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Energy Loss from Small Holes in the Vacuum Chamber

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Summary

A method is proposed for the calculation of the energy lost from the beam due to small holes in the wall of the vacuum chamber. For PEP the loss should be small.

I. Introduction

The present design of the PEP beam vacuum chamber consists of a more-or-less rectangular pipe roughly 55 mm high by 90 mm wide. In the bending magnets the inner wall (toward the bending center) is perforated to provide communication with an adjacent chamber containing the distributed pumps. The normal beam position is about 30 mm from the inner wall. (See Fig. 1.) This note shows how an estimate can be made of the possible energy loss of a bunched beam caused by the electromagnetic effects of the performations between the two chambers.

First, I make use of a calculation of H. A. Bethe which evaluates the diffraction of an electromagnetic wave through a small hole in a plane conducting sheet. (A variant of this method is in standard use for evaluating the coupling of two waveguides by a small hole. See e.g. Ref 2.). Bethe's theory is used to find the energy diffracted (that is, "scattered") from the field of the beam bunch as it passes a small circular hole in the wall. I argue that since the bunch length is shorter than the characteristic dimensions of the chamber, the energy in the diffracted fields will not be changed much by the presence of the other walls. I then make an estimate of a "coherence factor" which takes into account the interaction of nearby holes. These results are used to estimate the total diffraction loss for PEP.

II. Diffraction by a Single Hole

Bethe has shown that when a plane electromagnetic wave is incident on an infinite conducting plane with a small circular hole, the diffracted field is the sum of the fields that would be radiated by an electric and a magnetic dipole located at the hole. The electric dipole is directed perpendicular to this surface and has a dipole moment of:

$$\vec{d} = -\frac{2\varepsilon_0}{3} r^3 \vec{E}_0 \qquad (1)$$

where $\vec{E}_0$ is the incident electric field at the position of the hole -- by which is meant the field that would be present at the position of the hole if the hole were not there -- and r is the radius of the hole. The magnetic dipole is directed parallel to the plane of the hole and has a moment of:

$$\vec{m} = -\frac{4}{3\mu_0} r^3 \vec{B}_0, \tag{2}$$

where $\vec{B}_0$ is the undisturbed magnetic field at the position of the hole. It is assumed that the diffracting wall is a good conductor so that E_0 is perpendicular to the surface, and B_0 is parallel. It is further assumed that the wavelength of the incident field is much larger than r.

The varying dipole moments given by Eqs. (1) and (2) will radiate into the two half-spaces separated by the conducting plane. The total power being radiated into <u>both</u> half-spaces by a varying electric dipole is given by[2]

$$P(\text{elect}) = \frac{Z_0(\ddot{d})}{6\pi c^2} \tag{3}$$

where $Z_0 = 1/\varepsilon_0 c = 377$ ohms is the characteristic impedance of free space. Similarly, the power radiated by a magentic dipole is[2]

$$P(\text{mag}) = \frac{Z_0(\ddot{m})^2}{6\pi c^4} \tag{4}$$

I would now like to argue that the expressions (1) through (4) will also give a reasonably good result for the energy lost by the beam in passing a small hole in the chamber wall. First, the electric and magnetic fields produced at the wall by a relativistic beam bunch are like those of a free wave. Second, the bunch half-length σ_ℓ is shorter than the typical dimensions of the chamber. This means that during the passage of the beam bunch the fields will be radiated <u>as if</u> into free space. After the passage of the bunch the fields will "discover" the presence of the walls and have to adjust themselves into

