Resolution of the Quadratic Dependence of Tune on Sextupole Strength

(David Sagan figured it out.)

Tune change due to quad kick (Wille Eq. 3.272)

$$\beta \Delta K L = -2 \frac{\cos(2\pi(Q+\Delta Q)) - \cos(2\pi Q)}{\sin(2\pi Q)}$$

$$\simeq 4\pi \Delta Q$$

This approximation is not sufficiently accurate for our purposes !

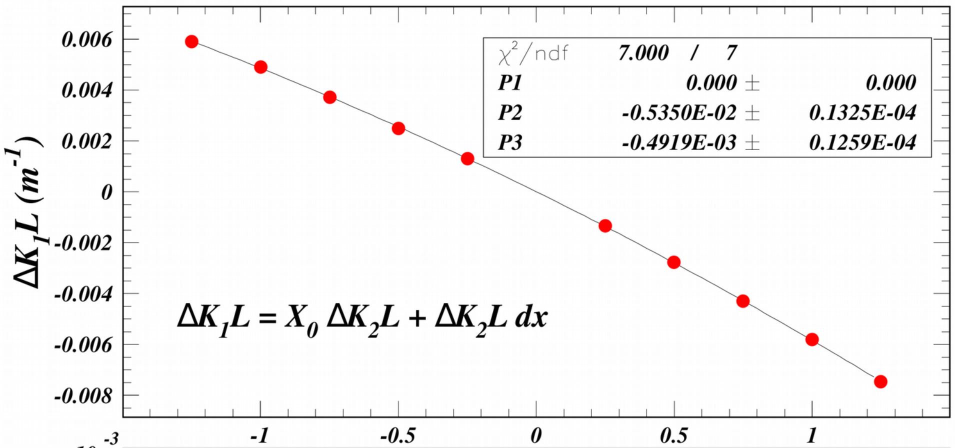
Jim Crittenden
Georg group meeting
14 October 2021

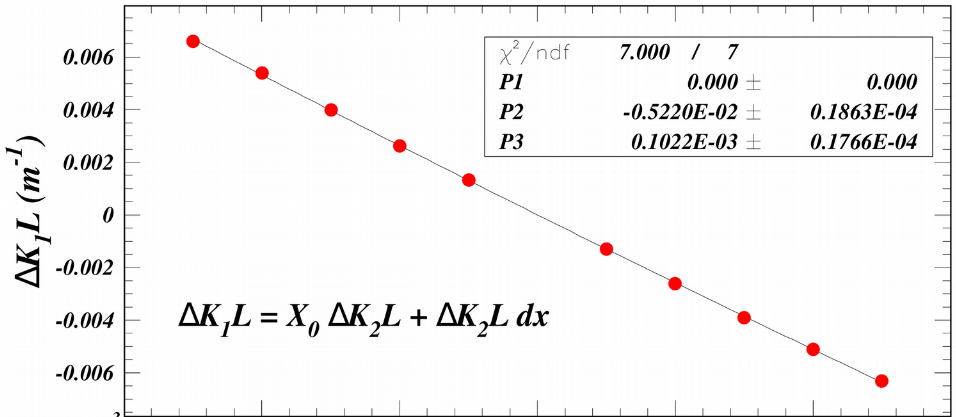
Quad kick from tune change
- Compare first order to exact -

