Electron-Cloud Intrabunch Density Modulation

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Abstract

During the passage of a proton bunch through an electron cloud, a complicated electron density modulation arises, with characteristic ring or stripe patterns that move radially outward along the bunch. We present simulation results for field-free and dipole regions, which reveal the morphology and main features of this phenomenon, explain the physical origin of the stripes in either case, and discuss the dependence on key parameters.

INTRODUCTION

When a proton or positron bunch passes through an electron cloud generated by preceding bunches, the cloud electrons are attracted towards the transverse center of the bunch or "pinched", resulting in regions of high electron density inside the bunch. This electron-cloud "pinch" gives rise to an incoherent betatron tune shift which varies with the longitudinal position and with transverse amplitude. Combined with synchrotron motion and together with the non-uniform distribution of the electron cloud around a storage ring (in the SPS, for example, the electron cloud builds up preferentially inside the dipole magnets [1]), this tune shift in turn leads to the excitation of betatron and synchro-betatron resonances [2, 3], as well as to "scattering" off these resonances [3]. For the LHC proton beam in the PS, SPS and LHC itself, these effects can be significant [4]. Some of their characteristics resemble space-charge phenomena [3, 5, 6].

Early models of the electron pinch assumed an electron density, or tune shift, that linearly increases along the bunch. Simulations and analytical treatments show that in reality, due to the nonlinear oscillation of electrons in the bunch potential, "stripes" of high density form close to the center of the bunch and then propagate outwards [7]. The presence of a dipole magnetic field restricts the horizontal motion of the electrons, and can lead to the appearance of different, "elliptical" stripes, that again start at the transverse bunch center and later shift outwards [8]. Recent studies using a refined pinch model with stripes have uncovered a complex phase-space structure, indicating the possibility of larger beam losses and stronger emittance growth than previously anticipated [6, 9].

ELECTRON MOTION

If the transverse beam size is much smaller than the vacuum chamber, we can approximate the electron cloud density in the vicinity of the beam prior to a bunch arrival by a uniform distribution. Under the influence of the electric field of the bunch, the electrons of the initially uniform cloud are perturbed and develop a structure with local density enhancements.

The electron motion in the bunch potential is characterized by the linear oscillation frequencies of electrons close to the transverse center of the beam, $\omega_{e;x,y}$. In the absence of an external magnetic field and for a round bunch the frequency is the same in both planes, and, assuming a transverse Gaussian density with rms size σ_r, equal to

$$\omega_e(z)\ \ [\mathrm{m}^{-1}] \equiv \lambda(z) r_e/\sigma_r^2\ , \qquad (1)$$

for an arbitrary longitudinal line density $\lambda(z)$. Introducing the radial coordinate $r = \sqrt{x^2+y^2}$, and its normalized counterpart $\tilde{r} \equiv r/\sigma_r$, the electron equation of motion is

$$\frac{d^2\tilde{r}}{dz^2} + \omega_e^2(z)\tilde{r} = -\frac{\omega_e^2(z)}{\tilde{r}}\left(2\left(1-e^{-\frac{\tilde{r}^2}{2}}\right) - \tilde{r}^2\right)\ , \qquad (2)$$

where the left-hand side represents the linear oscillation at small amplitudes, and the right-hand side the nonlinear terms. We observe that $\omega_e(z)$ defines the scaling of the pinch, so that e.g., for a fixed longitudinal shape, doubling the bunch intensity is equivalent to halving the bunch length, or to shrinking the transverse beam sizes by $\sqrt{2}$.

If the bunch is not round, the horizontal and vertical oscillation frequencies differ, and a second parameter or function is needed to characterize the pinch, e.g. either two frequencies $\omega_{e;x,y}$, or one together with the aspect ratio σ_y/σ_x.

In case of a strong dipole field oriented in the vertical direction, we can consider the electrons' horizontal position as frozen. The vertical force yields the equation of motion

$$\frac{d^2\tilde{y}}{dz^2} + \omega_e^2(z)\tilde{y} = -\frac{\omega_e^2(z)\tilde{y}}{\tilde{r}^2}\left(2\left(1-e^{-\frac{\tilde{r}^2}{2}}\right) - \tilde{r}^2\right)\ , \qquad (3)$$

where we have introduced a normalized vertical coordinate $\tilde{y} \equiv y/\sigma_r$, and again assumed a round beam ($\sigma_r \equiv \sigma_x = \sigma_y$). Also here $\omega_e(z)$ characterizes the electron motion completely, via (3).