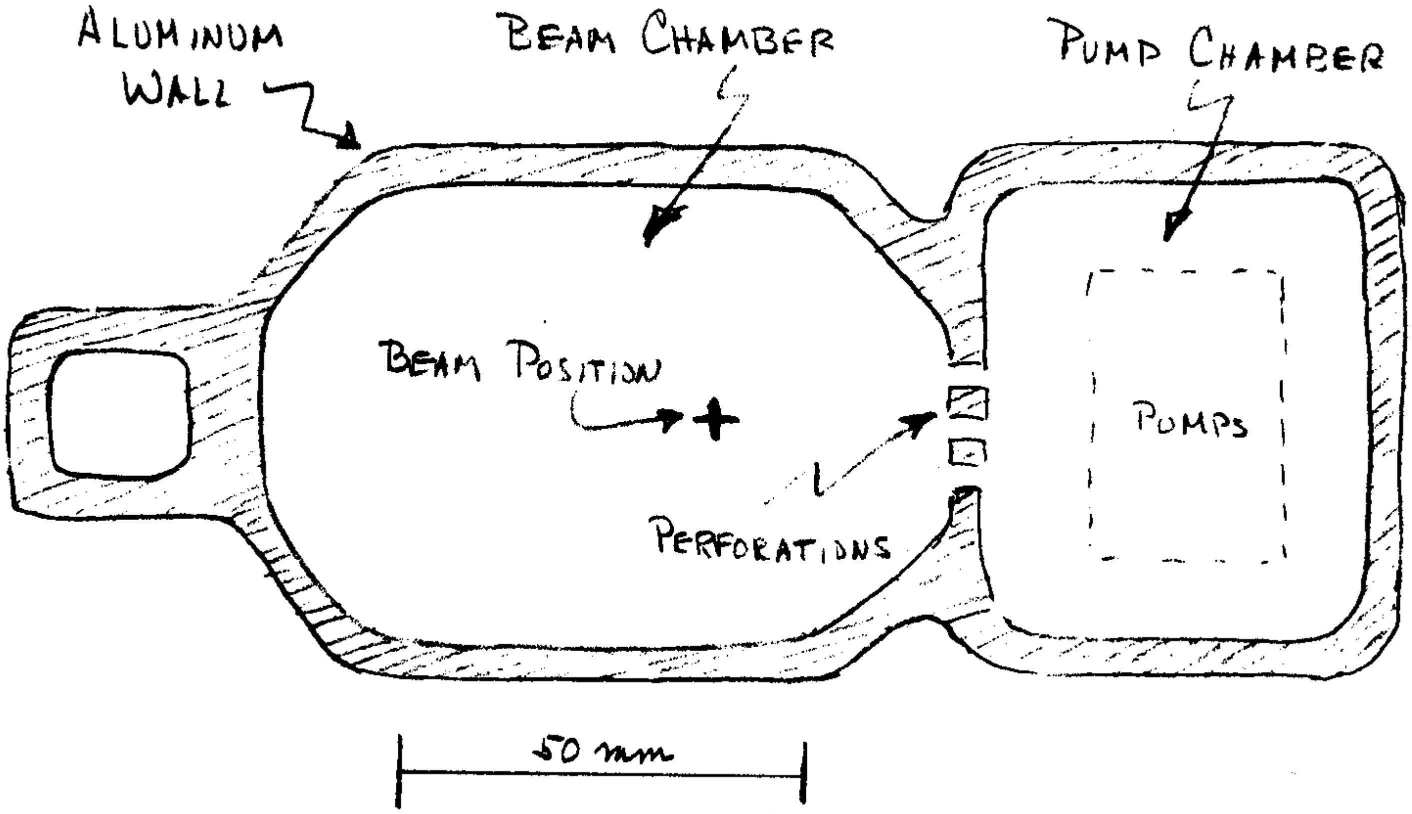

Fig. 1. Cross Section of the Vacuum Chamber

the various captured and propagating "waveguide modes" of the pipes. The total energy diffracted will, however, be the same. We could do a Fourier analysis of this bunch spectrum and calculate the coupling of each component to each of the possible waveguide modes, but this will, I maintain, give the same answer for the total energy loss as the radiation into free space.

There remains only to find the fields E_0 and B_0 that the beam produces at the chamber wall. I could solve for the field of the beam bunch in a rectangular tube, but I will be content to make the approximation that the fields are the same as would be found at the wall of a cylindrical beam pipe of radius "a" where a is the half-height of the beam pipe. Ignoring anomalous bunch lengthening, I take the linear charge density λ in the bunch at the axial position y to be given by:

$$\lambda = \frac{Q}{\sqrt{2\pi}\,\sigma_\ell} e^{-y^2/2\sigma_\ell^2} \tag{5}$$

where Q is the total charge of the bunch, and σ_ℓ is the bunch half-length. The electric field E_0 at a given point on the wall will, then vary with time as

$$E_0 = \hat{E}_0\, e^{-t^2/2\sigma_t^2}, \tag{6}$$

where $\sigma_t = \sigma_\ell/c$, and the peak field is

$$\hat{E}_0 = \frac{\hat{\lambda}}{2\pi\varepsilon_0 a} = \frac{1}{(2\pi)^{1.5}\,\varepsilon_0}\,\frac{Q}{a\sigma_\ell}\,. \tag{7}$$

Now Eqs. (6) and (7) can be used with Eq. (1) to find the varying dipole moment d(t) of the hole. (It's direction can be ignored), and this moment inserted into Eq. (3) to find the power being radiated. Here, I am interested only in the total energy loss U, which is the integral of P over all times. It is generally more convenient to express the loss in terms of an effective retarding potential differences V_{loss}. Then

$$V_{loss} = \frac{U}{Q} = \frac{1}{Q} \int_{-\infty}^{+\infty} P(t)dt. \tag{8}$$

Putting all of the pieces together (see Appendix A for details), I find that the loss potential through the electric dipole channel is

$$V_{loss}(\text{elect}) = \frac{Z_0 c}{144\ \pi^{3.5}} \frac{r^6 Q}{a^2 \sigma_\ell^{\ 5}} \tag{9}$$

To get the diffraction through the magnetic channel, note that for the fields of a relativistic bunch $B_0 = E_0/c$, so that P(mag) = 4P(elect). Further, recall that the radiation fields of electric and magnetic dipoles are given by orthogonal functions so that there is no interference in the total power radiated. We just add the powers, and, therefore, the total energy loss. The total loss potential for one hole is just five times larger than (9).

$$V_{loss}(\text{one hole}) = \frac{5 Z_0 c}{144\ \pi^{3.5}} \frac{r^6 Q}{a^2 \sigma_\ell^{\ 5}} \tag{10}$$

This diffraction loss includes both the energy left in the fields diffracted through the hole into the vacuum pump chamber, and the fields "scattered" back into the beam chamber

III. Enhancement by Coherence

As the beam makes one complete circuit of the storage ring it will pass each of the total of N holes in the chamber. If there were no interaction among the effects of nearby holes the total energy loss would be just N times larger than the single hole diffraction loss. If, on the other hand, there were full "coherence" in the diffracted fields we might expect the amplitudes of each mode to be N times the single hole fields and the energy would be N^2 times the single hole losses. Since $N \sim 10^5$ for PEP, the difference between N and N^2 is quite large.

