# STUDY OF THE SYSTEMATIC ERROR CONTRIBUTIONS TO THE MEASUREMENT OF BEAM SIZE USING SEXTUPOLE MAGNETS†

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***Abstract***

We present a study of the systematic uncertainties in beam size determination using sextupole strength variations. Variations in strength of a sextupole magnet in a storage ring result in changes to the closed orbit, phase functions and tunes which depend on the initial position of the beam relative to the center of the sextupole and on the beam size. Using the 6-GeV positron beam at the Cornell Electron-positron Storage Ring (CESR), we present two measurement methods for the position of the beam at the sextupole prior to the strength change: 1) using the horizontal and vertical betatron tune changes with sextupole strength, and 2) using the linear term in the dependence of quadrupole and skew quadrupole kicks. These kick values are determined from polynomial fits to the difference orbits and phase functions arising from the sextupole strength changes. Results for both horizontal and vertical misalignments are presented. Modeling studies to assess possible nonlinear effects are under development.

## Analytic Derivation of Beam Size Measurement Using Sextupole Strength Variations

The sextupole field components $\frac{qL}{p_0}B_X = K_2Lxy$ and $\frac{qL}{p_0}B_Y = \frac{1}{2}K_2L(x^2-y^2)$ can be used to derive expressions for the quadrupole kick $\Delta b_1$, the skew quadrupole kick $\Delta a_1$ and the dipole kicks $\Delta p_X$ and $\Delta p_Y$ from a change in sextupole strength $\Delta K_2L$ as follows. Assuming initial $K_2 = 0$ and including the parabolic and cubic terms,

$$\Delta b_1 = \Delta K_2L\,(X_0+\Delta x) \tag{1}$$

$$\Delta a_1 = \Delta K_2L\,(Y_0+\Delta y) \tag{2}$$

$$\Delta p_Y = \Delta K_2L\,(X_0+\Delta x)\,(Y_0+\Delta y) \tag{3}$$

$$\Delta p_X = \frac{1}{2}\Delta K_2L\left[(Y_0+\Delta y)^2+\sigma_Y^2-(X_0+\Delta x)^2-\sigma_X^2\right], \tag{4}$$

where we have integrated the Lorentz force over the transverse Gaussian bunch distribution of widths $\sigma_X$ and $\sigma_Y$. The quantities $X_0$ and $Y_0$ denote the initial horizontal and vertical positions of the beam relative to the center of the sextupole prior to the strength change. The sign of the horizontal orbit kick is given by the convention that it is positive toward the outside of the ring. Including only terms linear in $\Delta K_2L$, we have

$$\sigma_X^2-\sigma_Y^2 = -2\,\frac{\Delta p_X}{\Delta K_2L}+Y_0^2-X_0^2. \tag{5}$$

Since early 2021, we have performed a set of measurements of increasing sophistication and accuracy, presenting the results at IPAC'21 and IPAC'22. Here we present a status report on our investigations into the contributions to the precision of our beam size calculations. The requirements of micron- and sub-microradian-level orbit measurement accuracy entails a detailed model of the CESR optics. We begin with a means of measuring the sextupole alignments which is improved over that reported IPAC'22.

## Accuracy in the Determination of Horizontal and Vertical Misalignments

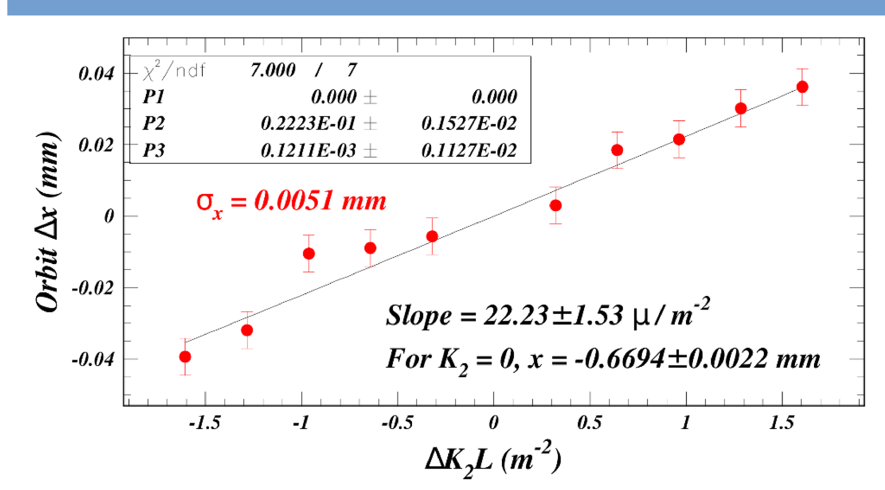



Figure 4: The horizontal orbit change $\Delta x$ as a function of sextupole strength change.

