Sextupole Alignment and Beam Size Measurement

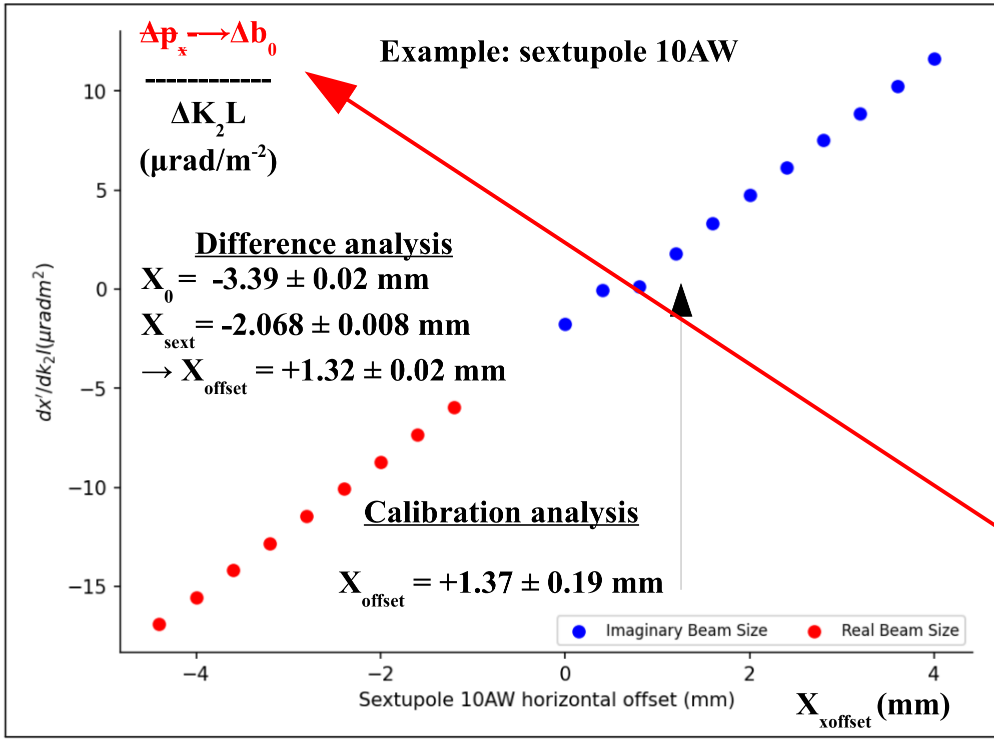

Figure 6: $dx'/dk_2 l$ plotted against the horizontal sextupole offset

Ishaan's systematic error discovery

2022 REU project final report

The horizontal orbit kick determined from the difference orbit is linearly correlated with the sextupole offset assumed in the lattice model.

Until now I have mistakenly used only the dipole term Δb_0 for Δp_x, omitting the contribution from $\Delta b_1 X_0$, where X_0 is the position of the beam relative to the sextupole center.

News flash

- **Complete data set re-analyzed**
- **Results independent of misalignment at machine accuracy**

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Optimization using difference orbits, phase and tunes

To measure the beam size in a sextupole we need $\frac{\Delta p_X}{\Delta K_2 L}$ and X_0:

$$\sigma_X^2 = -2\,\frac{\Delta p_X}{\Delta K_2 L} - X_0^2.$$

The optimization fits values for Δb_0 and Δb_1 at the sextupole to match the measured difference orbits, phase, and tunes for a given change ΔK_2. The reference orbits, phase and tunes are chosen to have $K_2 = 0$.

The optimization finds values for Δp_X (constrained primarily by the difference orbit) and Δb_1 (constrained primarily by the difference phase function and tunes) independently of any assumed sexupole misalignment x_{offset}, since $K_2 = 0$.

The position of the beam relative to the center of the sextupole is given directly by

$$X_0 = \frac{\Delta b_1}{\Delta K_2 L},$$

independent of any sextupole misalignment in the model.
The value for Δp_x is given by (Bmad manual Eq. 16.4 with sign adjusted to CESR left-handed coordinate convention)

$$\boxed{\Delta p_X = \Delta b_0 - X_0\,\Delta b_1.}$$

Since X_0 is given by the orbit position at the sextupole via $X_0 = x_{\mathrm{sext}} - x_{\mathrm{offset}}$, where x_{sext} is the orbit position at the sextupole and x_{offset} is the sextupole misalignment, we have

$$\Delta b_0 = \Delta p_X + (x_{\mathrm{sext}} - x_{\mathrm{offset}})\,\Delta b_1.$$

The slope of this linear dependence on the sextupole misalignment is

$$\frac{d\Delta b_0}{dx_{\mathrm{offset}}} = -\Delta b_1.$$

So the slope in Ishaan's plot is

$$\frac{d\Delta b_0/\Delta K_2 L}{dx_{\mathrm{offset}}} = \frac{-\Delta b_1}{\Delta K_2 L} = -X_0.$$

Example 10AW

$\Delta k_2 L = 1.82\ \mathrm{m}^{-2}$ and $X_{\mathrm{offset}} = 1\ .32$ mm

Optimization:

$\Delta b_0 = 5.43\text{e-}6$ radians

$\Delta b_1 = -6.07\text{e-}3\ \ \mathrm{mm}^{-1}$

So

$X_0 = \Delta b_1 / \Delta k_2 L = -\ 3.39$ mm

$\Delta b_1 X_0 = 20.58\ \mu$rad

$\Delta p_x = \Delta b_0 - \Delta b_1 X_0 = (5.43 - 20.58)\ \mu$rad

$\Delta p_x = -15.15\ \mu$rad

$\Delta p_x / \Delta k_2 L = \ -\ 8.32\ \mu\mathrm{rad/mm}^{-2}$

$X_0^{\ 2} = 11.15\ \mathrm{mm}^2$

$\sigma^2\ = -\ 2\ \Delta p_x / \Delta k_2 L - X_0^{\ 2} = 5.50\ \mathrm{mm}^2$