STUDY OF THE SYSTEMATIC ERROR CONTRIBUTIONS TO THE MEASUREMENT OF BEAM SIZE USING SEXTUPOLE MAGNETS

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Abstract

We present a study of the systematic uncertainties in beam size determination using sextupole strength variations. Variations in strength of a sextupole magnet in a storage ring result in changes to the closed orbit, phase functions and tunes which depend on the initial position of the beam relative to the center of the sextupole and on the beam size. Using the 6 GeV positron beam at the Cornell Electron-positron Storage Ring (CESR), we present two measurement methods for the position of the beam at the sextupole prior to its strength change: 1) using the horizontal and vertical betatron tune changes with sextupole strength, and 2) using the linear term in the dependence of quadrupole and skew quadrupole kicks produced by the sextupole. These kick values are determined from polynomial fits to the difference orbits and phase functions arising from the sextupole strength changes. Results for both horizontal and vertical misalignments are presented. Modeling studies to assess possible nonlinear effects are under development.

BEAM SIZE DETERMINATION USING SEXTUPOLE STRENGTH CHANGE

The sextupole field components $\frac{qL}{p_0}B_\mathrm{x} = K_2 L x y$ and $\frac{qL}{p_0}B_\mathrm{y} = \frac{1}{2}K_2 L(x^2 - y^2)$ can be used to derive expressions for the quadrupole kick Δb_1, the skew quadrupole kick Δa_1 and the dipole kicks Δp_x and Δp_y from a change in sextupole strength $\Delta K_2 L$ as follows. Assuming initially $K_2 = 0$, a change in sextupole strength leads to

$$\Delta b_1 = \Delta K_2 L\,(X_0 + \Delta\mathrm{x}) \quad (1)$$

$$\Delta a_1 = \Delta K_2 L\,(Y_0 + \Delta\mathrm{y}) \quad (2)$$

$$\Delta p_\mathrm{y} = \Delta K_2 L\,(X_0 + \Delta\mathrm{x})\,(Y_0 + \Delta\mathrm{y}) \quad (3)$$

$$\Delta p_\mathrm{x} = \frac{1}{2}\Delta K_2 L\left[(Y_0 + \Delta\mathrm{y})^2 + \sigma_\mathrm{y}^2 - (X_0 + \Delta\mathrm{x})^2 - \sigma_\mathrm{x}^2\right], \quad (4)$$

where we have integrated the Lorentz force over the transverse Gaussian bunch distribution of rms widths σ_x and σ_y. The quantities X_0 and Y_0 denote the initial horizontal and vertical positions of the beam relative to the center of the sextupole prior to the strength change. Including only terms linear in $\Delta K_2 L$, we have

$$\sigma_\mathrm{X}^2 - \sigma_\mathrm{Y}^2 = -2\,\frac{\Delta p_\mathrm{X}}{\Delta K_2 L} + Y_0^2 - X_0^2. \quad (5)$$

Since early 2021, we have performed a set of measurements of increasing sophistication and accuracy, presenting the results in Refs. [1] and [2]. Here we present a status report on our investigations into the precision of our beam size calculations. The requirements of micron- and sub-microradian-level orbit measurement accuracy entails a detailed model of the CESR optics. We begin with a means of measuring the sextupole alignments which is improved over that reported previously [2].

ACCURACY IN THE DETERMINATION OF HORIZONTAL AND VERTICAL MISALIGNMENTS

Our first method of determining X_0, the horizontal distance of the beam from the center of the sextupole prior to changing the strength of the sextupole K_2, is to derive the ΔK_1 value from the beta-weighted difference of horizontal and vertical tune measurements according to

$$\Delta K_1 L = \frac{\Delta\mu_y}{\beta_y} - \frac{\Delta\mu_x}{\beta_x} \quad (6)$$

derived in Ref. [2]. This calculation is more insensitive to skew quadrupole contributions than the value derived from either $\Delta\mu_x$ or $\Delta\mu_y$ alone.