For comparing results it is convenient to introduce the linear oscillation phase

$$\phi_e(z) \equiv \int_{-\infty}^{z} \omega_e(z')dz'\ . \qquad (4)$$

which for two specific longitudinal profiles translates to

$$\phi_e(z) = \frac{r_e N_b}{2\sigma_r^2}\left\{\begin{array}{ll} \left(1+\mathrm{erf}\left(\frac{z}{\sqrt{2}\sigma_z}\right)\right) & \text{(Gaussian)} \\ (1+2z/l_b) & \text{(uniform)} \end{array}\right. .$$

SIMULATION PARAMETERS

The simulation employs about 100,000 macro-electrons which are launched evenly distributed in the transverse space on a wide rectangular grid extending to $\pm 30\sigma_{x,y}$ with initial velocity. The initial electron energy of a few electron-volt can be neglected, since the typical electron kinetic energy acquired during the pinch is much larger, of order $m_e(c\omega_e\sigma_r)^2/2$, where m_e denotes the electron mass and c the speed of light. For simplicity, we will consider only circular symmetric bunches, for which the electron motion is described by (2) or (3). The bunch parameters in our simulation represent the LHC beam at injection: We consider $N_b = 1.15 \times 10^{11}$ protons per bunch, with transverse rms size $\sigma_{x,y} = 0.88$ mm and rms bunch length $\sigma_z = 11.4$ cm. The transverse beam distribution is taken to be Gaussian; in the longitudinal plane we choose either a Gaussian or a uniform shape. The zero of the longitudinal coordinate z coincides with the bunch center.

STRIPE STRUCTURE

Figure 1 presents the simulated density enhancement in the $x-z$ plane at $y = 0$ (left pictures) and also in a parallel plane with 2σ vertical offset (right pictures). The top pictures show results for a field-free region, the bottom ones for a dipole field. In all cases, about 4 stripes emerge during the passage of the bunch. For the field-free region the electron density at the center of the bunch, at $y = 0$, becomes very high (note the different density scale). For a plane with vertical offset, $y = 2\sigma_y$, the stripe structure becomes clearly visible also in the field-free case. Figure 2 shows the corresponding density in the $x-y$ plane at the longitudinal position $z = +\sigma_z$.

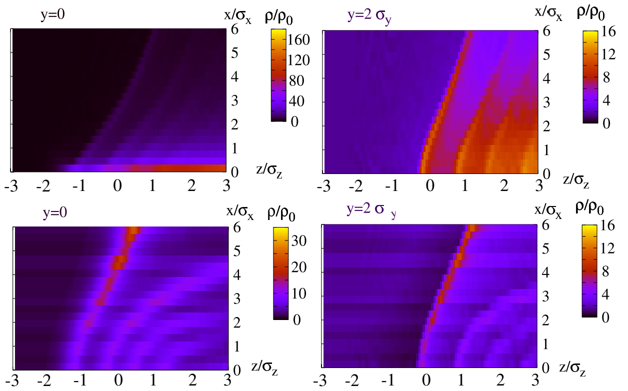

Figure 1: Electron density enhancement in the $x-z$ plane at $y = 0$ (left) and $y = 2\sigma_r$ (right) in a field-free region (top) and in a dipole (bottom), for a Gaussian bunch.

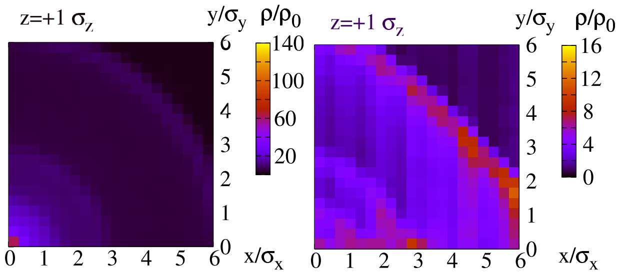

Figure 2: Electron density enhancement in the transverse plane at $z = \sigma_z$ without field (left) and in a dipole (right).

Analyzing these data, the left picture of Fig. 3 presents the simulated vertical position of the outermost (horizontal) stripe as a function of its horizontal position. The stripe for the dipole is almost of the same round circular shape in the $x-y$ plane as the one without field, and only slightly flatter. The right picture shows that the density in the stripe hardly varies with vertical position, both without field and in a dipole.

