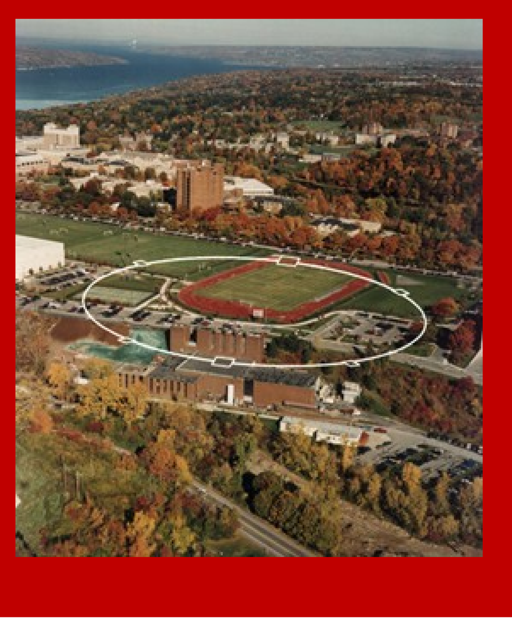
13th International Particle Accelerator Conference
Bangkok, Thailand
June 12th – 17th, 2022

Progress on the Measurement of Beam Size Using Sextupole Magnets †



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Abstract

Variations in strength of a sextupole magnet in a storage ring result in changes to the closed orbit, phase functions and tunes which depend on the position of the beam relative to the center of the sextupole and on the beam size. Such measurements have been carried out with 6 GeV positrons at the Cornell Electron Storage Ring. The initial analysis presented at IPAC21 has been extended to both transverse coordinates, introducing additional tune shifts and coupling kicks caused by skew quadrupole terms arising from the vertical position of the positron beam relative to the center of the sextupole. Variations of strength in each of the 76 sextupoles provide measurements of difference orbits, phase and coupling functions. An optimization procedure applied to these difference measurements determines the horizontal and vertical orbit kicks and the normal and skew quadrupole kicks corresponding to the the strength changes. Continuously monitored tune shifts during the sextupole strength scans provide a redundant, independent determination of the two quadrupole terms. Following the recognition that the calculated beam size is highly correlated with the calibration of the sextupole, a campaign was undertaken to calibrate the sextupoles and measure their offsets relative to the reference orbit, which is defined by the quadrupole centers. We present the measured distributions of calibration correction factors and sextupole offsets together with the accuracy in their determination.

2D Analytic Derivation

Following the line of argument of our IPAC21 paper to derive the quadrupole kick $\mathbf{dk_1}\boldsymbol{l}$ and the dipole kicks $\mathbf{dx'}$ and $\mathbf{dy'}$ from a change in sextupole strength $\mathbf{dk_2}\boldsymbol{l}$ using the sextupole field components $\frac{ql}{p_0}B_x = k_2xy$ and $\frac{ql}{p_0}B_y = \frac{1}{2}k_2(x^2-y^2)$, we obtain three equations with four unknowns:

$$\mathrm{d}k_1l = \mathrm{d}k_2l\,(X_0+\mathrm{d}x)$$

$$\mathrm{d}y' = \mathrm{d}k_2l\,(X_0+\mathrm{d}x)\,(Y_0+\mathrm{d}y)$$

$$2\,\mathrm{d}x' = -\mathrm{d}k_2l\left[-\left(\frac{\mathrm{d}y'}{\mathrm{d}k_2l}\right)^2\left(\frac{\mathrm{d}k_1l}{\mathrm{d}k_2l}\right)^{-2} - \sigma_Y^2 + \left(\frac{\mathrm{d}k_1l}{\mathrm{d}k_2l}\right)^2 + \sigma_X^2\right].$$

Assuming initial $k_2l = 0$ and including all terms:

