PROGRESS ON THE MEASUREMENT OF BEAM SIZE USING SEXTUPOLE MAGNETS

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Abstract

Variations in strength of a sextupole magnet in a storage ring result in changes to the closed orbit, phase functions and tunes which depend on the position of the beam relative to the center of the sextupole and on the beam size. Such measurements have been carried out with 6 GeV positrons at the Cornell Electron Storage Ring. The initial analysis presented at IPAC21 has been extended to both transverse coordinates, introducing additional tune shifts and coupling kicks caused by skew quadrupole terms arising from the vertical position of the positron beam relative to the center of the sextupole. Variations of strength in each of the 76 sextupoles provide measurements of difference orbits, phase and coupling functions. An optimization procedure applied to these difference measurements determines the horizontal and vertical orbit kicks and the normal and skew quadrupole kicks corresponding to the the strength changes. Continuously monitored tune shifts during the sextupole strength scans provide a redundant, independent determination of the two quadrupole terms. Following the recognition that the calculated beam size is highly correlated with the calibration of the sextupole, a campaign was undertaken to obtain precise calibrations of the sextupoles and to measure their offsets relative to the reference orbit, which is defined by the quadrupole centers. We present the measured distributions of calibration correction factors and sextupole offsets together with the accuracy in their determination.

2D ANALYTIC DERIVATION FOR BEAM SIZE DETERMINATION USING SEXTUPOLE STRENGTH CHANGE

Following the line of argument of our IPAC21 paper [1] to derive the quadrupole kick $\mathrm{d}k_1 l$ and the dipole kicks $\mathrm{d}x'$ and $\mathrm{d}y'$ from a change in sextupole strength $\mathrm{d}k_2 l$ using the sextupole field components $\frac{ql}{p_0} B_\mathrm{x} = k_2 l x y$ and $\frac{ql}{p_0} B_\mathrm{y} = \frac{1}{2} k_2 l (x^2 - y^2)$, we obtain three equations with four unknowns:

$$\mathrm{d}k_1 l = \mathrm{d}k_2 l \,(X_0 + \mathrm{d}x) \tag{1}$$

$$\mathrm{d}y' = \mathrm{d}k_2 l \,(X_0 + \mathrm{d}x)\,(Y_0 + \mathrm{d}y) \tag{2}$$

$$2\,\mathrm{d}x' = \mathrm{d}k_2 l \left[\left(\frac{\mathrm{d}y'}{\mathrm{d}k_2 l}\right)^2 \left(\frac{\mathrm{d}k_1 l}{\mathrm{d}k_2 l}\right)^{-2} + \sigma_\mathrm{Y}^2 - \left(\frac{\mathrm{d}k_1 l}{\mathrm{d}k_2 l}\right)^2 - \sigma_\mathrm{X}^2 \right] \tag{3}$$

We note that these quantities are differences, not differentials. The equations are exact; there is no expansion.

Assuming initial $k_2 l = 0$ and including all terms:

$$\sigma_\mathrm{X}^2 - \sigma_\mathrm{Y}^2 = -2\,\frac{\mathrm{d}x'}{\mathrm{d}k_2 l} + \left(\frac{\mathrm{d}y'}{\mathrm{d}k_2 l}\right)^2 \left(\frac{\mathrm{d}k_1 l}{\mathrm{d}k_2 l}\right)^{-2} - \left(\frac{\mathrm{d}k_1 l}{\mathrm{d}k_2 l}\right)^2 \tag{4}$$

Including only terms linear in $\mathrm{d}k_2 l$, we have:

$$\sigma_\mathrm{X}^2 - \sigma_\mathrm{Y}^2 = -2\,\frac{\mathrm{d}x'}{\mathrm{d}k_2 l} + Y_0^2 - X_0^2, \tag{5}$$

where X_0, Y_0 is the initial position of the beam relative to the center of the sextupole. The is the two-dimensional generalization of Eq. 5 in our IPAC21 paper.

ANALYSIS OF DIFFERENCE ORBIT AND PHASE MEASUREMENTS: EXAMPLE

Record phase and orbit measurements for eleven sextupole settings. Reference the ten sets of measurements with nonzero K_2 settings to the K_2=0 orbit and phase measurements. Fit for the linear terms in $\Delta b_1(\Delta k_2 l)$, $\Delta y'(\Delta k_2 l)$ and $\Delta x'(\Delta k_2 l)$. We obtain an estimate for the measurement uncertainties in Δb_1, $\Delta y'$ and $\Delta x'$ by setting them such that the χ^2/NDF is unity. The results are shown in Fig. 1 .