I define a coherence factor g such that the energy loss from N holes is gN times the single hole loss. As will be shown below, the loss is expected to be small, so we need only a rough estimate of g. For this estimate we may take that g is equal to the product of ν, the number of holes per unit length along the chamber, and σ_ℓ, the bunch half-length.

$$g \simeq \nu\sigma_\ell \tag{11}$$

This choice for g can be justified intuitively as follows. The typical (angular) frequency ω_0 in the spectrum of the bunch current is equal to c/σ_ℓ. So the typical reduced wavelength $\lambda\!\!\!^-$ of the propagating fields is about equal to σ_ℓ. We may expect coherence in the fields of different radiators if they are excited in place and are within a distance of $\lambda\!\!\!^-$ of each other. So I estimate that if there are g holes in the distance equal to $\lambda\!\!\!^-$ the fields will be g times larger than from a single hole and the power radiated will be g^2 times larger. Since there are N/g sets of such groups of holes, the total power is proportional to $g^2 \times N/g$ or gN; with g given by Eq. (11).

In Appendix B I give some perhaps more rigorous arguments for the choice of the coherence factor.

There is another possible coherence effect. Suppose that the field strength in the diffracted waves were to become comparable to the field E_0 that the beam produces near the wall. The induced dipole moment of the hole would be changed, and so would the radiation diffracted from a single hole. Using the results obtained below for the total energy in the diffracted fields one can show that even under the worst imaginable circumstances the diffracted fields are many orders of magnitude below E_0. There only remains the question of whether the near field of one hole is large near a neighboring hole. It is not, as shown in Appendix C.

IV. Estimate of the Energy Loss for PEP

The total energy loss for PEP is the single hole loss of Eq. (10) multiplied by the total number of holes N, and by the coherence enhancement factor g.

$$V_{loss} = \frac{5Z_0 c}{144\ \pi^{3.5}} \frac{gNr^6 Q}{a^2 \sigma_\ell^{\ 5}} \tag{12}$$

The planned current I for PEP is 55 mA per beam in three bunches (B=3), and the circulation time T_0 is 7.3 microseconds. The charge per bunch Q is then given by

$$Q = \frac{IT_0}{B} = 0.13 \text{ microcoulomb} \tag{13}$$

For a given area perforated in the wall the number N of holes would have to vary as $1/r^2$, and g would vary as, say, $1/r^2$. So the total loss would vary as $1/r^2$. We should use the smallest practical holes. The wall between the pump chamber and the beam chamber is 4 mm thick so that any hole smaller than this will not be an effective pumping passage. Let's assume, then, that the perforations will be circular holes with a radius r of 2 mm, and spaced on 6 mm centers. I assume also that there will be 3 rows of holes. There are, then, 500 holes per meter. The total length of bending magnets in PEP is about 10^3 meters so the chamber will have a total of 5×10^5 holes. The bunch half-length σ_ℓ is expected to be about 20 mm (any bunch-lengthening will reduce the loss). If follows that $g \simeq 10$. The half-height of the chamber "a" is 3 cm. In summary:

$I = 55$ mA	$N = 5 \times 10^5$
$B = 3$	$r = 0.002$ m
$Q = 0.13\ \mu C$	$\sigma_\ell = 0.02$ m
$T_0 = 7.3\ \mu s$	$g = 10$
$a = 0.03$ m	

from which, I get that, for nominal operating conditions, V_{loss} = 1030 volts. The loss is clearly quite negligible in comparison with the synchrotron energy loss of 27 MeV per turn.

V. Effect of Wall Thickness

The Bethe theory used in Section II assumed that the wall was thin in comparison with the hole radius. When the wall is thick, as here, the wall currents and charges induced by the beam will not be deviated as much, and the diffraction loss will be smaller. There will be, in fact, a large reduction of the energy radiated into the pump chamber, because the wall charges will hardly penetrate there at all.