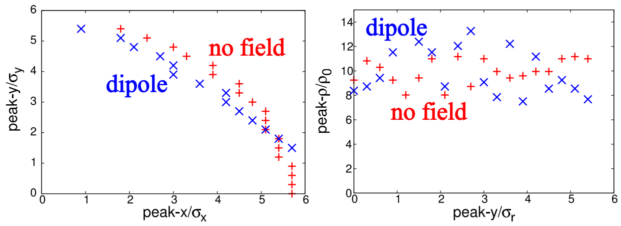

Figure 3: Vertical vs. horizontal position of the outermost stripe at $z = 1\sigma_z$, i.e. $\phi_e \approx 1.45 \times 2\pi$ (left), and the peak density in this stripe vs. the vertical position (right), comparing a field-free region and a dipole magnet.

Simulations were also performed for a longitudinal uniform bunch profile with a full bunch length chosen equal to $\sqrt{2\pi}\sigma_z$. Figure 4 demonstrates that, when plotted as a function of ϕ_e instead of z, the spatial distribution and magnitude of the density enhancement are similar, albeit not fully identical, for the Gaussian and uniform longitudinal profiles. Always a new "stripe" emerges on axis roughly at every half period of linear oscillation, starting from $\pi/2$, i.e. at $\phi_e = \pi/2, 3\pi/2$, etc.

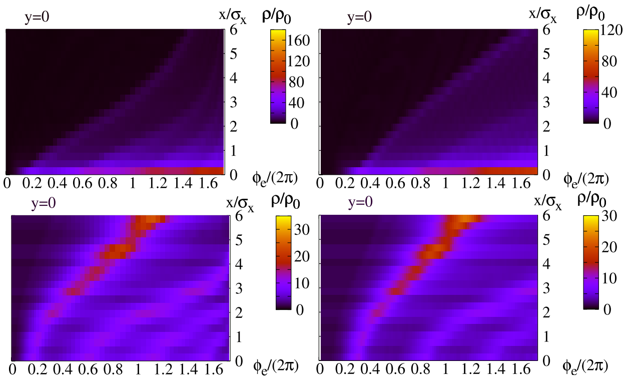

Figure 4: Electron density enhancement in the $x-\phi_e$ plane, at $y = 0$, with a Gaussian (left) or uniform longitudinal profile (right) without field (top) and in a dipole (bottom).

DISCUSSION

Comparing the top and bottom pictures in Figs. 1 or 4, we notice that while in a field-free region the electron density of a stripe decreases as the latter "moves" to larger amplitudes, in a dipole field the density increases instead. This is further illustrated in the right picture of Fig. 5 which shows the simulated density in the outermost stripe, in the plane $y = 0$, as a function of its horizontal position. The two lines were fitted by eye through the simulation data for amplitudes $\hat{x} > 1\sigma_r$. They correspond to a linear peak density evolution of $\hat{\rho}/\rho_0 \approx (17 - \hat{x}/\sigma_r)$ in a field-free region, and $\hat{\rho}/\rho_0 \approx (11 + \hat{x}/\sigma_r)$ inside a dipole.

The physical origin of the stripe patterns differs for the field-free region and for the dipole field, as is illustrated in Fig. 6. Without magnetic field, the electrons move radially. At large amplitudes they undergo a highly nonlinear

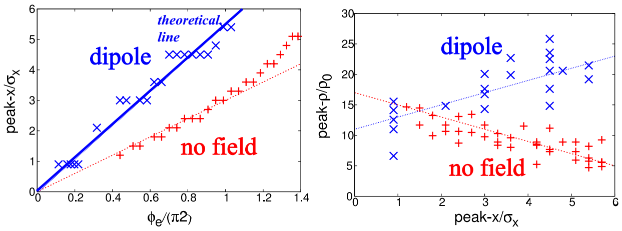

Figure 5: Horizontal position of the outermost stripe vs. electron oscillation phase ϕ_e along the bunch (left), and the density enhancement in this "stripe" as a function of horizontal stripe position (right), for a field-free region and a dipole. The solid line on the left is the prediction derived from (7); the other three lines are "eyeball" fits.

motion, losing their synchronization (left picture of Fig. 6). The stripes in physical space are the result of projecting the electron distribution winding around the phase-space origin onto the x axis. To estimate the amplitude of the turnover point we may consider electrons which started their journey at $\tilde{r}_0 \gg 1$, so that we can approximate the force in (3) by its asymptotic form $\propto 1/\tilde{r}$. Integration yields

$$\int_{\tilde{r}_0}^{\tilde{r}} \frac{dr'}{\sqrt{\ln\left(\tilde{r}_0/r'\right)}} = -\sqrt{\pi}\tilde{r}_0 \ \mathrm{erf}\left(\ln \frac{\tilde{r}_0}{\tilde{r}}\right) = 2\phi_e(z) \ . \quad (5)$$