The linear term for Δb_1 gives the initial horizontal position of the beam relative to the sextupole center: $X_0 = 4.943 \pm 0.029$ mm. The linear term for Δp_y gives the initial value for the product of horizontal and vertical beam positions relative to the sextupole center: $X_0 Y_0 = 3.79 \pm 0.13$ mm^2. From this we obtain $Y_0 = 0.766 \pm 0.26$ mm. The linear term for Δp_y ($-12.45 \pm 0.28 \times 10^{-6}$ rad/m^{-2}) and Eq. 5 are used to calculate the value $\sigma_x^2 - \sigma_y^2 = 1.03 \pm 0.53$ mm^2. The vertical beam size is typically 20x smaller than the horizontal, so we can deduce with good accuracy $\sigma_x = 1.01 \pm 0.26$ mm. This result is consistent with the value expected from the optics.

FIRST-ORDER 2D ANALYSIS OF TUNE SHIFTS FROM NORMAL (b_1) AND SKEW QUAD (a_1) TERMS

Defining the normal and skew quad multipole coefficients,

$$b_1 = \frac{qL}{P_0}\,\frac{\mathrm{d}B_\mathrm{Y}}{\mathrm{dx}} = \mathrm{K}_2 L\,\mathrm{x} \tag{6}$$

$$a_1 = \frac{qL}{P_0}\,\frac{\mathrm{d}B_\mathrm{X}}{\mathrm{dx}} = \mathrm{K}_2 L\,\mathrm{y} \tag{7}$$

we have the familiar results for the tune shifts from the normal quad term:

$$\Delta\mu_x = -b_1 \beta_x / 2 \tag{8}$$

$$\Delta\mu_y = b_1 \beta_y / 2 \tag{9}$$

The tune shifts from the skew quad terms can be shown [2] to be

$$\Delta\mu_x = -a_1^2 \frac{\beta_x \beta_y \sin\mu_y}{4\left(\cos\mu_x - \cos\mu_y\right)} \tag{10}$$

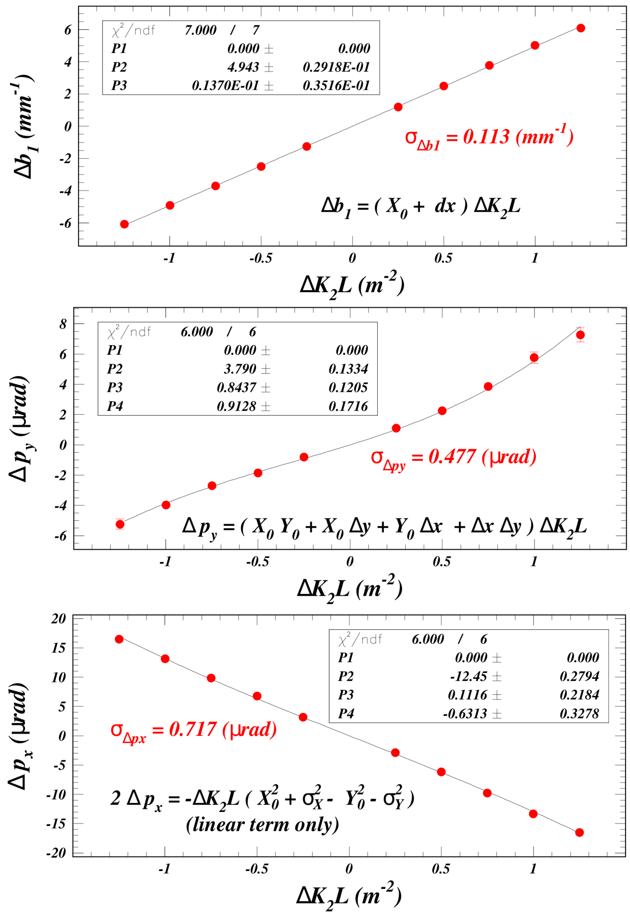

Figure 1: These three plots show the results of fitting difference orbit and phase measurements with values for the normal quadrupole kick (Δb_1, top), and the vertical ($\Delta y'$, middle) and horizontal ($\Delta x'$, bottom) dipole kicks at the sextupole when its strength dk_2l was varied. The annotations show that the linear terms in these polynomial fits provide values for the horizontal and vertical beam positions relative to the center of the sextupole, as well as for the squared beam size difference $\sigma_x^2 - \sigma_y^2$.

$$\Delta\mu_y = a_1^2 \frac{\beta_x \beta_y \sin\mu_x}{4\left(\cos\mu_x - \cos\mu_y\right)} \tag{11}$$