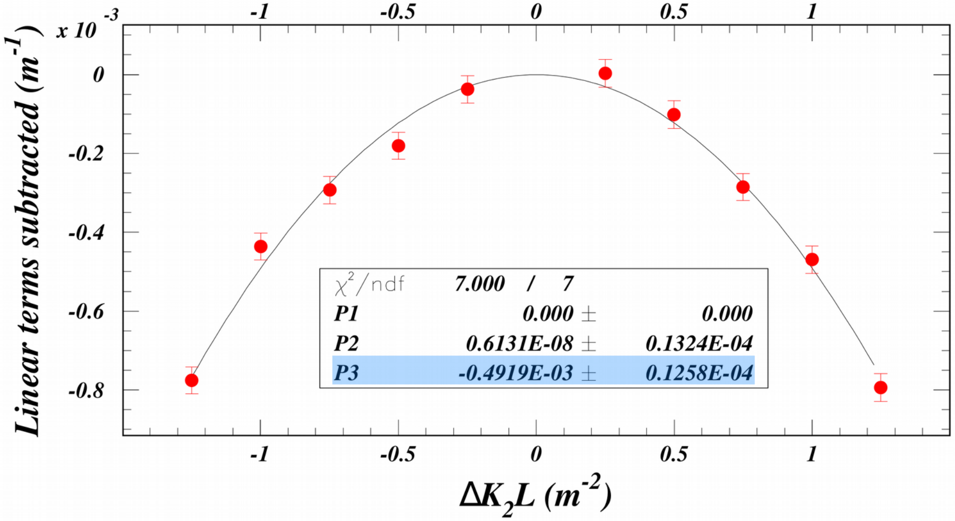
Sextupole 10AW

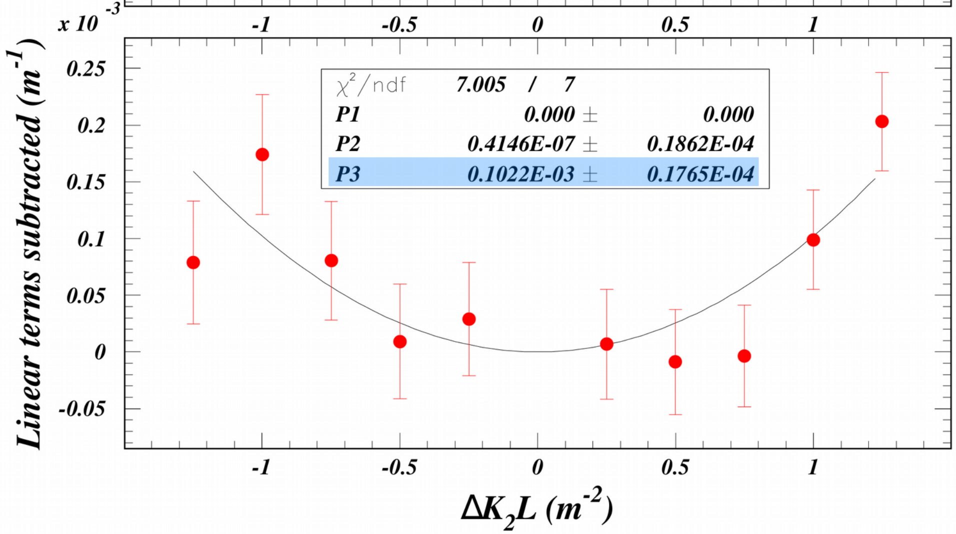
$$\beta \Delta K L = 4\pi \Delta Q$$

$$\beta \Delta K L = -2\frac{\cos(2\pi(Q+\Delta Q)) - \cos(2\pi Q)}{\sin(2\pi Q)}$$