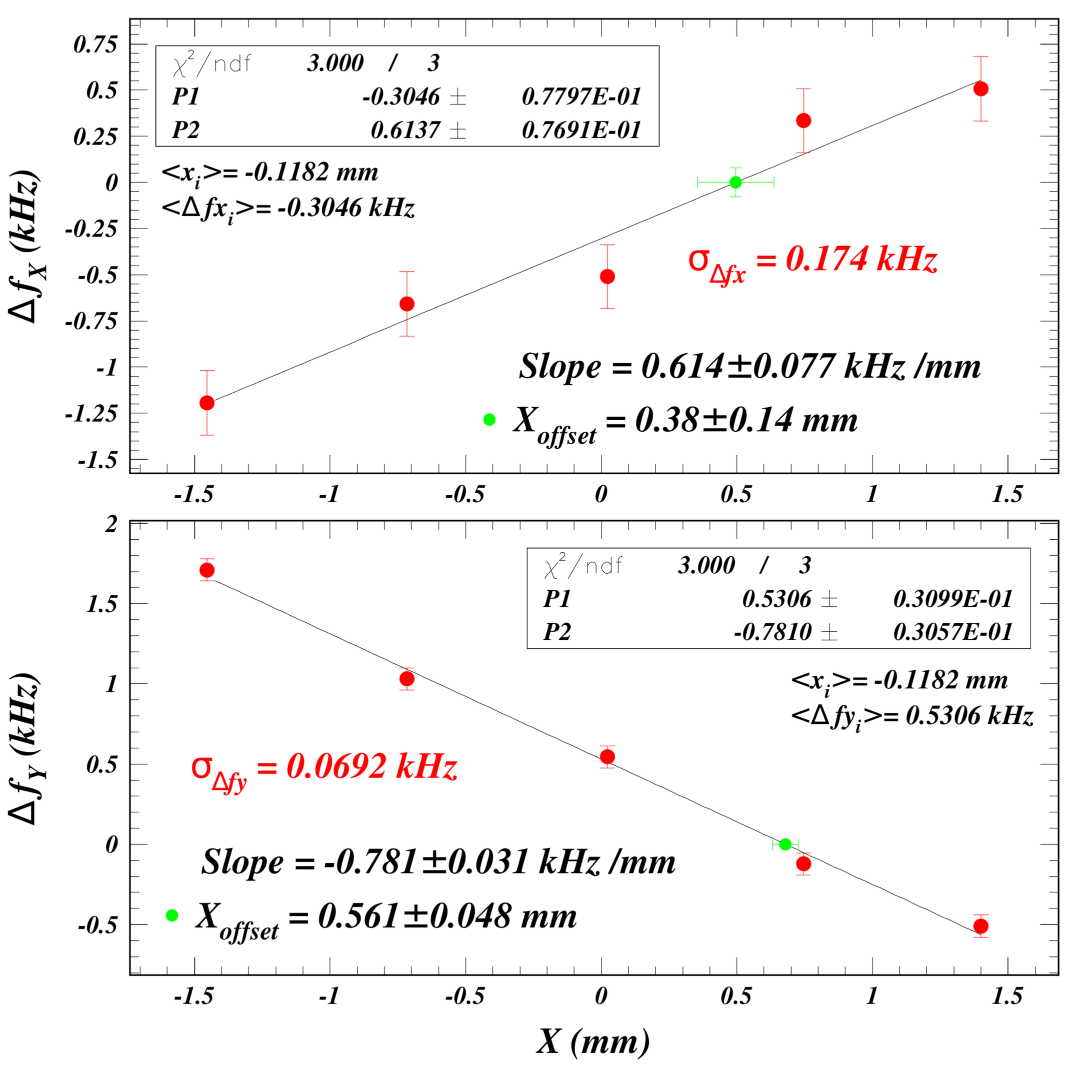
$$\sigma_X^2 - \sigma_Y^2 = -2\,\frac{\mathrm{d}x'}{\mathrm{d}k_2l} + \left(\frac{\mathrm{d}y'}{\mathrm{d}k_2l}\right)^2\left(\frac{\mathrm{d}k_1l}{\mathrm{d}k_2l}\right)^{-2} - \left(\frac{\mathrm{d}k_1l}{\mathrm{d}k_2l}\right)^2.$$

We note that these quantities are differences, not differentials. The equations are exact; there is no expansion. Including only terms linear in $\mathbf{dk_2}\boldsymbol{l}$ we have:

$$\sigma_X^2 - \sigma_Y^2 = -2\,\frac{\mathrm{d}x'}{\mathrm{d}k_2l} + Y_0^2 - X_0^2,$$

where X_0, Y_0 is the initial position of the beam relative to the center of the sextupole. This the two-dimensional generalization of Eq. 5 in our IPAC21 paper.