Superposing the two contributions to the tunes and isolating a_1 and b_1, we obtain their values as functions of known quantities when the tune shifts are measured:

$$\sin\mu_x \Delta\mu_x + \sin\mu_y \Delta\mu_y = \frac{-b_1}{2}\left(\beta_x \sin\mu_x - \beta_y \sin\mu_y\right) \tag{12}$$

$$\beta_y \Delta\mu_x + \beta_x \Delta\mu_y = a_1^2 \frac{\beta_x \beta_y \left(\beta_x \sin\mu_x - \beta_y \sin\mu_y\right)}{4\left(\cos\mu_x - \cos\mu_y\right)} \tag{13}$$

The second equation shows that

$$b_1 = \frac{\Delta\mu_y}{\beta_y} - \frac{\Delta\mu_x}{\beta_x} \tag{14}$$

is more independent of a_1 than b_1 derived from either $\Delta\mu_x$ or $\Delta\mu_y$ alone.

The 2D calculations of b_1 and a_1 are sensitive to cancellation divergences. The values of the initial tunes are such that $\sin\mu_x \simeq -0.4$ and $\sin\mu_y \simeq -0.8$. Thus the formula for b_1 and a_1 both diverge for $\beta_x \simeq 2\beta_y$.

For simplicity of presentation, we have used here the approximation $\cos(\mu + \Delta\mu) - \cos\mu = \Delta\mu \sin\mu$. This approximation breaks down near the half-integer resonance. In fact, the quadratic term in Fig. 1 of Ref. [1] was later shown to arise from this approximation, rather than from the quadratic term $dk_2l\ dx$. With this approximation removed, the quadratic term is consistent with the horizontal beam motion arising from the sextupole strength term. Our choice, however, is to use only linear terms in the calculation of beam size, since these are much more accurately determined.

SEXTUPOLE CALIBRATION AND OFFSET MEASUREMENT PROCEDURE

Inspection of the equation for the beam size shows 1) the most accurate beam size measurement is obtained when the beam is at the center of the sextupole, since the uncertainties in X_0 and Y_0 do not contribute, and 2) in that case, the value of the beam size and the uncertainty in dk_2l are perfectly correlated. The procedure for determining the beam size is at cross purposes to obtaining the sextupole calibration, since one cannot calibrate a sextupole with the beam at the center of it.

The procedure developed for obtaining the calibration of each sextupole consisted of measuring the horizontal and vertical tune changes for a given change in k_2 for five beam positions set by a closed bump. For this analysis we neglect small corrections arising from the beam motion consequential to the beam size.

The example of such measurements shown in Fig. 2 again uses the method of estimating uncertainties in the slope and offset determinations by adjusting the residual weights. The slopes are of opposite sign and approximately in the ratio of the beta values, modulo a coupling contribution (e.g. vertical sextupole offset.) The two measurements provide independent determinations of the horizontal sextupole offset relative to the reference orbit.

The calibration correction factors were obtained by using the beta-weighted difference of the horizontal and vertical tune shifts, which is insensitive to skew contributions and thus largely independent of vertical offset of the sextupole as can be inferred from the full 2D derivation of the tune shifts presented in the previous section.

RESULTS FOR SEXTUPOLE CALIBRATION CORRECTION FACTORS AND HORIZONTAL OFFSETS

The procedure described in the previous section resulted in the values for the calibration correction factors and horizontal offsets shown in Fig. 3 for the 76 sextupoles in the

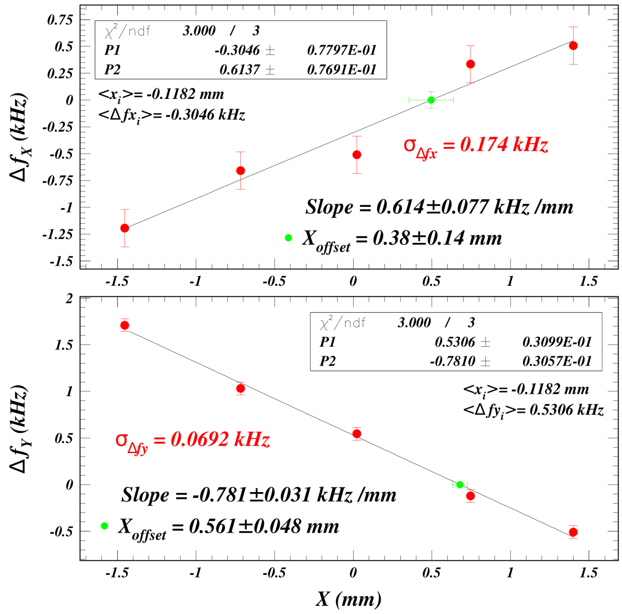

Figure 2: Example of the measurement and analysis procedure to obtain the calibration correction factor and horizontal offset relative to the reference orbit, which is defined by the quadrupole centers. See text for details.