We can estimate this reduction by thinking of the hole as a circular waveguide. (This method has been used to evaluate the effect of the wall thickness on the aperture coupling between two waveguides. See, e.g., Ref. 3.) Assuming that the hole is much smaller than the bunch half length σ_ℓ, the dominate frequencies in the beam pulse will be well below the cut-off frequency of the circular guide. The fields in the hole will then decay exponentially as

$$\exp\left[-\frac{2\pi\delta}{\lambda_c}\right] \tag{14}$$

where δ is the wall thickness (length of the hole) and λ_c is the cut-off wavelength of a circular guide of radius r. For the electric dipole field (E parallel to the axis) λ_c = 2.6 r, and for the magnetic dipole field (B transverse) λ_c = 3.4 r. Since the magnetic dipole term dominates, let's take c = 3.4a. Then the reduction factor is

$$\exp\left[-\frac{2\pi\delta}{3.4r}\right] = \exp\left[-3.7\,\frac{\delta}{r}\right] \tag{15}$$

If we take $\delta = 2r$ then the reduction factor is $e^{-7.4} \simeq 10^{-3}$. For small holes the fields in the pump chamber are much smaller than estimated earlier.

The total loss is reduced by about one half.

Actually, the reduction will be even more because the diffracted fields in the beam chamber will also be reduced somewhat. I do not see how to make quantitative estimates, but would guess it will not be a large reduction.

IV. Diffraction by Slots

It may be desirable, for technical reasons, to perforate the chamber wall by slots instead of holes. Referring to Eqs. (1) and (2), let me define the electric and magnetic polarizibilities of a hole by

$$\alpha_e = \frac{d}{\varepsilon_o E_o} \quad ; \quad \alpha_m = \frac{\mu_o m}{B_o} \tag{16}$$

The polarizibilities of some non-circular holes has been worked out.[2] I report in the table below, the results for a narrow ellipse (semi-major axis p >> semi-minor axis r) and a narrow rectangular slot (length l >> width w). The electric polarizability is independent of the orientation of the slots, but the magnetic polarization depends on the orientation of the slot with respect to B_o. All of these polarizibilities assume that the major dimension of the slot is shorter than the wavelength of the incident field -- which means that here they should be used only for slots not as long as the bunch.

Polarizibilities of Ellipses and Slots

	α_e	α_m
Circle, radius r	$\frac{2}{3} r^3$	$\frac{4}{3} r^3$
Thin ellipse, major axis perpendicular to B_o	$\frac{\pi}{3} pr^2$	$\frac{\pi}{3} pr^2$
Thin ellipse, major axis parallel to B_o	$\frac{\pi}{3} pr^2$	$\frac{\pi}{3} \frac{p^3}{\ln(\frac{4p}{r})-1}$
Thin slot, perpendicular to B_o	$\frac{\pi}{16} lw^2$	$\frac{\pi}{16} lw^2$

First, notice that slots transverse to B_0 will have much less loss than slots parallel to B_0. If slots are used they should have their long axes parallel to the beam axis. (This is the same answer as one would get from an intuitive feeling that the slot should "interrupt" as little as possible the induced wall currents.)

Next, suppose we compare the diffraction of a slot to that from a circular hole <u>of the same area</u>. Remembering that the diffracted energy is proportional to the square of the polarizabilities, and that we add electric and magnetic effects, we see that

$$\text{Ratio} = \frac{V(\text{slot})}{V(\text{circle})} = \frac{\frac{\pi^2}{128}\, l^2 d^4}{\frac{20}{9}\, r^6} \qquad (17)$$

which with $\pi r^2 = ld$, gives

$$\text{Ratio} = \frac{9\pi^3}{2560}\,\frac{d}{l} \simeq \frac{d}{l} \qquad (18)$$

Not very much penalty is paid for distorting a hole to an equal area slot.

Alternatively, let's compare the loss from a long slot with the loss from n holes whose diameter is equal to the slot width, $2r = w$. and choosing $n = l/w$, so that the n holes cover roughly the same area as the slot. The single hole or slot losses are proportional $\alpha_e^2 + \alpha_m^2$, so the loss for the slot will be larger than the loss from a single hole by the factor

$$\frac{9\pi^2}{40}\,\frac{l^2}{w^2} \simeq 2\,\frac{l^2}{w^2}$$

There are, however, fewed slots than holes by the factor w/l, and the coherence factor g is smaller for the holes by an additional factor of w/l. So the effective loss from slots and holes of the same "width" which cover the same area are roughly equal.

I remind you that this result is true for slots shorter than the bunch half-length σ_ℓ. For slots longer than σ_ℓ the diffraction loss probably increases more slowly than ℓ^2, and becomes [4] independent of ℓ. I would expect, however, that for slots longer than σ_ℓ, one might have losses caused by interactions of the induction field of the slot with structures in the pump chamber.

VII. Conclusions

For perforations whose width (dimension at right angles to the beam axis) is 0.5 cm or so, and whose length is no longer than the bunch half-length -- say 2 cm -- the energy loss from the beam due to diffraction from the perforations is estimated for PEP to be about 10^3 to 10^4 electron-volts per turn. It is, therefore, negligible in comparison with the energy loss by synchrotron radiation.

References

1. H. A. Bethe, Phys. Rev. 66, 163 (1944).
2. Montgomery, et al., "Principles of Microwave Circuits", M.I.T. Rad. Lab. Series, Vol. 8 (1948).
3. Ramo and Whinnery, "Fields and Waves in Modern Radio", (1954).
4. Perry Wilson, private communication.