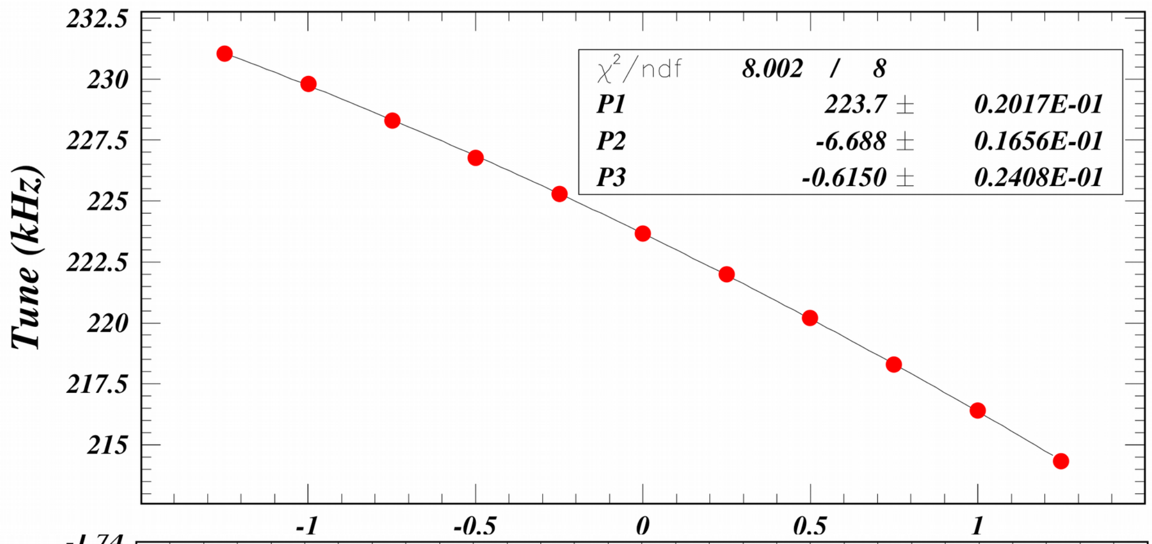


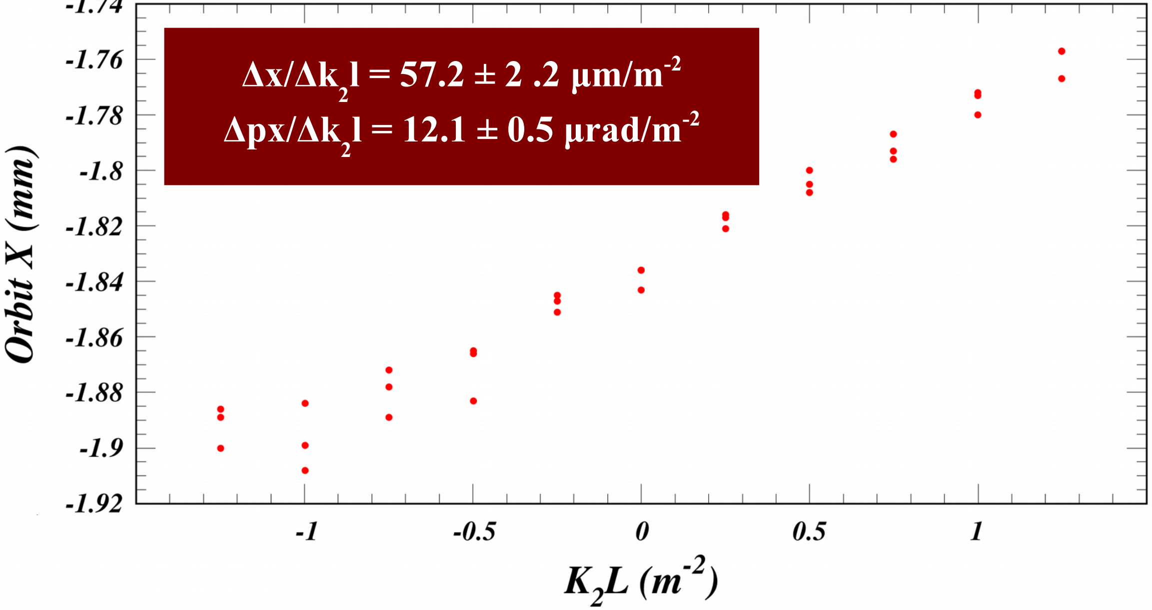


Quad kick $\Delta K_1 L$ is now more symmetric around $\Delta K_2 L$=0. Sign now agrees with single kick analysis.

Magnitude of 102 ± 18 μrad/m² is about 2σ too large.

Sign disambiguation





The tune is decreasing, so the beam must be moving toward the center of the sextupole.

The orbit is clearly moving toward the outside of the ring.

So the sextupole must be displaced toward the outside of the ring.