Sextupole Calibration and Offset Measurement



First-order 2D analysis of tune shifts from normal (b_1) and skew quad (a_1) terms

Defining the normal annd skew quad multipole coefficients,

$$b_1 = \frac{1}{2!}\frac{qL}{P_0}\frac{\mathrm{d}B_Y}{\mathrm{dx}} = K_2L\,x \qquad a_1 = \frac{1}{2!}\frac{qL}{P_0}\frac{\mathrm{d}B_X}{\mathrm{dx}} = K_2L\,y$$

we have the familiar results for the tune shifts from the normal quad term:

$$\Delta\mu_x = -b_1\beta_x/2 \qquad \Delta\mu_y = b_1\beta_y/2$$

The tune shifts from the skew quad terms can be show to be (see, for example, Linear analysis of coupled lattices, D.Sagan and D.Rubin,PRSTAB volume 2, 074001 (1999))

$$\Delta\mu_x = -a_1^2\frac{\beta_x\beta_y\sin\mu_y}{4\,(\cos\mu_x-\cos\mu_y)} \qquad \Delta\mu_y = a_1^2\frac{\beta_x\beta_y\sin\mu_x}{4\,(\cos\mu_x-\cos\mu_y)}$$

Superposing the two contributions to the tunes and isolating $a1$ and $b1$, we obtain their values as functions of known quantities when the tune shifts are measured:

$$\sin\mu_x\Delta\mu_x + \sin\mu_y\Delta\mu_y = \frac{-b_1}{2}\,(\beta_x\sin\mu_x - \beta_y\sin\mu_y)$$

$$\beta_y\Delta\mu_x + \beta_x\Delta\mu_y = a_1^2\,\frac{\beta_x\beta_y\,(\beta_x\sin\mu_x - \beta_y\sin\mu_y)}{4\,(\cos\mu_x-\cos\mu_y)}$$

The second equation shows that $b_1 = 2\left(\frac{\Delta\mu_y}{\beta_y} - \frac{\Delta\mu_x}{\beta_x}\right)$ is much more independent of a_1 than b_1 derived from either $\Delta\mu_x$ or $\Delta\mu_y$ alone.

Inspection of the equation for the beam size shows 1) the most accurate beam size measurement is obtained when the beam is at the center of the sextupole, since the uncertainties in X_0 and Y_0 do not contribute, and 2) in that case the value of the beam size and the uncertainty in dk_2l are perfectly correlated. The procedure for determining the beam size is at cross purposes to obtaining the sextupole calibration, since one cannot calibrate a sextupole with the beam at the center of it.

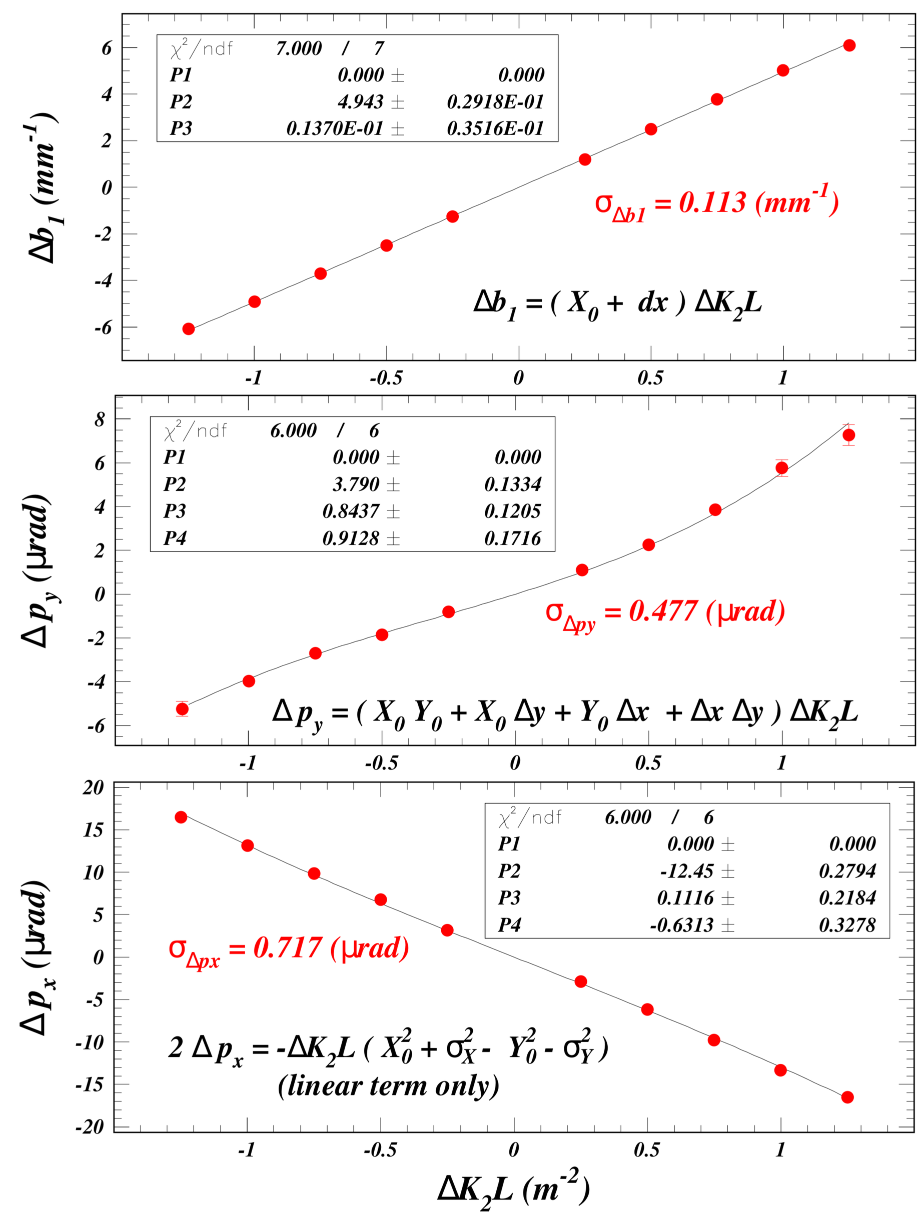
The procedure developed for obtaining the calibration of each sextupole consisted of measuring the horizontal and vertical tune changes for a given change in k_2 for five beam positions set by a closed bump. For this analysis we neglect small corrections arising from the beam motion consequential to the beam size.

