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SEXTUPOLE FOR CESR

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3D properties of the sextupole lens are investigated with the help of numerical code in comparison with the measurements done.

In [1] a sextupole upgrade was considered. After 7.36 *mm* cut from the pole tip made, the next allowed for sextupole harmonic corresponding to 18 pole component decreased about 7 times. In [1] some of the longitudinal measurements, done at that time, were represented also. On the basis of these measurements the conclusion was made that the effective length of the sextupole became equal to $278 \pm 1mm$.

All calculations were carried out at that time with 2D code MERMAID [2]. Meanwhile the 3D version of MERMAID appeared [3], and there was interesting to compare the measurements done about two and half years ago with newest calculations with 3D version. On this basis some new predictions could be made.

The sketch of the sextupole with some dimensions is represented in Fig.1.

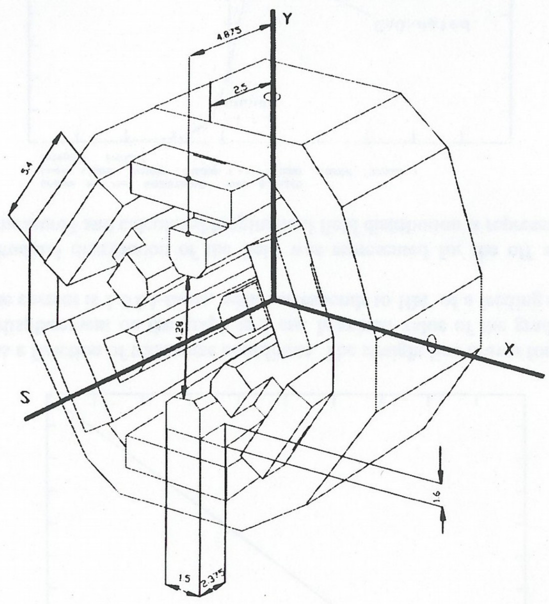

Fig.1. Sextupole. Dimensions are given in inches. The coils for vertical steering are not shown.

The field, generated by ideal sextupole can be described as the following (see for example, [4])

$$B_x(x,y,s) = 2S(s)xy - \frac{3x^3y + xy^3}{12}S''(s) + \ldots,$$
$$B_y(x,y,s) = S(s)(x^2 - y^2) - \frac{3x^4 + 6x^2y^2 - 5y^4}{12\cdot 4}S''(s) + \ldots, \quad (1)$$
$$B_s(x,y,s) = \frac{3x^2y - y^3}{3}S'(s) - \ldots,$$

where defined

$$S(s) \equiv \frac{1}{2!}\frac{\partial^2}{\partial x^2}B_y(x,y,s)\Big|_{x,y=0}, \quad S'(s) \equiv \frac{\partial S(s)}{\partial s}, \quad S''(s) \equiv \frac{\partial^2 S(s)}{\partial s^2}, \ldots$$

The harmonics $\sim x^8, x^{14}, x^{20}, \ldots$ also allowed by sextupole symmetry.

3D mesh in MERMAID constructed with layers filled with triangle prisms with the bases parallel to transverse plane. MERMAID treats the straight parts of the coils as having uniform current density. The current density in the bending region approximated on the basis of calculation of electrical potential. So with this method the thick coils with non uniform current density could be taken into account. We used in our modeling the coil with single turn, which fills the geometry of full multi turn coil. This was made for simplified input of the coil. The specific geometry of the poles, when they are screening the coils, (see Fig. 1) also makes this type of modeling adequate.

For modeling 1/8 of the 3D lens geometry was inputted into the code. The magnetic lines in the midplane of the sextupole are represented in Fig.2.