Appendix A. Details of Calculations for a Single Hole

From Eqs. (1), (6) and (7)

$$d = \hat{d}\, e^{-t^2/2\sigma_t^2} \qquad \text{(A.1)}$$

with

$$\hat{d} = \frac{2\epsilon_o r^3}{3} \hat{E}_o = \frac{\sqrt{2}\, r^3 Q}{3\pi\sqrt{\pi}\, a\, \sigma_\ell} \qquad \text{(A.2)}$$

The power radiated depends on $(\ddot{d})^2$ which goes as

$$(\ddot{d})^2 = \left\{ 1 - \frac{2t^2}{\sigma_t^2} + \frac{t^4}{\sigma_t^4} \right\} \frac{\hat{d}^2}{\sigma_t^4} e^{-t^2/\sigma_t^2} \qquad \text{(A.3)}$$

The integral of $(\ddot{d})^2$ over all times is easily evaluated by reference to a table of definite integrals.

$$\int (\ddot{d})^2\, dt = \frac{3\sqrt{\pi}}{4} \frac{\hat{d}^2}{\sigma_t^3}$$

So V_{loss} as defined by Eq. (8) is

$$V_{loss} = \frac{1}{Q} \frac{Z_o}{6\pi c^2} \frac{3\sqrt{\pi}}{4} \frac{\hat{d}^2}{\sigma_t^3}$$

Taking $\hat{d}$ from (A.2) and remembering that $\sigma_\ell = c\sigma_t$, I get the result shown in Eq. (9).

Appendix B. Estimate of Enhancement of the Loss Due to Coherent Effects

B 1. Radiators in free space with same phase

When two antennas are nearby, their combined radiation pattern may be quite different from the pattern of each antenna working alone. Assuming that the charge and current distribution of each antenna is not affected by the other, it is relatively easy to calculate the interference of the individual radiation fields and to obtain by integration the total power radiated by the system. If one does so, one finds that for parallel dipole antennas (dimensions small compared with $ƛ = c/\omega$), excited with the same phase, the total power radiated varies from $2P_0$ when the antennas are far apart to $4P_0$ when they are close together, where P_0 is the power radiated by each dipole acting along. The larger power is obtained if the separation of the two antennas is less than about $ƛ$.

When several antennas are involved this method becomse cumbersome, and is not, for me at least, helpful for intuitive, qualitative calculations. An alternative method seems more useful. It is based on a direct calculation of the interaction of the antennas, and it corresponds to one of the methods (the so-called "e.m.f. method") of evaluating the radiation resistance of an antenna. See, for instance, Ref. 3. I will use this method for estimating the coherence effects of diffraction from an array of holes.

Consider, first, two parallel electric dipole antennas whose lengths h are shorter than both the $ƛ = c/\omega$ of the radiation field and the separation x between the antennas and take the separation to be perpendicular to the dipole axes. Let each antenna carry the current

$$I(t) = I_0 e^{j\omega t}. \tag{B.1}$$

As is well known such an antenna acting alone will radiate energy at the average rate:

$$P_0 = \frac{Z_0}{12\cdot\pi} I_0^2 h^2 k^2 , \tag{B.2}$$

where $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ = 377 ohms, and $k = 1/\lambda\!\!\!^{-} = c/\omega$.

This power can be obtained by integrating the Poynting vector over a sphere (or any closed surface). It can also be obtained by considering the field produced by each infinitesimal current element at all other current elements and by adding up the rate at which work is done by each current element.

Now consider the two dipoles described above. The total power P being radiated will be $2P_0 + P_{21} + P_{12}$ where P_{21} is the rate at which the current in dipole number 2 is doing work against the electric force (field) of dipole number 1, and P_{12} is the obverse. In the present instance the symmetry demands that $P_{12} = P_{21}$ so that

$$P_{total} = 2(P_0 + P_{21}) \tag{B.3}$$

To evaluate P_{21} we need only calculate the field at "2" from "1" -- <u>including also the near field terms</u> -- and evaluate the work done. The field at the distance x from the dipole in the plane perpendicular to the dipole axis -- and in the direction of the positive direction of the moment -- is $E(x) = \tilde{E}(x)e^{j\omega t}$; with

$$\tilde{E}(x) = \frac{I_0 h Z_0}{4\pi x^2} e^{-jkx} \left\{ 1 + jkx + \frac{1}{jkx} \right\} . \tag{B.4}$$

The <u>average</u> rate of work done against this field by the <u>in-phase</u> current of dipole number 2 at x is

$$P_{21} = \frac{1}{2} I_0 h \,\mathcal{R}e(\tilde{E}) \tag{B.5}$$

(The average power is different from zero only for components of E(t) that are <u>in phase</u> with I(t).) It will be convenient to express P_{21} as a product of P_0 with an "enhancement factor" F.

$$P_{21} = F\, P_0 \tag{B.6}$$

I get that

$$F = \frac{3}{2k^2x^2} \left\{\cos kx + (kx - 1/kx) \sin kx\right\} \quad (B.7)$$

It looks, at first sight, as though F diverges for small x. On closer examination, however, it is found that $F \to 1$ as $x \to 0$. This is comforting, because, for very small x, the two dipoles blend into one dipole of twice the current, and from Eq. (B.2) the power should quadruple -- as it does if F = 1. For large x,

$$F \to \frac{3}{2} \frac{\sin kx}{kx} \quad (kx >> 1) \quad (B.8)$$

The form of F for intermediate x is shown by the curve of Fig. B.1. Speaking very roughly, we can say that $F \approx 1$ for $kx < 1$ and $F << 1$ for $kx > 1$. Or, physically, we conclude that the radiation from two dipoles is increased by a factor of 2 ($P \simeq 4P_0$) if that separation is less than $\lambda = 1/k$ and that, otherwise, they radiate nearly independently ($P \simeq 2P_0$).

