MEASUREMENT OF HORIZONTAL BEAM SIZE USING SEXTUPOLE MAGNETS

J.A. Crittenden, K. Deitrick, H. Duan, G.H. Hoffstaetter, V. Khachatryan and D.C. Sagan
CLASSE, Cornell University, Ithaca NY 14853

Abstract

The quadratic dependence of sextupole fields on position results in a beam-size-dependent kick on a beam traversing a sextupole magnet. A change in sextupole strength changes the closed orbit and the tune of the beam in a storage ring. Measuring both therefore allows conclusions about the beam size in the sextupole. Here we derive the pertinent formula and discuss the applicability to storage rings. In particular we investigate the measurement accuracy that can be achieved at the Cornell High Energy Synchrotron Source. The Cornell Electron-positron Storage Ring underwent a major upgrade in 2018 with the goal of reducing the emittance by a factor of four. A variety of beam size measurement methods have been developed to monitor the positron beam size, including visible synchrotron light and interferometry. We investigate the sensitivity of the sextupole method and compare to other measurement techniques. The design horizontal emittance of the 6-GeV positron beam is about 30 nm-rad with typical beam sizes of about 1 mm, setting the scale for the required accuracy in the beam-size measurement.

CONCEPT

Our method for measuring beam sizes in sextupole magnets was inspired by a private communication,[1] and has not, to our knowledge, been developed elsewhere.

The variation of a sextupole strength by an amount $\mathrm{d}k_2 l$ in a storage ring introduces

1. a quadrupole kick $\mathrm{d}k_1 l$

$$\mathrm{d}k_1 l = X_0\ \mathrm{d}k_2 l + (k_2 l + \mathrm{d}k_2 l)\,\mathrm{d}x, \tag{1}$$

2. and a dipole kick $\mathrm{d}x'$

$$\begin{aligned}\mathrm{d}x' &= \frac{1}{2}\left(X_0^2 + \sigma^2\right)\mathrm{d}k_2 l \\ &+ \frac{1}{2}\left(2\ X_0\ \mathrm{d}x + \mathrm{d}x^2\right)(k_2 l + \mathrm{d}k_2 l),\end{aligned} \tag{2}$$

where l is the length of the sextupole, $\mathrm{d}x$ is the displacement of the beam from the initial offset of the beam from the center of the sextupole, X_0 is that initial offset, and σ is the beam size.

[1] Reinhard Brinkman, private communication to Georg Hoffstaetter (2001)

ANALYTIC SOLUTION

The two equations permit the elimination of the unknown value X_0 to determine the beam size from the measured values of $\frac{\mathrm{d}k_1 l}{\mathrm{d}k_2 l}$ and $\frac{\mathrm{d}x}{\mathrm{d}k_2 l}$:

$$\begin{aligned}\sigma^2 &= \frac{4\tan(\pi Q)}{\beta}\frac{\mathrm{d}x}{\mathrm{d}k_2 l} - \left(\frac{\mathrm{d}k_1 l}{\mathrm{d}k_2 l}\right)^2 \\ &+ (k_2 l\frac{\mathrm{d}x}{\mathrm{d}k_2 l})^2\left(1 + \frac{\mathrm{d}k_2 l}{k_2 l}\right),\end{aligned} \tag{3}$$

where the values prior to the variation are the fractional tune Q, the sextupole strength $k_2 l$, and β. We note that no terms have been neglected in this derivation.

The judicious choice for the initial value of the sextupole strength $k_2 l = 0$, together with the fact that a kick $\mathrm{d}x'$ causes a closed orbit change at the location of the kick $\mathrm{d}x$ given by

$$\mathrm{d}x = \frac{\beta\ \cot(\pi Q)}{2}\mathrm{d}x', \tag{4}$$

results in the simple relationship

$$\sigma^2 = 2\frac{\mathrm{d}x'}{\mathrm{d}k_2 l} - \left(\frac{\mathrm{d}k_1 l}{\mathrm{d}k_2 l}\right)^2. \tag{5}$$

MEASUREMENTS

At the Cornell Electron-positron Storage Ring, values for $\frac{\mathrm{d}k_1 l}{\mathrm{d}k_2 l}$ of a few mm infer values for $\frac{\mathrm{d}x'}{\mathrm{d}k_2 l}$ of about 10^{-5} radians/m^{-2}, or, equivalently, values for $\frac{\mathrm{d}x}{\mathrm{d}k_2 l}$ of about 5×10^{-5} m/m^{-2}.