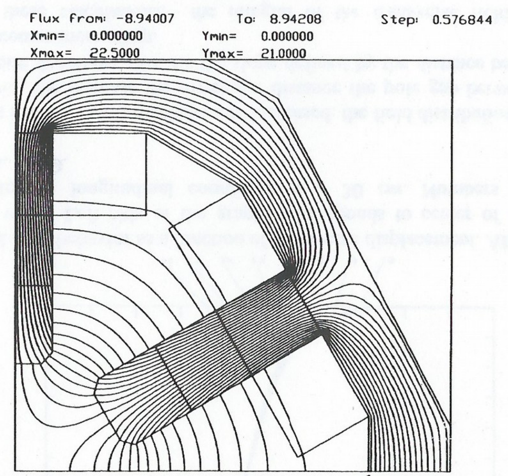

Fig.2 Magnetic field lines in the midplane of sextupole. $^1/_4$ of a cross section is shown.

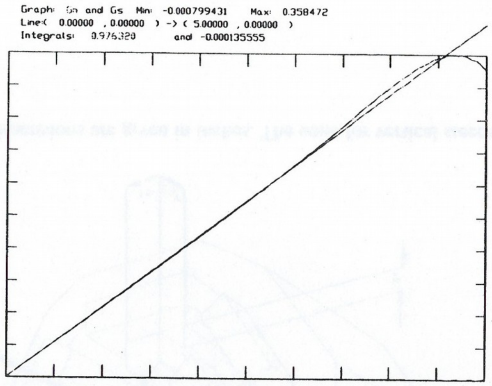

Fig.3. Gradient as a function of transverse coordinate. The straight line drawn for the reference. Maximal displacement on the graph is 5 *cm*. Maximal value of the gradient is 0.3584 *kG/cm*. The current is 1.6 *kA-turns*, what corresponds to 10*A* of a feeding current.

In [1] the longitudinal distribution of the field was represented for the off axis position 4 *cm*. Comparison of measured and calculated longitudinal field distribution is represented in Fig.4.

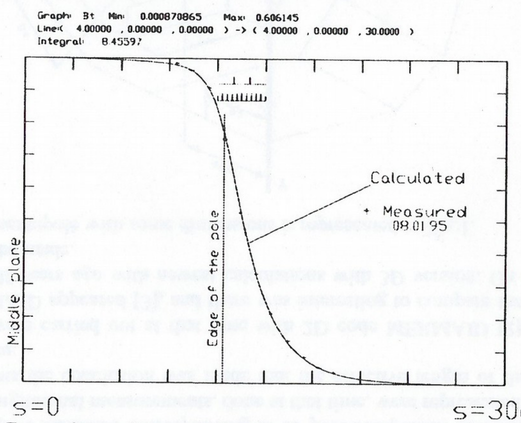

Fig. 4. Comparison of measured and calculated longitudinal field distribution.

So as the agreement looks excellent, we can move further with 3D calculations. In Fig. 5, the series of longitudinal field distributions are represented as a function of transverse position.

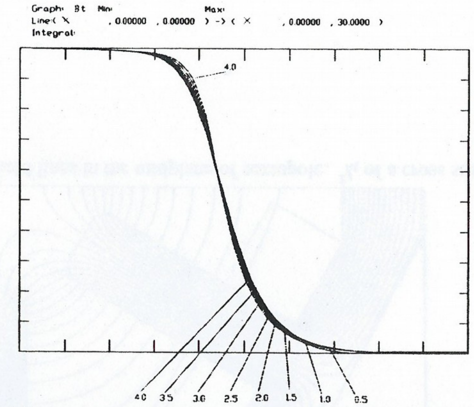

Fig.5. Longitudinal field behavior as a function of transverse displacement. All curves normalized to it's maximal value. Left side of the graphs corresponds to center of sextupole. Right side corresponds to the longitudinal coordinate s = 30 *cm*. Numbers mark the transverse displacements, $y = 0$.

One can see that as the transverse displacement increased, the field distribution becomes more steep. This is evident: with the increase the off center distance the pole gap between neighbor poles in selected cross section decreases also. As the slope defined by the distance between poles, the field drop, naturally, becomes more steep.