An inversion of this relation would give $\tilde{r}$ as a function of r_0, and the extreme point of this function $\tilde{r}(\tilde{r}_0)$ be an estimate for the location $|x|$ of the outermost stripe. More empirically, the simulation data for an intermediate range of ϕ_e values, $0.4 \lesssim \phi_e \lesssim 1$, can be described by $|x|/\sigma_r \approx 3\phi_e(z)/(2\pi)$, which is shown by a thin dashed line in the left picture of Fig. 5; for larger values of ϕ_e the stripe distance from the origin grows faster than linearly. After crossing the beam axis the electrons spread out uniformly in all directions, and the peak density decreases inversely with distance $\tilde{r}$, which may explain the shrinking electron density in the right picture of Fig. 5.

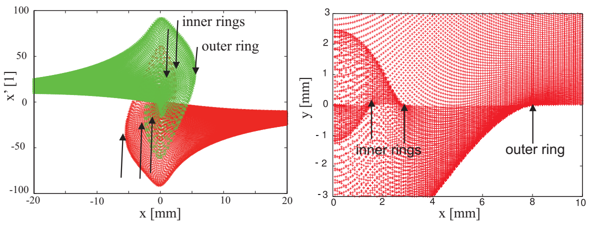

Figure 6: Snapshot at $z = 1\sigma_z$, of electrons in horizontal phase space for a field-free region (left) and in the $x - y$ plane for a dipole field (right). The two colors on the left distinguish electrons which started on either side of the beam. The right picture shows electrons launched at $y > 0$.

In a dipole field, as the horizontal amplitude $|x|$ increases the vertical electron motion becomes approximately linear over a larger and larger vertical range, $|y| \lesssim |x|$, and, therefore, an ever greater number of electrons reach the central plane $y = 0$ simultaneously. The regions of increased density on the horizontal axis correspond to the crossing of $y = 0$ by large groups of electrons oscillating in the linear vertical beam field at x positions where the local vertical phase advance $\phi_e(x, z)$ is equal to $\pi/2$ plus a multiple of π (right picture of Fig. 6). More specifically, for amplitudes $|y| \lesssim |x|$ and $|x| \gg \sigma_r$, (3) can be approximated as

$$\frac{d^2\tilde{y}}{dz^2} + \frac{2\omega_e^2(z)}{(x/\sigma_r)^2}\tilde{y} \approx 0 \ , \quad (6)$$

i.e., the local vertical oscillation frequency depends on x as

$$\omega_{e,y}(z, x) \approx \sqrt{2}\omega_e(z)\sigma_r/|x| \ , \quad (7)$$

where $\omega_e(z)$ is the central oscillation frequency (1). Accordingly, the location of a "stripe" starting at $\phi_e(z) \approx k\pi/2$ (k integer) near the origin, should later, at larger horizontal amplitudes, be described by $|x|/\sigma_r \approx 4\sqrt{2}\phi_e(z)/(2\pi)/k$. The location of the outermost stripe ($k = 1$) is expected at $|x|/\sigma_r \approx 5.66\phi_e(z)/(2\pi)$, which, in the left picture of Fig. 5, is superimposed on the dipole simulation data as a thick solid line. Data and analytical curve are in nearly perfect agreement. The region where the vertical motion is approximately linear and, therefore, also the number of "synchronized" electrons crossing the $y = 0$ plane at the same time grow in proportion to the distance $|x|$, which may explain the density evolution for a dipole field seen in the right picture of Fig. 5.

CONCLUSIONS

The accumulated phase advance of the linear electron oscillation $\phi_e(z)$ determines the spatial structure of the electron pinch, almost independently of the longitudinal bunch profile. The pinch structure is also affected by the presence or absence of a magnetic field. In all cases considered, the high-density "stripes" are approximately circular rings in the $x - y$ plane. At every phase advance value ϕ_e equal to a $\pi/2$ pluS a multiple of π a new stripe emerges close to the beam axis. The physical origin of the stripes is different in the field-free and dipole case, which explains why in a field-free region the peak electron density decreases as a "stripe" shifts outwards, while in a magnetic field the peak electron density grows at larger amplitudes. For either case the simulation indicates a linear dependence of the stripe density on its distance from the axis, with a slope of $+1$ or -1, respectively, in normalized units. An analytical function describes the variation of the horizontal stripe position with ϕ_e for a dipole field in good agreement with the simulation. The same dependence for the field-free case can be modelled in general analytical terms and by an eyeball fit.

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