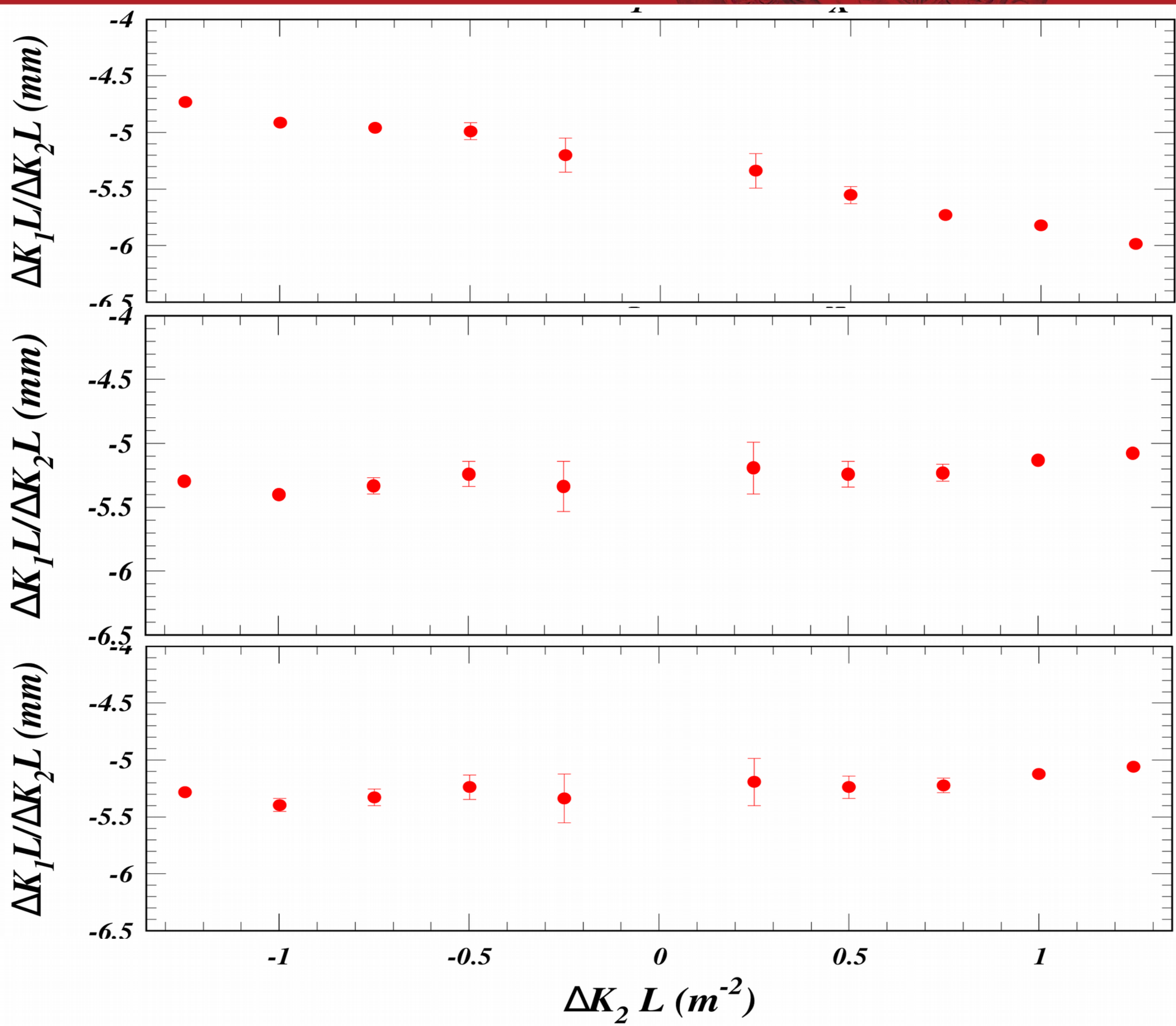
$$\Delta x = \Delta p_x \beta_x / (2 \tan(\pi Q))$$

(Wille Eq. 3.259)