On the basis of these calculations, the integral of the transverse field $B_y(x,y,s)$ along the longitudinal axis could be found. The integrals $I(x)=\int_0^{30} B_y(x,y=0,s)ds\ [kG\cdot cm]$ are represented as a functions of transverse displacement in the Table below. The current is the same as in Fig.3 what is 1.6 *kA-turns*, which corresponds to 10*A* of a *feeding* current.

x, cm	$I_{half\ lens}(x)=\int_0^{30cm} B_y(x,s)ds$, kG·cm	B_{ymax}, kG	$I(x)/B_{ymax}=\frac{1}{2}L_{eff}$, cm	L_{eff}, cm
0.0	0.0	0.0002	-	
0.5	0.1272	0.00916	13.888	27.77
1.0	0.5265	0.03778	13.937	27.87
1.5	1.1934	0.08550	13.957	27.91
2.0	2.1277	0.15237	13.964	27.93
2.5	3.3305	0.23849	13.965	27.93
3.0	4.8079	0.34435	13.962	27.92
3.5	6.5698	0.47083	13.954	27.91
4.0	8.6291	0.61927	13.934	27.87
4.5	10.9647	0.78811	13.913	27.83
5.0	13.410	0.96583	13.884	27.77

In this Table in position for B_{ymax} at zero displacement (x = 0.0) the field value indicated B_{ymax}=0.0002 kG, generated by numerical noise arising from the accuracy of geometry input. In Fig. 6 the *full* effective length as a function of transverse displacement is represented. The accuracy calculated as a ratio of residual field of 0.0002 kG to the maximal field value for the displacement indicated.

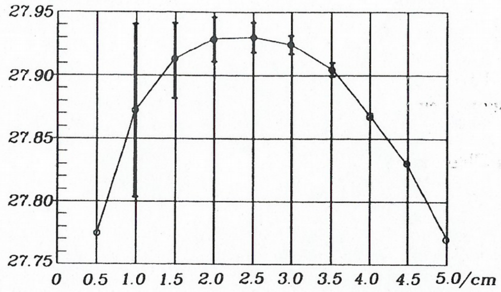

Fig. 6. The full effective length as a function of transverse displacement. All dimensions are in *cm*. The effective length for the point with displacement $x = 0.5$ *cm* is 27.77 ± 0.3 *cm*

This accuracy of calculations about 10^{-4} does not allow to calculate the fields closer than 0.25 *cm* (field is too small). The accuracy of positioning of the poles and mechanical assembling the parts of the lens remains within 0.5 *mm*, what was measured with few lenses.

Integral of magnetic field over all lens $I_{all\ lens}(x) = 2 \cdot I_{half\ lens} = 2 \cdot \int_0^{30cm} B_y(x,s)ds$ could be represented, according the values from Table as

$$I_{all\ lens}[kG \cdot cm] = \int_{lens} B_y(x,s)ds = 1.06553 \cdot x^2 + 4.023 \cdot 10^{-6} \cdot x^8 - 2.278 \cdot 10^{-10} \cdot x^{16}, \quad (2)$$

where x measured in *cm*, and for 10*A* of a *feeding* current. So each coefficient represents the integral of corresponding multipole over the lens. For example, the first term from (2) gives

$$\int_{lens} S(s)ds = 1.06553\,[kG / cm],$$

as we represented the field with $y = 0$ according to (1) as $B_y(x,s) = S(s) \cdot x^2 + ...$.

References

[1] A. Mikhailichenko, "CESR Sextupole upgrade", CON 96-05, March 9, 1996.

[2] A.N. Dubrovin, E.A. Simonov, "**MERMAID-MEsh** oriented **R**outine for **MA**gnetic **I**nteractive **D**esign", User's guide, BINP, Novosibirsk, 1990.

[3] **MERMAID/3D,** SIM Inc., Novosibirsk, P.O. Box 402, Russia.

[4] A.A. Mikhailichenko, "3-D electromagnetic Field. Representation and measurements", CBN 95-16, Cornell, 1995, 42 pp.