east and west arcs of the CESR ring. Horizontal and vertical tunes were measured by shaking the beam and locking to the tune [3]. Thirty-two single-pole-filtered 60-Hz samples were averaged, resulting in tune measurement RMS fluctuations between about 20 and 200 Hz. Those values are 174 Hz (horizontal) and 69 Hz (vertical) in the example shown in the previous section. This procedure required about 3 minutes per sextupole. The accuracy was shown to improve when additional 1-second averaging was included. For an additional 16 measurements, the tune accuracy improved by nearly a factor of four and the duration increased from three to ten minutes.

The calibration correction factor is derived from a measured/theory ratio for the beta-weighted tune shift differences, where the theory value assumes the nominal calibration value used for the sextupoles during operations. A rough estimate of 5% for the variations due to construction tolerances was made during the initial field measurements in 1998 [4]. Our measurements show an RMS deviation of 9.5% with a mean value of 1.009 ± 0.010. The uncertainties average 2.9% with an RMS spread of 2.0%.

The values of the horizontal offsets relative to the quadrupole centers are obtained by identifying the horizontal position at the sextupole which results in zero tune shift. The weighted average of the two values shown in the example in the previous section provides the value 0.543 ± 0.045 mm The values for all sextupoles are shown in Fig 3. With several exceptions, the RMS spread in the offsets is found to be 0.83 mm. The uncertainties average 43 microns with an RMS spread of 28 microns.

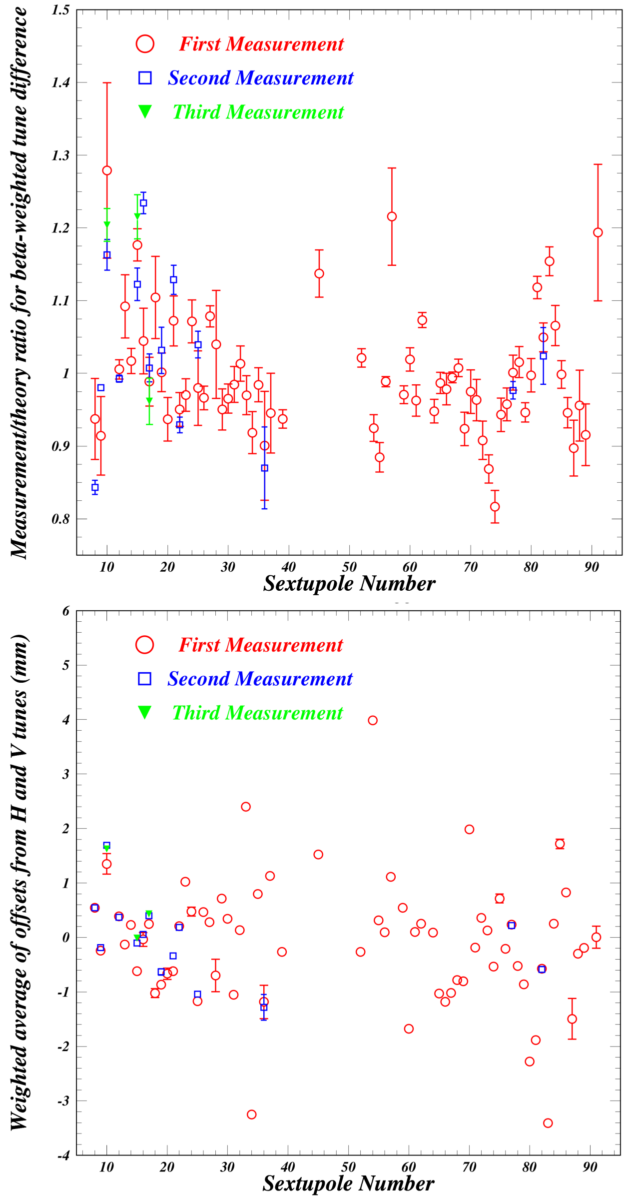

Figure 3: Results for the calibration correction factor (top) and horizontal offset (bottom) for each of the 76 CESR sextupoles. The RMS spread in correction factors (offsets) is 9.5% (0.83 mm).

SUMMARY

These measurements improve our model of the CESR ring optics and provide valuable information to our beam size analysis.

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