We are investigating the uncertainties associated with three types of measurement:

1. betatron tunes to obtain the quadrupole kick via $\mathrm{d}\nu = \beta\ \mathrm{d}k_1 l$. Our tune measurements derive from two sources: 1) we operate the Digital Tune Tracker [1] continuously during the measurements, obtaining about 20 measurements at intervals of 3 seconds for each sextupole setting, 2) following three phase function measurements at each sextupole setting, we record 32k turns for each of 100 beam position monitors (BPMs). This data is post-processed to obtain tune measurements with accuracy of about one part in 10^4. The combination of these two tune measurement methods provides an accuracy of about 0.003%. Figure 1 shows an example of ten difference measurements obtained from eleven sextupole settings. The value of $\Delta k_2 l$ ranges 0.1248 to 2.496 m^{-2}. The value of $\Delta k_1 l$

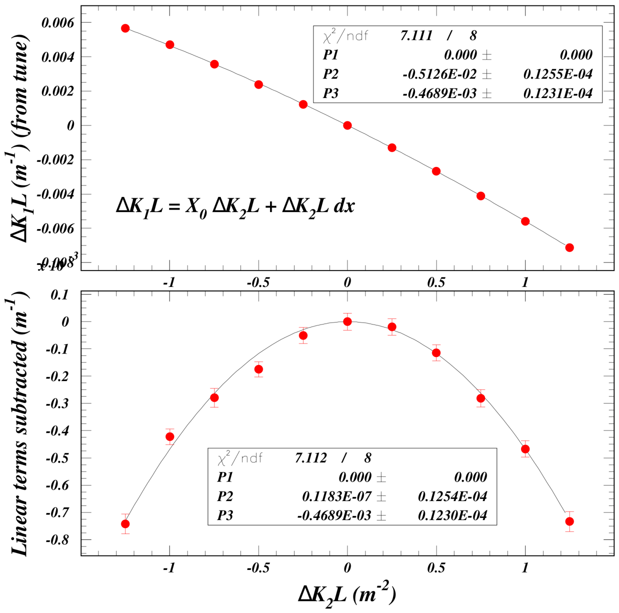

Figure 1: Quadrupole kick values calculated via $\Delta k_1 l = \nu/\beta$ as a function of the change in sextupole strength $\Delta k_2 l$. The initial offset X_0 is determined to be 5.126 ± 0.013 mm. The quadratic term shows the dependence of the orbit change in the sextupole on the change in its strength. The lower plot shows the dependence with the linear term subtracted, observed to be purely parabolic to good accuracy.

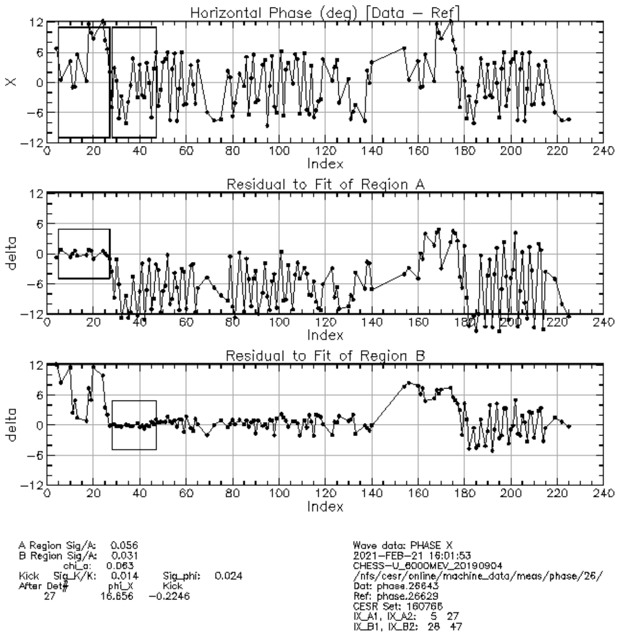

Figure 2: Results of a wave analysis applied to the difference in measured phase function for two sextupole strength settings which differ by $\Delta k_2 l = 1.25$ m^{-2}. The upper plot shows the difference in the phase function, the middle plot show the residuals of the fit to the upstream 23 BPMs and the bottom plot shows the residuals of the fit to the 19 downstream BPMs.

shows a quite pure quadratic dependence on $\Delta k_2 l$. Recalling Eq. 1, we can identify the linear coefficient as the initial offset X_0, and the quadratic term as relating to dx. The initial offset X_0 is determined to be 5.126 ± 0.013 mm. The lower plot shows the result for the fit when the linear terms are constrained to be small. The weights have been chosen to give χ^2/NDF=1, which gives relative uncertainties in the measurements of about 1.5×10^{-4}. The determination of the offset is accurate to 0.2%, and the second order coefficient is accurate to 3%.