Now consider what happens if there are n parallel dipoles in the same plane. If we can make the assumption that the current of each dipole is not changed by the presence of the other (see Appendix C), then the field seen by dipole number 1 is its own field plus the linear superposition of the fields of the other $(n-1)$ dipoles,and the total power-- from Eq. (B.5) -- is just the sum of the powers obtained by considering each dipole pair. That is, we may write that

$$P_1(\text{total}) = P_0 + P_{12} + P_{13} \cdots P_{1n}$$

$$P_2(\text{total}) = P_0 + P_{21} + P_{23} \cdots P_{2n}$$

etc...

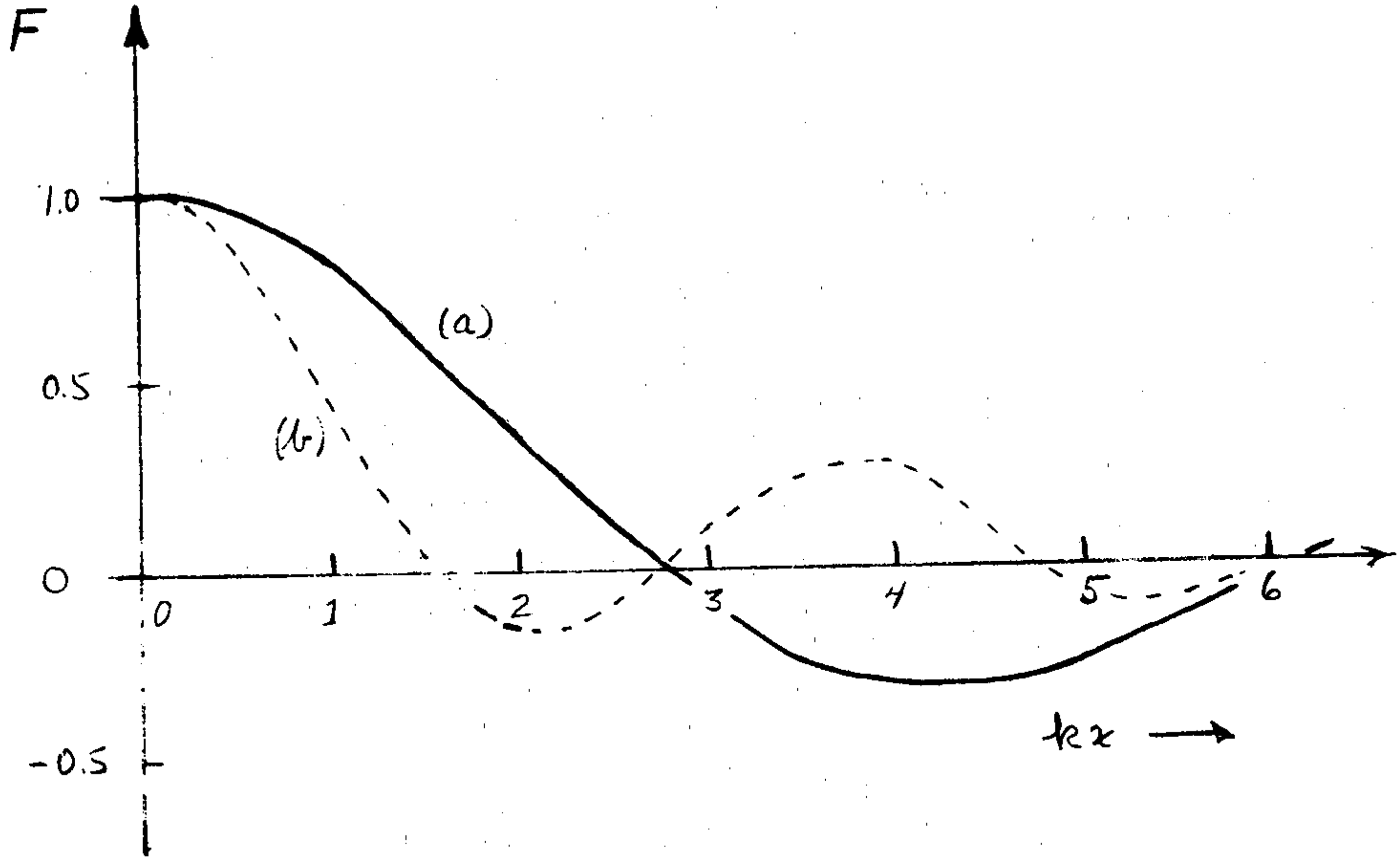

Fig. B1. Form of the Enhancement Function

(a) The function F for each radiator of a symmetric pair

(b) The average enhancement $\bar{F}$ for two radiators with a phase shift $\delta = kx$

Suppose that all n dipoles are grouped together with the maximum separation less than $\lambda\!\!\!^{-}$; then the enhancement factor F for all pairs will be equal to 1 and $P_{1(total)} = nP_0$, and similarly for the other dipoles. The total power radiated by all n dipoles is n^2P_0, as we expect for full coherence.

