Jim,

It seems that you are correct. Indeed there are two regimes of operation: dilute cloud (free-particle approximation) and dense cloud (collective effect). According to a rough estimate I just made (see next pages), the transition between the two regimes depends on the typical kinetic energy of the photo-electrons (subscript pe) and it occurs at

$$\boxed{n_{\mathrm{cr}} = 3.47\times10^{12}[m^{-3}]\frac{E_{\mathrm{pe}}\left[eV\right]}{\mathrm{Pipe\ Area}\left[cm^{2}\right]}}$$

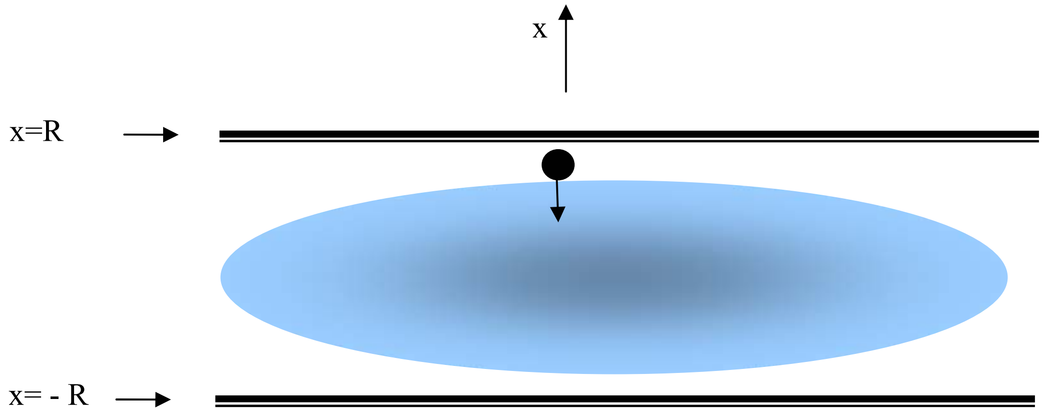
For example, if the pipe area is $\pi\times4\times9=113[cm^2]$ and $E_{\mathrm{pe}}\sim100[eV]$ the critical density is of the order of $n_{cr}\simeq3\times10^{12}[m^{-3}]$. Since in my early estimates, the density was of the order of $10^{11}[m^{-3}]$ the assumption of collective effect dominating the dynamics was clearly not justified. I already fixed that and in the framework of free-particle approximation, the density of photo-electrons is indeed proportional to the number of electrons in the ring. I did not include that yet in my draft – but I will soon.

Please also note that the density you have found in the vicinity of the wiggler (bend or straight) is of the order of $10^{13}[m^{-3}]$ therefore, it is possible that *collective* effects may be observed?!

Best regards, Levi

Regime of Operation: free-particle vs. collective effect

Consider two metallic plates with an electron cloud of density n_0 in between; The distance between the two plates is $2R$. At $t=0$ an electron leaves the upper plate inwards with an instantaneous kinetic energy E; both plates are grounded therefore the electric field is given by $E=-en_0 x/\varepsilon_0$.



Assuming 1D motion, the dynamics is determined by

$$\frac{d^2x}{dt^2}=-\frac{e}{m}E=\frac{e^2n_0}{m\varepsilon_0}x=\omega_p^2x \tag{1}$$

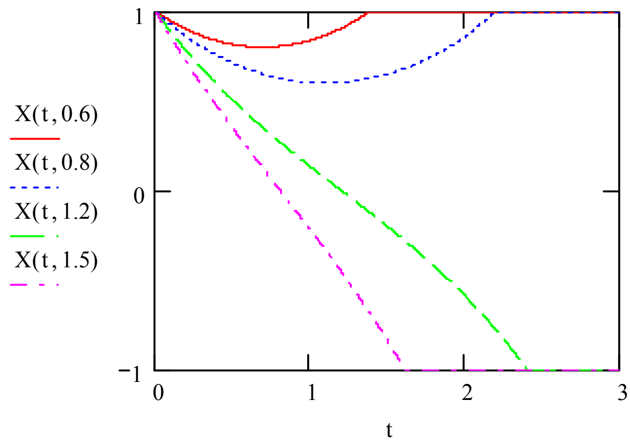
wherein the plasma frequency is

$$\omega_p^2=\frac{e^2n_0}{m\varepsilon_0} \tag{2}$$

Its solution, subject to the initial conditions $\left[x(t=0)=R,\ \dot{x}(t=0)=-V_0=-\sqrt{2E/m}\right]$ is

$$x(t) = R\cosh\left(\omega_p t\right) - \frac{V_0}{\omega_p}\sinh\left(\omega_p t\right) \Rightarrow$$
$$X\left(\tau \equiv \omega_p t, \xi \equiv \frac{V_0}{\omega_p R}\right) \equiv \frac{x(t)}{R} = \cosh\left(\tau\right) - \xi\sinh\left(\tau\right) \tag{3}$$