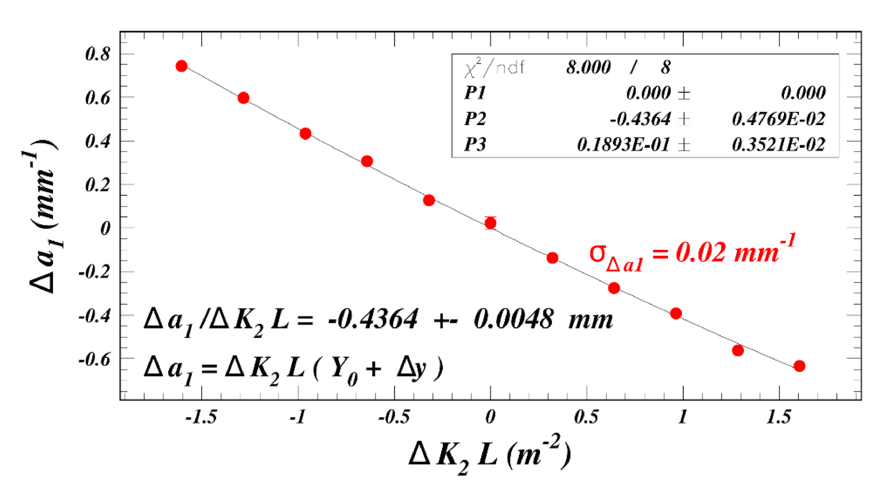



Figure 5: Skew quadrupole kick values $\Delta a_1$ determined using fits to phase function and orbit differences.