Next suppose that some fraction n_1 of the n dipoles are within the radius $\lambda\!\!\!^{-}$ of dipole "1", and the remainder $(n - n_1)$ are dispersed beyond $\lambda\!\!\!^{-}$. The fields of the n_1 dipoles will add at dipole "1", but the rest will make, generally, a negligible contribution. The power radiated by dipole "1" will be increased by the coherence factor $g = n_1$.

These results justify the intuitive arguments of Sect. III that were used to obtain Eq. (11).

In the remainder of this appendix I wish only to tie up a few loose ends. I will next show that roughly the same results are obtained even when the radiators are not excited in phase (as they are not in our problem). Then I will show that the results are not sensitive to the fact that the radiators are not, in fact, in free space -- that similar results hold for an array of radiators in a waveguide.

B.2 Radiators in free space with phase shift

Since the radiating dipole that represents the effect of a hole in the chamber wall is in phase with the beam current, there will be an additional phase shift in the radiation from two holes with different axial coordinates. In fact, if two holes are separated by the axial distance x, the phase shift of the Fourier component at frequency $\omega = kc$ will be $\delta = kx$. Let's say that dipole "2" is downstream of dipole "1" by the distance x. Then the field at "2" from "1" is the field of Eq. (B.4). But the current of "2" is retarded by the phase shift δ so Eq. (B.5) should be changed to

$$P_{21} = \frac{1}{2} I_0 h \, \mathcal{Re}(e^{i\delta}\hat{E}) \qquad \text{(B.9)}$$

Then we get for F_{21}

$$F_{21} = \frac{3}{2k^2x^2} \left\{ \cos (kn-\delta) + (kx - \frac{1}{kx}) \sin (kx - \delta) \right\} \qquad (B.10)$$

Now if kx = δ the term in curly brackets becomes exactly 1, and

$$F_{21} = \frac{3}{2k^2x^2} \qquad (B.11)$$

This time F_{21} does indeed become very large for small kx. For dipoles close together the energy from dipole "2" can be much larger than P_o.

Remember, however, that we no longer have symmetry between "1" and "2". In fact, the coherence factor F_{12} in the energy taken from "1" by the field of "2" is just Eq. (B.10) with the sign of δ reversed. Specifically,

$$F_{12} = \frac{3}{2k^2x^2} \left\{ \cos (kx + \delta) + (kx - \frac{1}{kx}) \sin (kx + \delta) \right\} \qquad (B.12)$$

If we look at the behavior of F_{12} for small kx when σ = kx, we find that

$$F_{12} \simeq \frac{3}{2k^2x^2} \left\{ -1 + \frac{4}{3} k^2x^2 \right\} \qquad (B.13)$$

Comparing this equation with (B.12) we see that the large power from dipole "2" at small x is compensated by an equally large power <u>given to</u> dipole "1". The dipoles are not radiating larger power only exchanging it.

For this situation it becomes convenient to use an average enhancement factor $\bar{F}$, defined by

$$\bar{F} = \frac{F_{21} + F_{12}}{2} \qquad \text{(B.14)}$$

Using (B.11) and (B.13) the average enhancement is found to be, for small kx, just equal to 1 as for F in the symmetric case. In general, when $\delta = kx$,

$$\bar{F} = \frac{3}{4k^2x^2} \left\{ 1 + \cos 2kx + (kx - \frac{1}{kx}) \sin 2\,kx \right\} \qquad \text{(B.15)}$$

The behavior of $\bar{F}$ is shown by curve (b) of Fig. B.1. It is qualtatively similar to F, so the conclusions of the preceeding subsection are still valid.

B.3 Coherence effects in propragating waveguide modes

There may be skeptics who are suspicious that interference effects among radiators may be different when they are radiating into a pipe rather than into free space. The following is for them.

Consider first a single radiator at x = 0, and consider a particular Fourier component at ω of its exciting current. For the typical frequencies in the bunch spectrum the radiator will couple into many propagating waveguide modes (because σ_ℓ = 20 mm is much less than the 100 mm width of the chamber). At each frequency the radiator will launch waves going in both directions. Consider only the waves going in the +x direction; the field amplitudes will be proportioned to

$$a_o e^{j(\omega t - kx)}. \qquad \text{(B.16)}$$

And the Poynting vector in the wave will be proportional to a_o^2 for small x.

(Note that k is now the propagation coefficient of the guide, not of free space.)

