE \sqrt{E} f_{\text{pe}}(E)$$
(40)

And again, using previous definitions $E_{\text{eff},v} = \left(\varepsilon_0 \mathcal{E}_{\text{dec}}^2 / 2\right) \pi r_e^3$ and $\mathcal{E}_{\text{dec}} \simeq enR / 2\varepsilon_0$ we obtain

$$\overline{n}_{ec,pe} \simeq 8.428 \times 10^4 \sqrt{\frac{\gamma N_e N_b \overline{\delta}_{pe}}{R^3 [cm] \rho^2 [m]}} \overline{E_{pe}^{1/2}} [eV]$$
(41)

Two conclusions are evident, the average density of electrons in the e-cloud is:

- Proportional to the square root of the total charge in the ring.
- Inversely proportional to the area of the vacuum pipe $1/R^{3/2} \sim 1/(ab)^{3/4}$

It is difficult to draw a conclusion regarding the scaling with the energy or radius of curvature since the photo-electrons are expected to be dependent on both.

With the upper limit for the photo-electrons yield evaluated in Eq.(26) based on energy conservation, we get the following estimate for the maximum average density of generated photo-electrons

$$\overline{n}_{ec,pe} \le \overline{n}_{ec,pe}^{(max)} \cong 10.113 \gamma^2 \sqrt{\frac{N_e N_b}{R^3 [cm] \rho^3 [m]}} \frac{\overline{E_{pe}^{1/2}}[eV]}{\overline{E}_{pe}[eV]}$$
(42)

As a typical example consider $R = \sqrt{\frac{9}{2} \frac{4}{2}} = 3[cm], \rho = 80[m], N_e = 5 \times 10^{10}, N_b = 100,$ $\gamma = 10,000$, and $\overline{E_{pe}^{1/2}}[eV] \simeq 30, \overline{E}_{pe}[eV] \simeq 600$ thus $\overline{n}_{ec,pe}^{(max)} \simeq 1.36 \times 10^{11} [m^{-3}]$ (43)

APPENDIX: Evaluation of the delay time in a uniform cloud

Cloud density is assumed to be uniform $n(r) = n_0$ implying an electric field

$$\frac{1}{r}\frac{\partial}{\partial r}(rE_r) = -\frac{e}{\varepsilon_0}n_0 \Longrightarrow E_r(r) = -\frac{e n_0}{2\varepsilon_0}r \tag{A.1}$$

The dynamics of an electron injected inwards with a velocity V_0 into the cloud is given by

$$\frac{d^2r}{dt^2} = \frac{1}{2}\omega_p^2 r \tag{A.2}$$

where $\omega_p^2 = e^2 n_0 / m\varepsilon_0$ is the plasma frequency. From this last expression, energy conservation entails

$$\frac{d}{dt}\left[\frac{1}{2}\left(\frac{dr}{dt}\right)^2 - \frac{1}{4}\omega_p^2 r^2\right] = 0$$
(A.3)

which together with the initial condition reads

$$\frac{1}{2} \left(\frac{dr}{dt}\right)^2 - \frac{1}{4}\omega_p^2 r^2 = \frac{1}{2}V_0^2 - \frac{1}{4}\omega_p^2 R^2$$
(A.4)

The electron is assumed to reach zero velocity at a radius $r = R_0$ therefore

$$V_0^2 = \frac{1}{2}\omega_p^2 \left(R^2 - R_0^2\right)$$
(A.5)

Defining $\overline{r_0} = R_0 / R$ and the normalized velocity $\overline{V} = \frac{\sqrt{2}V_0}{\omega_p R}$ we get

$$\overline{\overline{r_0} = \sqrt{1 - \overline{V}^2}}$$
(A.6)

We now proceed to evaluate the time it takes an electron to reach the edge of the cloud. For this purpose we integrate Eq.(B.4)

$$\left(\frac{d\,\overline{r}}{d\overline{t}}\right)^2 = \overline{V}^2 - \left(1 - \overline{r}^2\right) \Longrightarrow \frac{d\,\overline{r}}{\sqrt{\overline{V}^2 - 1 + \overline{r}^2}} = -d\overline{t} \tag{A.7}$$

where $\overline{r} = r/R$ and $\overline{t} = \omega_p t/\sqrt{2}$. For convenience we substitute Eq.(B.6) to get

$$\frac{d\,\overline{r}}{\sqrt{\overline{r}^2 - \overline{r_0}^2}} = -d\overline{t} \tag{A.8}$$

Consequently, the normalized round trip (delay) time is

$$\overline{\tau}_{ES} = -2\int_{1}^{\overline{r}_{0}} \frac{d\overline{r}}{\sqrt{\overline{r}^{2} - \overline{r}_{0}^{2}}} = -2\ln\left(\frac{\overline{r}_{0}}{1 + \sqrt{1 - \overline{r}_{0}^{2}}}\right) = \ln\left(\frac{1 + \overline{V}}{1 - \overline{V}}\right)$$
(A.9)

or explicitly

$$\tau_{ES}\omega_p \frac{1}{\sqrt{2}} = \ln \left(\frac{1 + \frac{\sqrt{2}V_0}{\omega_p R}}{1 - \frac{\sqrt{2}V_0}{\omega_p R}} \right) \underset{V \ll 1}{\approx} 2 \frac{\sqrt{2}V_0}{\omega_p R} \Rightarrow \left[\tau_{ES} \approx 4 \frac{V_0}{\omega_p^2 R} \right]$$
(A.10)

Further defining $\mathcal{E}_{dec} \simeq enR/2\varepsilon_0$ and denoting by *E* the energy of the emitted electron we finally get

$$\tau_{ES}(E) \simeq \frac{r_e}{c} \sqrt{\frac{E}{E_{\text{eff},v}}}$$
(A.11)

wherein $E_{\text{eff},v} = (\varepsilon_0 \mathcal{E}_{\text{dec}}^2 / 2) \pi r_e^3$ is the effective energy associated with the decelerating processes and r_e is the classical radius of the electron.

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