Four trajectories are plotted below



We clearly observe that if $\xi \equiv V_0 / \omega_p R < 1$, the electron returns to the same electrode where it was injected. In the opposite case, the electron reaches the opposite plate. This example reveals the difference between two regimes!! If the cloud controls the dynamics of the photo-electrons in other words, *collective* effect of many electrons are dominant, $\xi \le 1$ then the photo-electrons are reflected back to the original electrode. In case of a dilute cloud, the photo-electron traverses the pipe with virtually no effect of the cloud, thus this is the *free-particle* regime. Based on this observation we may determine a simple criterion enabling to distinguish between the two regimes for this purpose, let us define the critical density (from the condition $\xi = 1$)

$$n_{cr} = \frac{E}{e^2 R^2 / 2\varepsilon_0} = 3.47 \times 10^{12}[m^{-3}] \frac{E_{\text{pe}}\left[eV\right]}{\text{Pipe Area}\left[cm^2\right]}. \tag{4}$$

For example if the pipe area is $\pi \times 4 \times 9 = 113[cm^2]$ and $E_{\text{pe}} \sim 100[eV]$ the critical density is of the order of $n_{cr} \simeq 3 \times 10^{12}[m^{-3}]$.

A better insight may be achieved by realizing that the condition for the electron to return is clearly that under the effect of the electron cloud force, this decelerating force will bring its velocity to zero. This happens at a time

$$\tanh\left(\omega_p t\right) = \xi \equiv \frac{V_0}{\omega_p R} \tag{5}$$

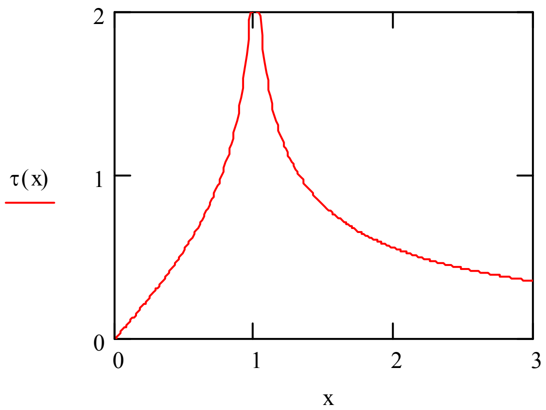
and it is evident that solution is possible only if $\xi < 1$ in which case the delay is

$$\tau_{ES}(E) = \frac{2}{\omega_p}\text{atanh}\left(\sqrt{\frac{E}{\frac{1}{2}m\,\omega_p^2 R^2}}\right) \tag{6}$$

and in the opposite case, the time it takes the electron to reach the opposite electrode $x = -R$ is given by

$$\tau_{ES}(E) = \frac{2}{\omega_p}\text{atanh}\left(\sqrt{\frac{\frac{1}{2}m\,\omega_p^2 R^2}{E}}\right) \tag{7}$$

Figure xx illustrates the normalized delay time $\overline{\tau}_{ES} = \omega_p \tau_{ES} / 2$ as function of the normalized energy ξ.



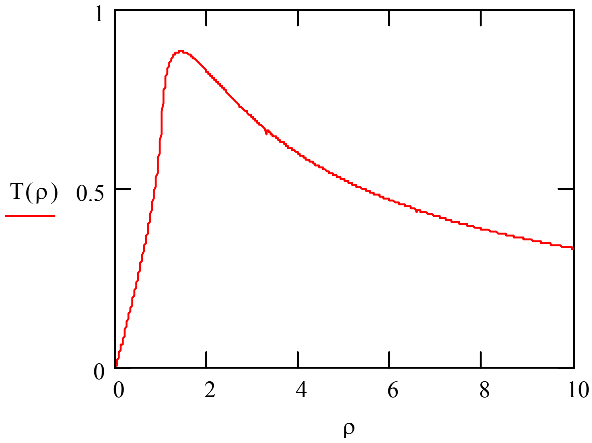
For an estimate of what happens in an elliptical cross-section let us denote $R_0 = \sqrt{2E / m\omega_p^2}$ which entails that the delay is a function of R is

$$\frac{1}{2}\omega_p \tau_{ES}(R) = \begin{cases} \text{atanh}(R/R_0) & R < R_0 \\ \text{atanh}(R_0/R) & R > R_0 \end{cases} \tag{8}$$

Since in an elliptic pipe the radius varies from zero to a typical value R_c we get

$$\tilde{\tau}_{ES} = \frac{1}{2}\omega_p \overline{\tau}_{ES} = \frac{1}{2}\omega_p \frac{1}{R_c}\int_0^{R_c} \tau_{ES}(R)\,dR \tag{9}$$

which is illustrated next as a function of $\rho = R_c / R_0$



In the framework of the present approximation maximum average delay occurs for $\rho_{\max} \simeq 1.44$ in which case $\tilde{\tau}_{ES}^{(\max)} = 0.882$ thus

$$n_{\max}[m^{-3}] = 7\times 10^{12}\,\frac{E[eV]}{Pipe\ Area[cm^{-2}]}$$

$$\bar{\tau}_{ES}^{(\max)} = \frac{11.8[n\sec]}{\sqrt{\dfrac{E[eV]}{Pipe\ Area[cm^{-2}]}}} \qquad (10)$$

which is roughly twice the value of the critical density. Note that this may be fairly close to the bunch spacing!!