Our tune measurements derive from two sources: 1) we operate the Digital Tune Tracker [3] continuously during the measurements, obtaining about 20 measurements at intervals of 3 seconds for each sextupole setting, 2) following three phase function measurements at each sextupole setting, we record turn-by-turn orbit data, 32k turns for each of 126 beam position monitors (BPMs). This data is post-processed to obtain tune measurements with an accuracy of about one part in 10^4. The combination of these two tune measurement methods provides an accuracy of about 0.003‰. Figure 1 shows an example of ten difference measurements obtained from eleven sextupole settings. We employ a method for estimating uncertainties in the polynomial coefficients by adjusting the residual weights to obtain χ^2/NDF=1. The linear term provides us with a value for X_0 of -2.3532 ± 0.0092 mm. The estimate for the $\Delta K_1 L$ uncertainty in each point is 0.03 mm^{-1}.

A second, independent, means of determining X_0 is to record phase function and orbit measurements at each sextupole setting, then to fit the difference functions with multipole values b_1, a_1 and horizontal and vertical dipole kicks superposed on the sextupole. The fit procedure described above provides a value for X_0 of -2.4346 ± 0.0070 mm, as shown in Fig. 2.

These two methods for determining X_0 are compared in the correlation plot in Fig. 3, which includes all measurements to date. The RMS of the difference distribution (not shown here) is 0.132 mm, showing sufficient precision for measuring beam sizes of 1-2-mm. The significance of the

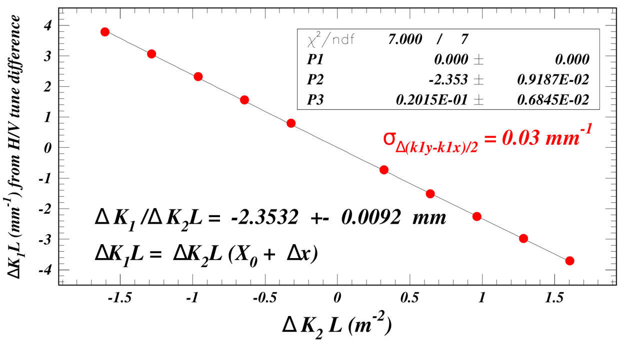

Figure 1: Quadrupole kick values K_1 derived from betatron tune changes as a function of sextupole strength change.

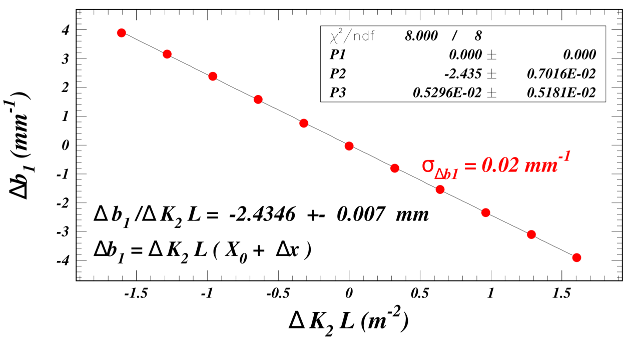

Figure 2: Quadrupole kick values Δb_1 determined using fits to phase function and orbit differences.

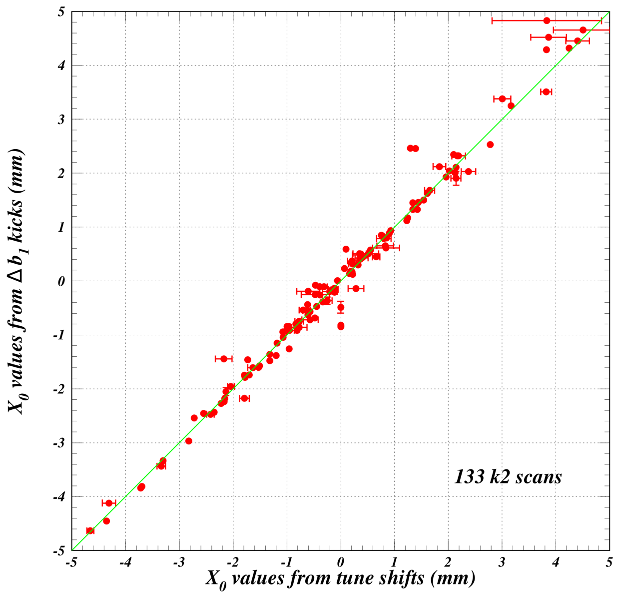

Figure 3: Degree of correlation obtained from the values for X_0 derived from tune changes and from fits to phase function and orbit differences.

good agreement between the local kick result and the ring-wide tune measurement is that the underlying assumption of linear optics is sufficiently accurate for our purposes.