Now consider the waves of this mode that arrive at x = 0 from all the radiators upstream of x - 0. Let all the radiators be identical, and with an even spacing Δx. The wave from the n-th radiator will arrive at x = 0 with a phase shift $n\theta$, where θ is the net shift due to (a) the delay of the beam in traveling the distance Δx, and (b) the propagation delay of the wave in the guide. Namely,

$$\theta = (\frac{\omega}{c} - k)\ \Delta x. \tag{B.17}$$

In addition, the wave from each radiator will be attenuated by $e^{-n\delta}$. The complex amplitude of the wave arriving at x = 0 from the n-th radiator is then

$$a_o\ e^{-n(\delta + j\theta)} \tag{B.18}$$

The total amplitude from a large number N of radiators is then

$$\tilde{A} = \sum_{n=o}^{N} a_o\ e^{-n(\delta + j\theta)} = a_o \frac{1 - e^{-N(\delta + j\theta)}}{1 - e^{-(\delta + j\theta)}} \tag{B.19}$$

I assume that $N\delta$ is much larger than 1. Then the amplitude $\tilde{A}$ is independent of N

$$\tilde{A} = \frac{a_o}{1 - e^{-\delta - j\theta}} \tag{B.20}$$

The power P in this wave is proportional to $|\tilde{A}|^2$. Its ratio to P_o, the power in the wave from a single radiator, is

$$\frac{P}{P_o} = \frac{1}{1 + e^{-2\delta} - 2e^{-\delta}\cos\theta} \tag{B.21}$$

We may expect that for the different frequencies of the bunch k will, generally, be of the same order as ω/c, so that θ will be of the same order as $\omega\Delta x/c$ -- which is, say ≈ 0.2 for the mid-frequency of the bunch. Let's just say that θ is distributed more or less uniformly from $-\theta_0$ to $+\theta_0$ with $\theta_0 \approx \Delta x/\sigma_e$ which we assume is less than 1. With both σ and θ less than 1 we can rewrite (B.21) as

$$\frac{P}{P_0} = \frac{1}{\theta^2 + \delta^2} \tag{B.22}$$

We expect, generally, that δ will be a very small number (10^{-3} or less) while θ may be typically 0.1 or so. For such frequency components $P/P_0 \sim 1/\theta^2$. For those frequencies that are "on resonance" $\theta = 0$ and $P/P_0 \sim 1/\sigma^2$ which is a much larger number. For a best estimate of the energy loss we should take the average of P/P_0 for overall relevant frequencies. I do this roughly by assuming that θ is uniformly distributed from $-\theta_0$ to $+\theta_0$, and find that the average $\langle P/P_0 \rangle$ is

$$\langle P/P_0 \rangle = \frac{1}{\theta_0 \delta} \tag{B.23}$$

Observe that if there were no coherence at all we would expect $P_0 \approx 1/\delta$ because $1/\delta = n_{abs}$ is the number of radiators in one absorbtion distance. The wave at $x = o$ will have a power equal to $n_{abs} \cdot P_0$. On resonance ($\theta = 0$) there is an enhancement by an additional factor of n_{abs} (not suprising, since all n_{abs} radiator work in phase). Off resonance ($\theta \gg \delta$), $P/P_0 \approx 1/\theta_0^2 \approx n_\sigma^2$ which will be generally much less than n_{abs}. On the average, Eq. (B.23) says that $\langle P/P_0 \rangle = n_{abs} \cdot n$. The "normal" (non-coherent) power $n_{abs}P_0$ is increased by an enhancement factor n_σ, which is roughly the number of radiators in a bunch length. This is the enhancement factor assumed in Section III.

Appendix C. The Influence of Nearby Holes on Dipole Moments

I have assumed throughout the paper that the dipole moment of each hole was not affected by the presence of other holes. I now justify this assumption.

The dipole moment is -- see Eq. (1) -- given by

$$d_o = \frac{2\varepsilon_o r^3}{3} E_o, \tag{C.1}$$

so is proportional to the total electric field at the hole. Let E_o be of two parts E_B, the field from the beam and E' the field from a nearby hole. I will show that the assumption that $E' \ll E_B$ is self-consistent.

Take $E_o = E_B$ in Eq. (C.1). The dipole current $I_o = j\omega d_o/h$. The field E' of a nearby hole will be given by Eq. (B.4) of Appendix B. The most significant component will be the one that is <u>out of phase</u> with I_o, and, therefore, <u>in phase</u> with d_o and E_o. For small separations ($kx \ll 1$)

$$E' = \frac{I_o h\, Z_o}{4\pi x^2} \frac{1}{kx}. \tag{C.2}$$

Taking $\quad I_o h = \omega d_o = kcd_o = \dfrac{2\varepsilon_o r^3 kc}{3} E_B$

I get that (using $\varepsilon_o c\, Z_o = 1$)

$$\frac{E'}{E_B} = \frac{1}{6\pi} \frac{r^3}{x^3} \tag{C.3}$$

Since the center-to-center separation x of two holes must be at least twice the hole radius r, I get that $E'/E_B < 1/48\pi$. There are,at most,six such nearest neighbors For farther neighbors the fields are much less. So $E' \ll E_B$, always.