Two-dimensional Beam Size Analysis with Sextupoles

(Include
$$Y_0 \neq 0$$
, $\sigma_Y \neq 0$)

Bmad manual section 16

Assuming initial $K_2L=0$ and including second and third order terms:

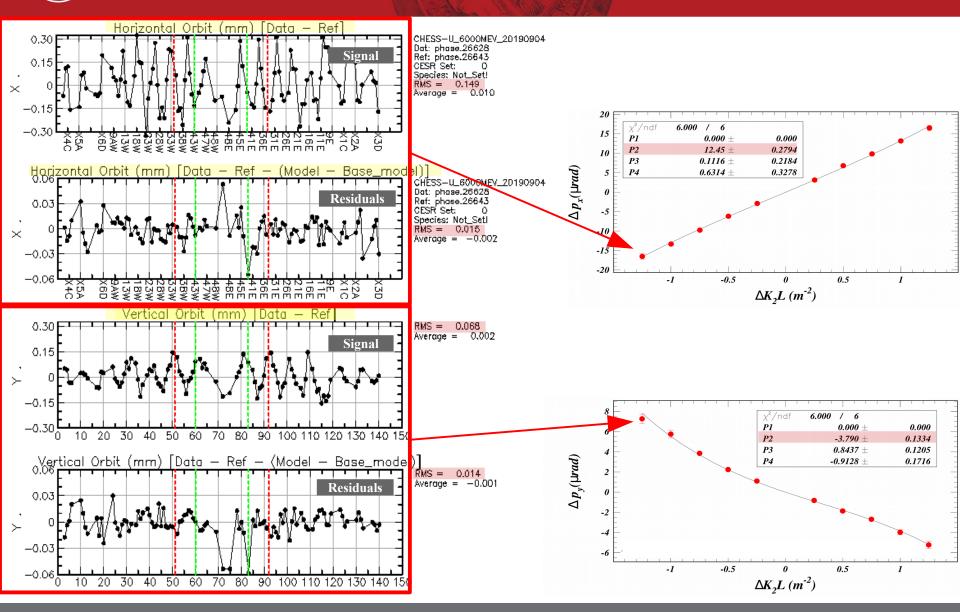
$$\sigma_{ ext{X}}^2 - \sigma_{ ext{Y}}^2 = -2 \; rac{\Delta p_{ ext{X}}}{\Delta K_2 L} + \left(rac{\Delta p_{ ext{Y}}}{\Delta K_2 L}
ight)^2 \left(rac{\Delta K_1 L}{\Delta K_2 L}
ight)^{-2} - \left(rac{\Delta K_1 L}{\Delta K_2 L}
ight)^2$$

Linear terms only:

$$\sigma_{
m X}^2 - \sigma_{
m Y}^2 = -2 \, rac{\Delta p_{
m X}}{\Delta K_2 L} + Y_0^2 - X_0^2$$

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Bazarov/Rubin group meeting
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H and V Orbit Kicks



Beam size Example of 10aw February data

Three equations, four unknowns:

$$\Delta K_1 L = \Delta K_2 L \left(X_0 + \Delta \mathbf{x} \right)$$

$$\Delta p_{\mathrm{Y}} = \Delta K_2 L \left(X_0 + \Delta \mathrm{x} \right) \left(Y_0 + \Delta \mathrm{Y} \right)$$

$$2 \; \Delta p_{ ext{X}} = \Delta K_2 L \left[\left(rac{\Delta p_{ ext{Y}}}{\Delta K_2 L}
ight)^2 \left(rac{\Delta K_1 L}{\Delta K_2 L}
ight)^{-2} + \sigma_{ ext{Y}}^2 - \left(rac{\Delta K_1 L}{\Delta K_2 L}
ight)^2 - \sigma_{ ext{X}}^2
ight]$$

 $X_0 = -4.945 + -0.023$ mm (from $\Delta K_1 L$ fit to H and V phase difference)

$$X_0Y_0 = -3.79 + -0.13 \text{ mm}^2$$

$$Y_0 = 0.766 + 0.26 \text{ mm}$$

$$\sigma_{\rm X}^2 - \sigma_{\rm Y}^2 = 2*12.45 + Y_0^2 - X_0^2 = 1.03 + 0.53 \text{ mm}^2$$