The horizontal misalignment X_{offset} of the sextupole relative to the BPM coordinate system, which defines the origin as the centers of the quadrupole magnets, can now be found by determining the horizontal orbit position measurement prior to the sextupole strength change. The full statistical power of the measurements at eleven sextupole settings is shown in Fig. 4. The value for x at $K_2 = 0$ of

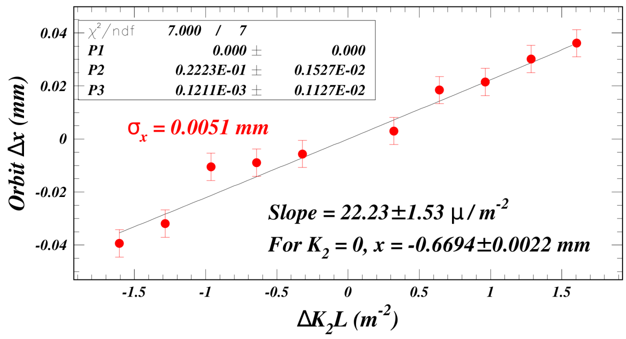

Figure 4: The horizontal orbit change Δx as a function of sextupole strength change.

-0.6694 ± 0.0022 mm yields a value for the horizontal misalignment $X_{\text{offset}} = 1.7652 \pm 0.0075$ mm. This means of determining the horizontal misalignment has two advantages over the method presented in Ref. [2], which entailed measuring tune changes with sextupole strength at prescribed orbit positions. The first is precision, since the present method uses multi-parameter fits to the entire-ring phase functions and orbit. Secondly, this method can also be used to determine vertical misalignments. Just as Eq. 1 was used above for finding the values of X_0, Eq. 2 can be used to find the value for Y_0. The corresponding analysis is shown in Fig. 5. The vertical distance of the beam from the center of the sextupole is found to be -0.4364 ± 0.0048 mm.

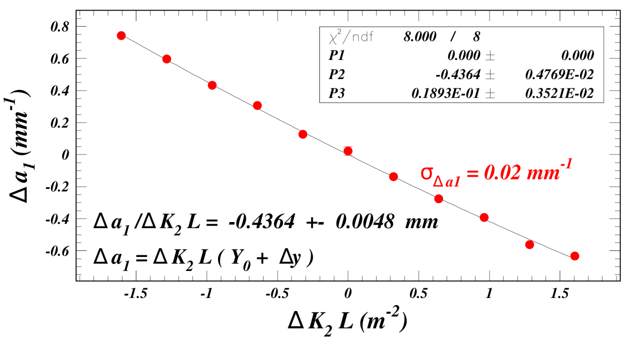

Figure 5: Skew quadrupole kick values Δa_1 determined using fits to phase function and orbit differences.

The measurement of the vertical motion of the beam in the sextupole is shown in Fig. 6. The value for y at $K_2 = 0$ of 0.1063 ± 0.0012 mm yields a value for the vertical misalignment $Y_{\text{offset}} = 0.54268 \pm 0.0049$ mm.

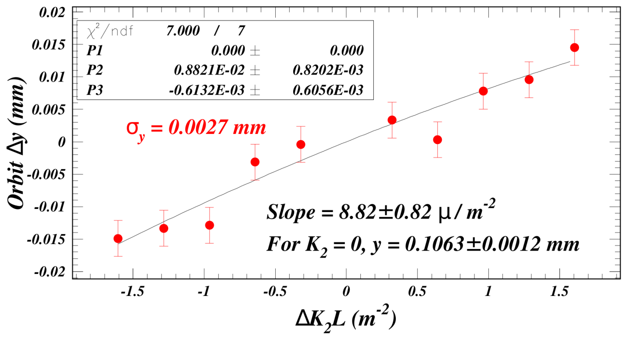

Figure 6: The vertical orbit change Δy as a function of sextupole strength change $\Delta K_2 L$.