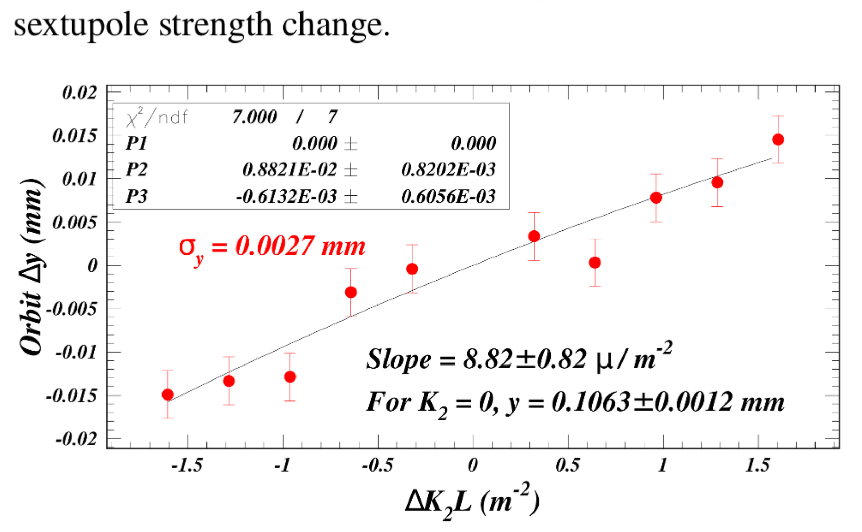
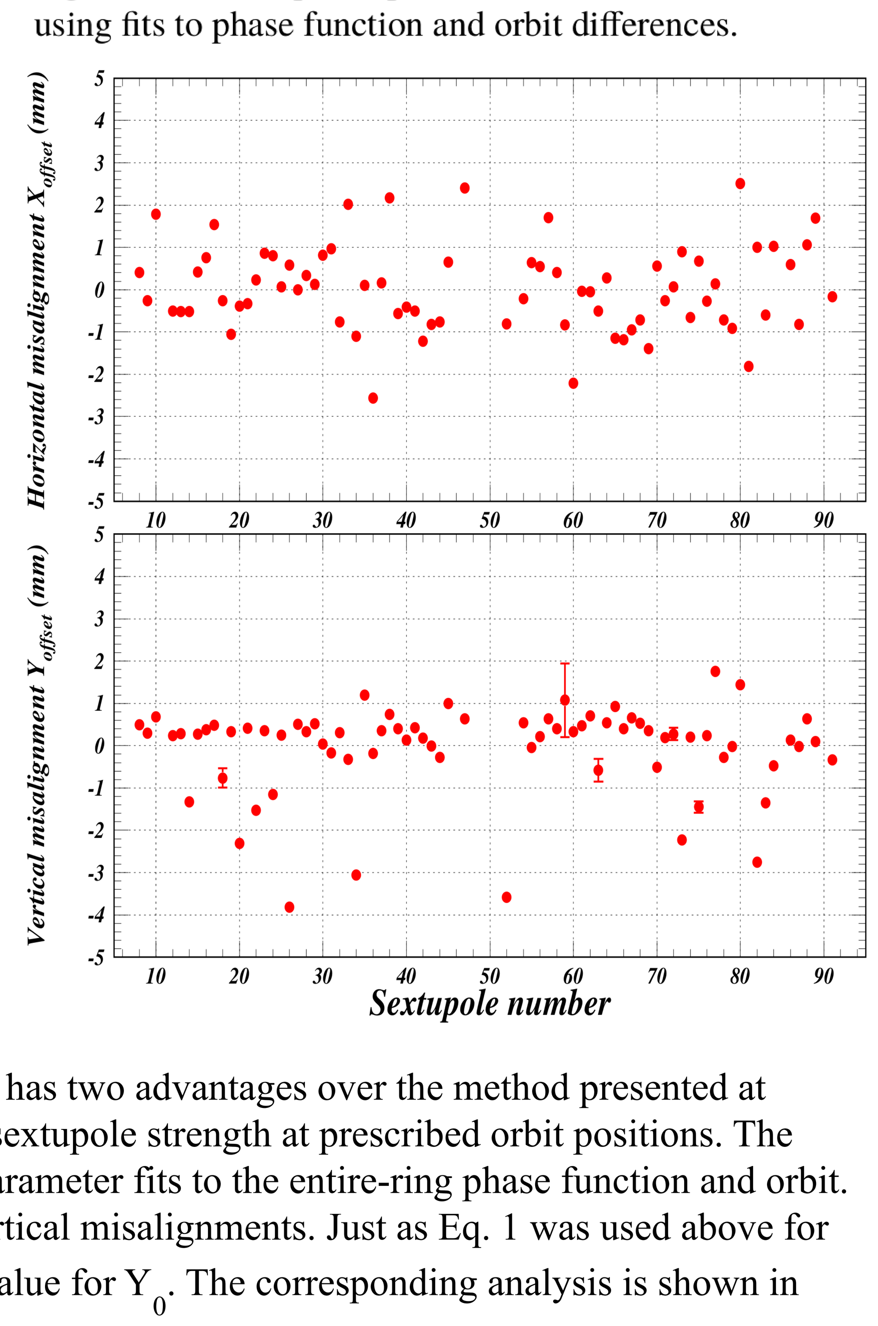



Figure 6: The vertical orbit change $\Delta y$ as a function of sextupole strength change $\Delta K_2L$.




The horizontal misalignment $X_{offset}$ of the sextupole relative to the BPM coordinate system can now be found by determining the horizontal orbit position measurement prior to the sextupole strength change. The full statistical power of the measurements at eleven sextupole settings is shown in Fig. 4. The value for $x$ at $K_2$=0 of -0.6694 ± 0.0022 mm yields a value for the misalignment of 1.7652 ± 0.0075 mm.

This means of determining the horizontal misalignment has two advantages over the method presented at IPAC'22, which entailed measuring tune changes with sextupole strength at prescribed orbit positions. The first is precision, since the present method uses multi-parameter fits to the entire-ring phase function and orbit. Secondly, this method can also be used to determine vertical misalignments. Just as Eq. 1 was used above for finding the values of $X_0$, Eq. 2 can be used to find the value for $Y_0$. The corresponding analysis is shown in Figs. 5 and 6. The vertical misalignment is found to be −0.4364 ± 0.0048 mm. The weighted averages of multiple measurements for each of the 76 sextupoles are shown in Fig. 7. The uncertainties range from 0.01 to 0.1 mm. These misalignments must be included in any detailed model of the CESR optics.