The example of such measurements shown here again uses the method of estimating uncertainties in the slope and offset determinations by adjusting the residual weights. The slopes are of opposite sign and approximately in the ratio of the beta values, modulo a coupling contribution (e.g. vertical sextupole offset.) The two measurements provide independent determinations of the horizontal sextupole offset relative to the reference orbit.

The calibration correction factors were obtained by using the beta-weighted difference of the horizontal and vertical tune shifts, which is insensitive to skew contributions and thus largely independent of vertical offset of the sextupole, as can be inferred from the full 2D derivation of the tune shifts shown on the left.

The calculations of b_1 and a_1 are sensitive to cancellation divergences. The values of the tunes at the operations working point are such that $\sin(\mu_x) \simeq -0.39$ and $\sin(\mu_y) \simeq -0.77$. Thus both b_1 and a_1 diverge for $\beta_x \simeq 2\ \beta_y$.

Analysis of Difference Orbit and Phase Measurements



Record phase and orbit measurements for eleven sextupole settings. Reference the ten sets of measurements with nonzero K_2 settings to the K_2=0 orbit and phase measurements. Fit for the linear terms in $\Delta b_1(\Delta k_2 l)$, $\Delta x'(\Delta k_2 l)$, $\Delta y'(\Delta dk_2 l)$.

We obtain an estimate for the measurement uncertainties in Δb_1, $\Delta x'$ and $\Delta y'$ by setting them such that the λ^2/NDF is unity.

The linear term for Δb_1 gives the initial horizontal position of the beam relative to the sextupole center.

$$X_0 = 4.943 \pm 0.029 \text{ mm}$$

The linear term for Δp_y gives the initial value for the product of horizontal and vertical beam positions relative to the sextupole center.