One approach we are considering is an optimization of $\Delta k_1 l(\Delta k_2 l)$ with fit parameters such as the beam size σ, the initial offset X_0, the initial value for the beta function at the sextupole β, the sextupole strength calibration constant, and the initial fractional tune value Q_0. Correlations and sensitivities remain to be studied.

2. orbit measurements at the BPMs to infer the beam position at the sextupole as a function of sextupole strength using our model of the optics. While providing an independent measure, the required accuracy of a few microns can be expected to present significant challenges to both the BPM measurement accuracy and the accuracy of the optics model.

3. wave analyses of measured phase and orbit functions to obtain the kicks directly, as described in Ref. [2]. Figure 2 shows the results for such an analysis of the difference of the measured phase function for a sextupole strength change of $\Delta k_2 l = 1.25$ m^{-2}. The upper plot shows the difference in the phase function, the middle plot show the residuals of the fit to the upstream 23 BPMs and the bottom plot shows the residuals of the fit to the 19 downstream BPMs. The location of the kick is correctly identified as the sextupole, which is immediately downstream of BPM 27. The quadrupole kick is determined to be $\Delta k_1 l = -0.2240/\beta$ m^{-1} with an accuracy of 1.4%. The dipole kick is obtained through the analogous optimization applied to the measured orbit difference.

Figure 3 shows the quadrupole and dipole kicks obtained from the wave analyses as functions of sextupole strength, the central point corresponding to the analysis shown in Fig. 2 above. The polynomial fits are constrained to zero for zero $k_2 l$ difference and the scale chosen to by symmetric about that point to reduce correlations.

This means of measuring the quadrupole kick does not show the clean parabolic behavior observed for the tune dependence. We have found that the results depend on the choice of fit ranges at the level of a few percent. This systematic effect must be eliminated by further investigations into the choice of fit range, tuned individually to each sextupole.

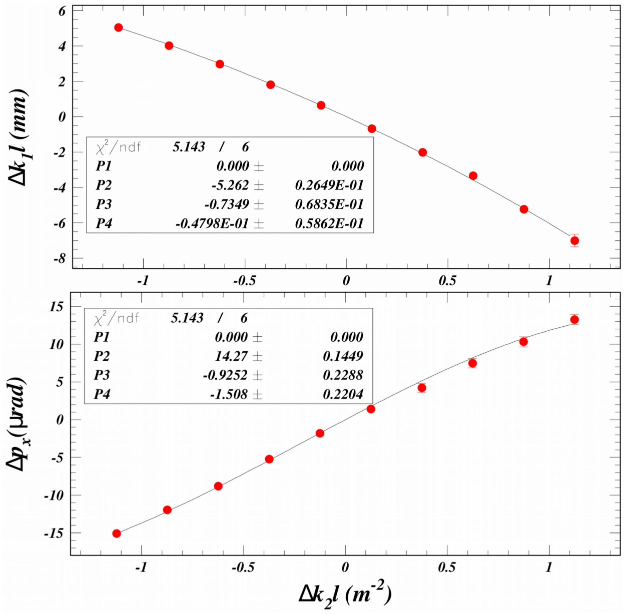

Figure 3: Quadrupole and dipole kick values determined from the wave analyses of the difference phase and orbit measurements as functions of sextupole strength. The polynomial fits are been constrained to zero for zero $k_2 l$ difference. The complicated polynomial behavior points to systematic fluctuations likely associated with the choice of fit ranges.

SUMMARY

We present a status report on measurements to determine the beam size at each of the 76 sextupoles in the Cornell Electron-positron Storage Ring. After describing the concept behind the method and deriving the beam size as a function of dependence of the quadrupole and dipole kicks on the change in sextupole strength, we enumerate three types of measurements we expect to prove useful in the analysis. These are 1) betatron tune measurements, 2) orbit difference measurements and 3) wave analyses of measured phase and orbit differences. Such measurements serve to determine not only the beam sizes, but also the beam position relative to the center of the sextupole and the sextupole calibration in each case. While the precision obtained in these measurements appears quite promising, systematic effects and optimization techniques remain to be investigated.

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