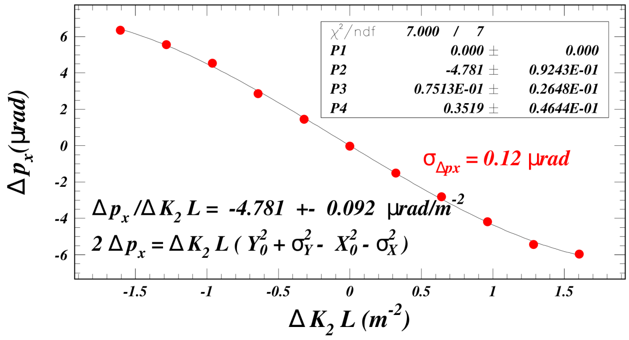

Figure 8: The horizontal orbit kick change Δp_x as a function of sextupole strength change $\Delta K_2 L$.

RESULTS FOR THE DETERMINATION OF MISALIGNMENTS

We have recorded 145 sextupole strength scans for the 76 sextupoles in the ring. The error-weighted averages of all measurements are shown in Fig. 7. Typical values for

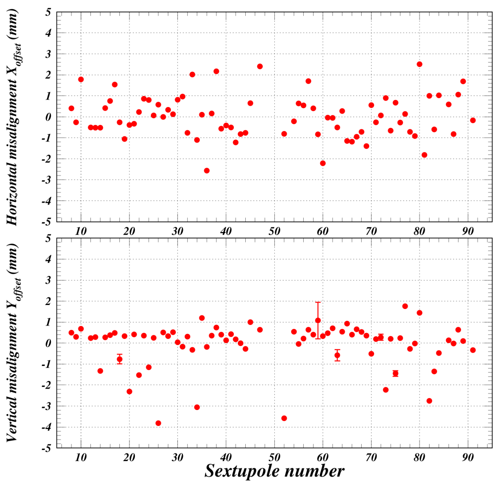

Figure 7: Weighted averages of horizontal and vertical sextupole misalignments derived from 145 sets of sextupole strength scan data.

the horizontal misalignments are 1-2 mm. The vertical misalignments are generally smaller, less than 1 mm, but with a number of exceptions up to 4 mm. The statistical uncertainties in their determination are typically 0.01 to 0.1 mm. Since beam motion in these sextupoles will lead to deviations from the assumption of linear optics implicit in our derivations, these misalignments must be included in an accurate model of the ring optics.

BEAM SIZE CALCULATION

The measurement for the remaining term in the beam size calculation (Eq. (5)) is shown in Fig. 8. The value for the horizontal orbit kick slope of -4.781 ± 0.092 µrad/m^{-2} results in a calculated value for the horizontal beam size of $\sigma_x = 1.955 \pm 0.048$ mm when neglecting the vertical beam size, which is a factor of 5 smaller according to the optics functions. We obtain at present typical beam size calculations which are significant overestimates when compared to the values expected from the emittance and Twiss functions, which give $\sigma_x = 1.09$ mm for the example at hand.

DISCUSSION

An improvement over the analysis presented in Ref. [2] achieved during the past year is our ability to perform a complete analysis of all our data sets in a few hours, thus showing results for all sextupoles, such as in Fig. 7. Our calculations of beam size generally give beam size values greater than those expected from the optics, i.e. our measured values of $\Delta p_x/\Delta K_2 L$ appear to have contributions other than those we have considered here.The prime suspects are nonlinear effects stemming from beam movement in the sextupoles around the ring when the strength of the sextupole under study changes. We are now mounting a modeling campaign to understand these effects.

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REFERENCES

[1] J. Crittenden *et al.*, "Measurement of Horizontal Beam Size Using Sextupole Magnets", in *Proc. IPAC'21*, Campinas, Brazil, May 2021, pp. 802–804. doi:10.18429/JACoW-IPAC2021-MOPAB254.

[2] J. Crittenden *et al.*, "Progress on the Measurement of Beam Size Using Sextupole Magnets", in *Proc. IPAC'22*, Bangkok, Thailand, Jun. 2022, pp. 550–552. doi:10.18429/JACoW-IPAC2022-MOPOTK040

[3] R. E. Meller and M. A. Palmer, "Digital Tune Tracker for CESR", in *Proc. PAC'11*, New York, NY, USA, Mar.-Apr. 2011, paper MOP215, pp. 504–506.