$$X_0 Y_0 = 3.79 \pm 0.13 \text{ mm}^2$$

From this we obtain

$$Y_0 = 0.766 \pm 0.26 \text{ mm}$$

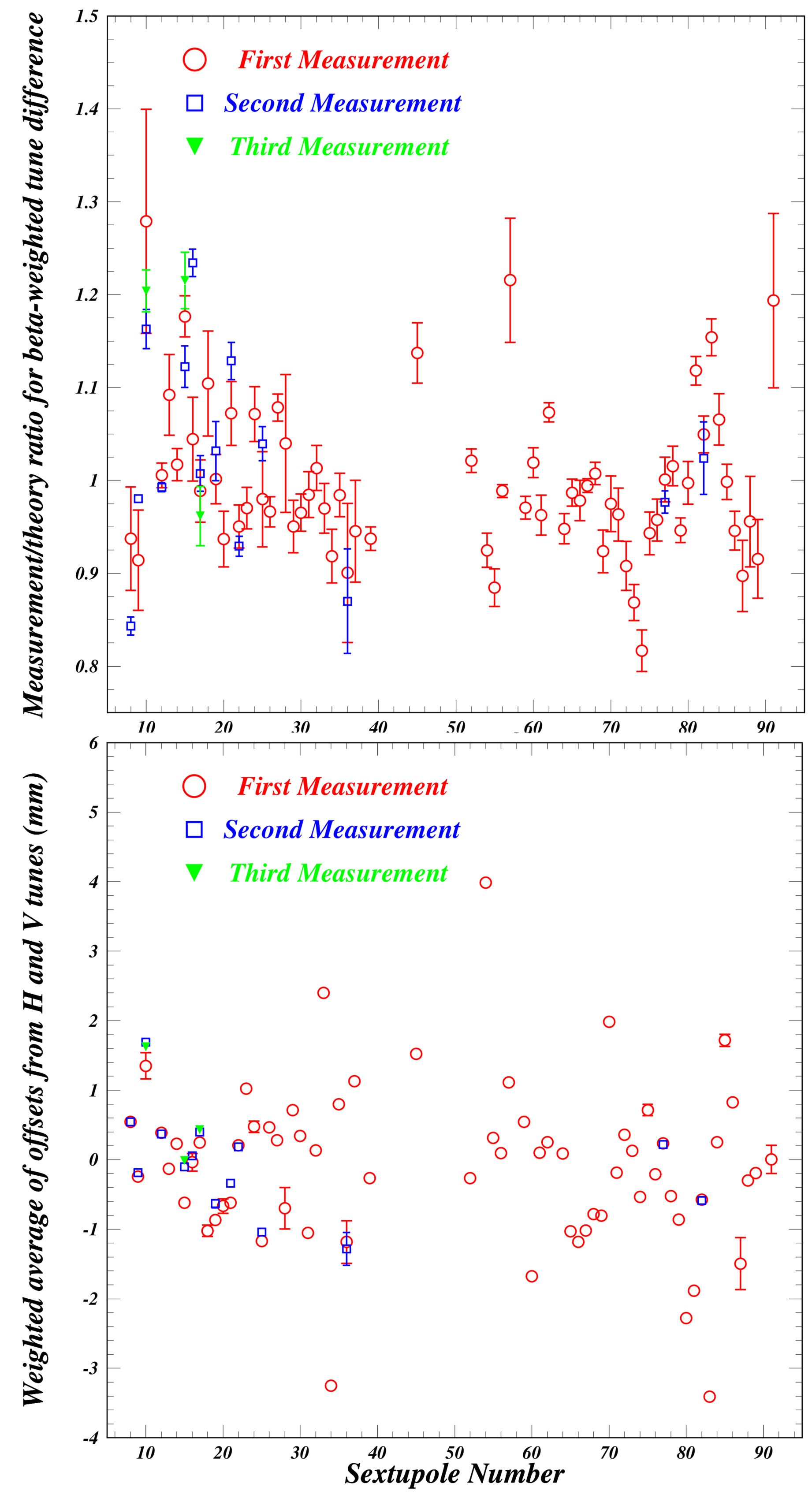
The linear term for Δp_x ($-12.45 \pm 0.28\times10^{-6}$ rad/m^{-2}) can be used to calculate the value

$$\sigma_x^2 - \sigma_y^2 = 1.03 \pm 0.53 \text{ mm}^2.$$

The vertical beam size is typically 20x smaller than the horizontal, so we can deduce with good accuracy

$$\sigma_x = 1.01 \pm 0.26 \text{ mm}$$

Results of the Calibration and Offset Measurements



The procedure described in the previous section resulted in the values for the calibration correction factors and horizontal offsets shown on the left for the 76 sextupoles in the east and west arcs of the CESR ring. Horizontal and vertical tunes were measured by shaking the beam and locking to the tune. Thirty-two single-pole-filtered 60-Hz samples were averaged, resulting in tune measurement fluctuations between about 20 and 200 Hz. Those values are 174 Hz (horizontal) and 69 Hz (vertical) in the example shown in the previous section. This procedure required about 3 minutes per sextupole. The accuracy was shown to improve when additional 1-second averaging was included. For an additional 16 measurements, the tune accuracy improved by nearly a factor of four and the duration increased from three to ten minutes.

The calibration correction factor is derived from a measured/theory ratio for the beta-weighted tune shift differences, where the theory value assumes the nominal calibration value used for the sextupoles during operations. A rough estimate of 5% for the variations due to construction tolerances was made during the initial field measurements in 1998. Our measurements show an RMS deviation of 9.5% with a mean value of 1.009 ± 0.010. The uncertainties average 2.9% with an RMS spread of 2.0%.

The values of the horizontal offsets relative to the quadrupole centers are obtained by identifying the horizontal position at the sextupole which results in zero tune shift. The weighted average of the two values shown in the example in the previous section provide the value 0.543 ± 0.045 mm. The values for all sextupoles are shown in the plot on the left. With several exceptions, the RMS spread in the offsets is found to be 0.83 mm. The uncertainties average 43 microns with an RMS spread of 28 microns.

These measurements improve our model of the CESR ring optics and provide valuable information to our beam size analysis.

† This work is supported by National Science Foundation award number DMR-1829070.

CLASSE facilities are operated with major support from the National Science Foundation.

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