$$Q = 0.5733 \quad \beta_x = 40.3 \text{ m}$$

$$\Delta x/\Delta p_x = - 4.722 \text{ m/radian}$$

Measurements to be used in the optimization to obtain the beam size



First order

$$\beta \Delta K L = 4\pi \Delta Q$$

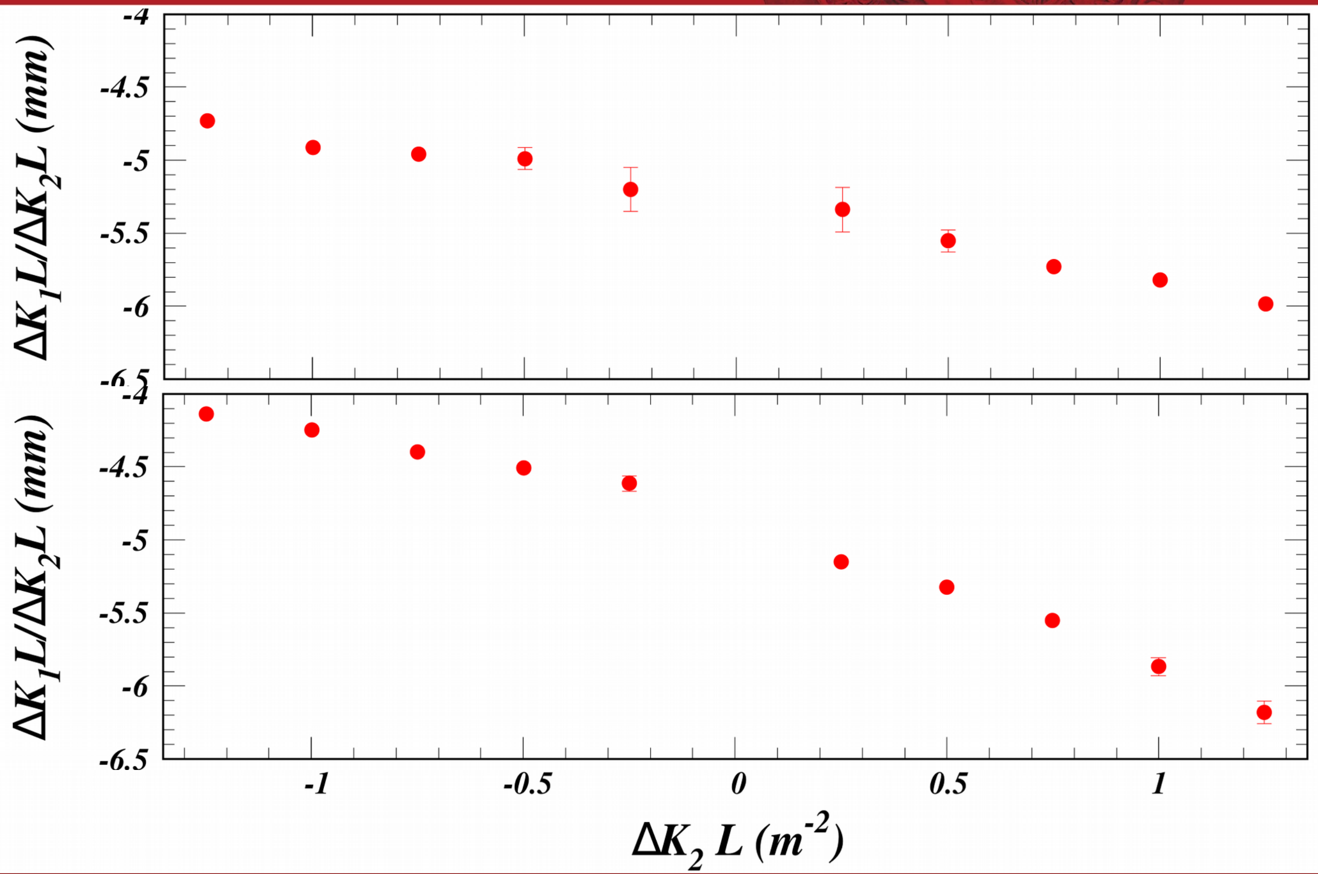
Second order

$$\beta \Delta K L = 4\pi \Delta Q \left(1 + \frac{2\pi \Delta Q}{\tan(2\pi Q)}\right)$$

All orders

$$\beta \Delta K L = -2 \frac{\cos(2\pi(Q+\Delta Q)) - \cos(2\pi Q)}{\sin(2\pi Q)}$$

Measurements to be used in the optimization to obtain the beam size



First order tune analysis

$$\beta \Delta K L = 4\pi \Delta Q$$

Wave analysis

Question for David

Does it make sense to "blindly" make this correction to the wave analysis?
Does the wave analysis use this approximation?

However, it appears that the correction will not be large enough.