## Accuracy in the Determination of $X_0$

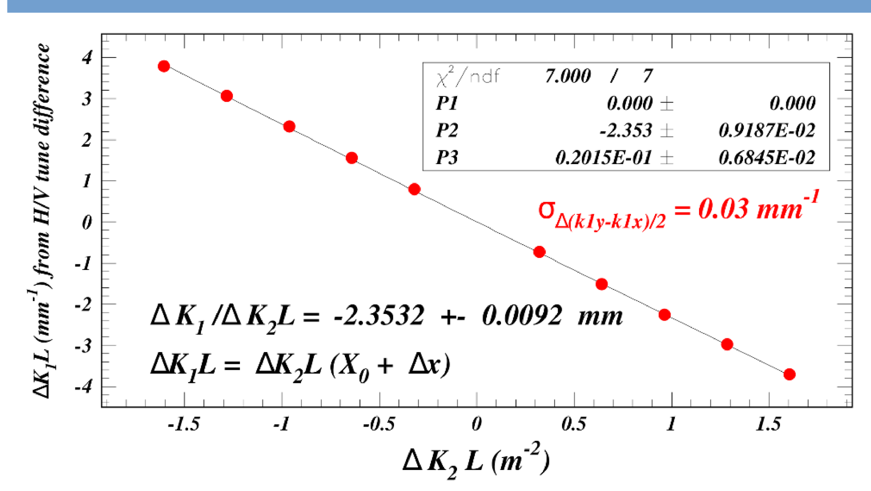



Figure 1: Quadrupole kick values $K_1$ derived from betatron tune changes as a function of sextupole strength change.

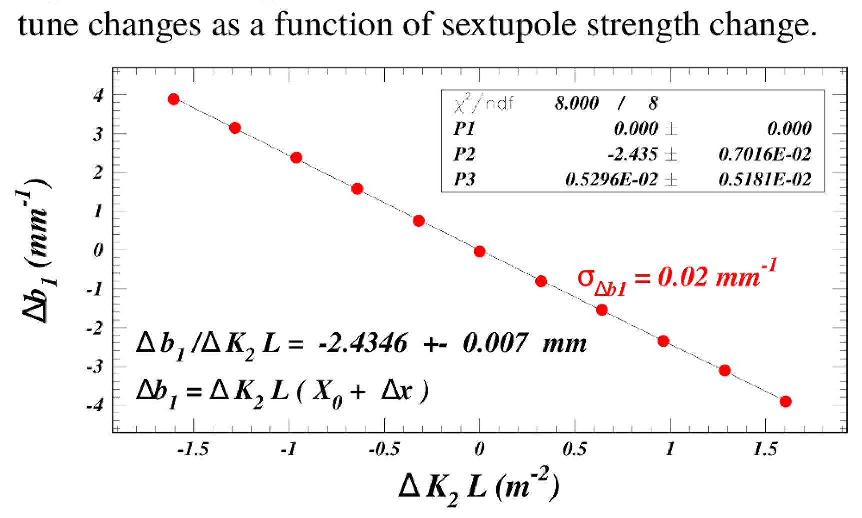



Figure 2: Quadrupole kick values $\Delta b_1$ determined using fits to phase function and orbit differences.

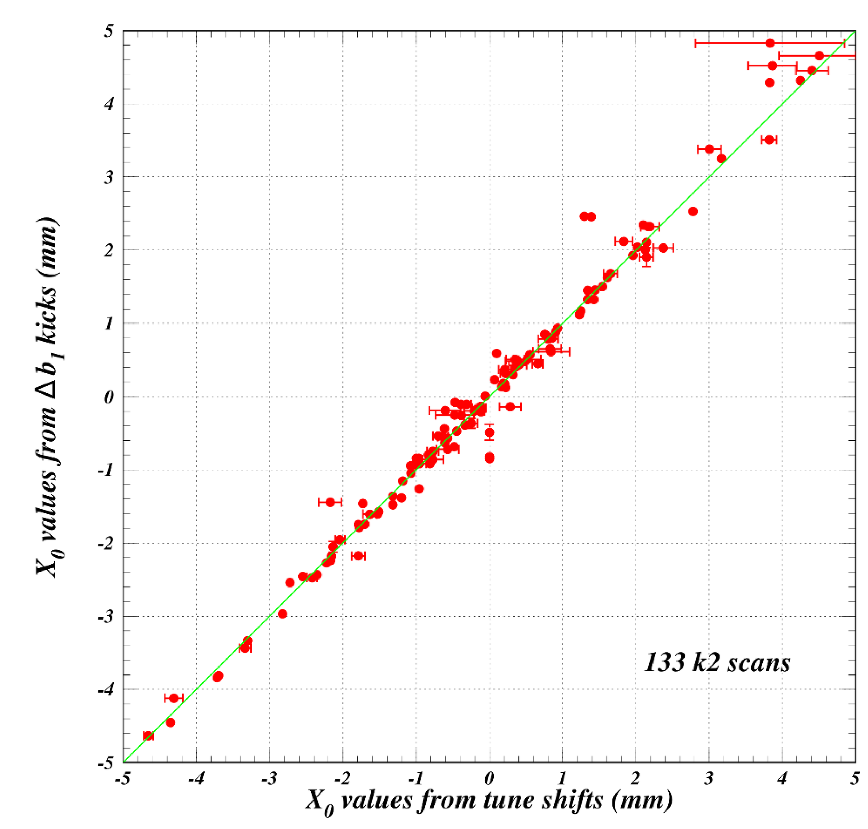



Figure 3: Degree of correlation obtained from the values for $X_0$ derived from tune changes and fits to phase function and orbit differences.

We compare determinations of $X_0$ using the ring-wide betatron tune change with the local quad kick optimized to fit difference phase functions and orbits.

We obtain an estimate for the measurement uncertainties in $\Delta K_1L$ and $\Delta b_1$ by setting them such that the $\lambda^2$/NDF of the polynomial fit is unity. The high degree of correlation and agreement at the 0.13 mm level shows that the underlying assumption of linear optics is sufficiently accurate to allow measurement of 1-2-mm beam sizes.

## Beam Size Calculation

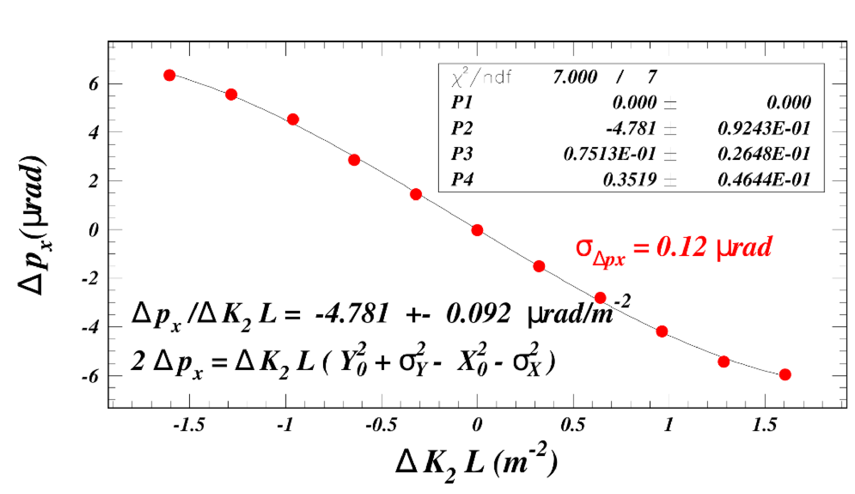



Figure 8: The horizontal orbit kick change $\Delta p_x$ as a function of sextupole strength change $\Delta K_2L$.

The measurement for the remaining term in the beam size calculation (Eq. 5) is shown in Fig. 8. The value for the horizontal orbit kick slope of −4.781 ± 0.092 µrad/m$^{-2}$ results in a calculated value for the horizontal beam size of $\sigma_x$ = 1.955 ± 0.048 mm when neglecting the vertical beam size, which is a factor of 5 smaller according to the optics functions. We obtain at present typical beam size calculations which are significant overestimates when compared to the values expected from the emittance and Twiss functions, which give $\sigma_x$ = 1.09 mm for the example at hand.

## Discussion

An improvement over the analysis presented at IPAC'22 achieved during the past year is our ability to perform a complete re-analysis of all our data sets in a few hours, thus showing results for all data sets and all sextupoles, such as in Fig. 7. Our calculations of beam size generally give beam size values greater than those expected from the optics, i.e. our measured values of $\Delta p_X/\Delta K_2L$ appear to have contributions other than those we have considered here. The prime suspects are nonlinear effects stemming from beam movement in the sextupoles around the ring when the strength of the sextupole under study changes. We are now mounting a modeling campaign to understand these effects.

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