Calculating Sextupole Strength and Placement From Shaking Data

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1 General Resonance Analysis

Given a second order one-turn map, we want to find the resonance amplitudes and phases. The one-turn map is of the form

$$\mathbf{x}(n+1) = \mathbf{M}_x\, \mathbf{x}(n) + \mathbf{T}_x\, \mathbf{x}(n)\, \mathbf{x}(n) \tag{1}$$

$\mathbf{x} = (x, p_x, y, p_y)$ are the transverse beam coordinates in phase space, n is the turn number, $[\mathbf{M}_x, \mathbf{T}_x]$ are the first and second order parts of the map. The linear matrix $\mathbf{M}_x$ can be written in normalized form[3]

$$\mathbf{M}_a = \mathbf{V}^{-1}\, \mathbf{M}_x\, \mathbf{V} \tag{2}$$

where $\mathbf{V}$ is a symplectic matrix, and $\mathbf{M}_a$ is block diagonal

$$\mathbf{M}_a = \begin{pmatrix} \mathbf{R}(\omega_a) & 0 \\ 0 & \mathbf{R}(\omega_b) \end{pmatrix} \tag{3}$$

ω_a and ω_b are the a and b eigen mode tunes and $\mathbf{R}$ is a rotation matrix

$$\mathbf{R}(\omega) = \begin{pmatrix} \cos\omega & \sin\omega \\ -\sin\omega & \cos\omega \end{pmatrix} \tag{4}$$

The normalized coordinates $\mathbf{a} = (a, p_a, b, p_b)$ are related to the laboratory coordinates via

$$\mathbf{x} = \mathbf{V}\, \mathbf{a}. \tag{5}$$

The one-turn map $[\mathbf{M}_a, \mathbf{T}_a]$ in normalized coordinates is related to the laboratory map via concatenation with bfV

$$[\mathbf{M}_a, \mathbf{T}_a] = \mathbf{V}^{-1} \circ [\mathbf{M}_x, \mathbf{T}_x] \circ \mathbf{V} \tag{6}$$

Explicitly, using the implied summation convention, the second order transformation is

$$\mathbf{T}_{a,ijk} = V^{-1}_{ip}\, T_{x,pqr}\, V_{qj}\, V_{rk} \tag{7}$$

The one-turn map can now be analyzed in the resonance basis[4] $\mathbf{z} = (z_a, z_a^*, z_b, z_b^*)$ where

$$\begin{aligned} z_a &\equiv a - i\, p_a \\ z_b &\equiv b - i\, p_b \end{aligned} \tag{8}$$

In the resonance basis the equation of motion of the one-turn map $[\mathbf{M}_z, \mathbf{T}_z]$ is

$$\mathbf{z}(n+1) = \mathbf{M}_z\, \mathbf{z}(n) + \mathbf{T}_z\, \mathbf{z}(n)\, \mathbf{z}(n) \tag{9}$$

where

$$
\begin{aligned}
\mathbf{M}_z &= \mathbf{W}^{-1}\,\mathbf{M}_a\,\mathbf{W} = \begin{pmatrix} e^{i\,\omega_a} & 0 & 0 & 0 \\ 0 & e^{-i\,\omega_a} & 0 & 0 \\ 0 & 0 & e^{i\,\omega_b} & 0 \\ 0 & 0 & 0 & e^{-i\,\omega_b} \end{pmatrix} \\
\mathbf{T}_z &= \mathbf{W}^{-1} \circ \mathbf{T}_a \circ \mathbf{W}
\end{aligned} \tag{10}
$$

with

$$
\mathbf{W} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ i & -i & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & i & -i \end{pmatrix} \tag{11}
$$

The analysis uses first order perturbation theory considering $\mathbf{T}_z$ as the perturbation. The solution is written in the form

$$
\mathbf{z} = \overline{\mathbf{z}} + \widehat{\mathbf{z}} \tag{12}
$$

where $\widehat{\mathbf{z}}$ is the first order solution and $\overline{\mathbf{z}}$ is the zeroth order solution which is readily seen to be

$$
\begin{aligned}
\overline{z}_a(n) &= A_a\, e^{i\,\omega_a\, n} \\
\overline{z}_b(n) &= A_b\, e^{i\,\omega_b\, n}
\end{aligned} \tag{13}
$$

where the complex amplitudes A_a and A_b are determined by the initial conditions. Using Eq. (12) and Eq. (13) in Eq. (9), and ignoring higher order terms, the equation for $\widehat{\mathbf{z}}_a$ is

$$
\widehat{z}_a(n+1) - \widehat{z}_a(n)\, e^{i\,\omega_a} = T_{z,1jk}\, \overline{z}_j(n)\, \overline{z}_k(n) \tag{14}
$$

with a similar equation for $\widehat{z}_b$. The solution of $\widehat{z}_a$ is

$$
\widehat{z}_a(n) = \sum_{j,k} \widehat{z}_{a,jk}\; e^{i(\omega_j+\omega_k)\,n} = \sum_{j,k} \frac{A_j\, A_k\, T_{z,1jk}\; e^{i\,(\omega_j+\omega_k)\,n}}{e^{i(\omega_j+\omega_k)} - e^{i\,\omega_a}} \tag{15}
$$

where

$$
\begin{aligned}
(A_1, A_2, A_3, A_4) &= (A_a, A_a^*, A_b, A_b^*) \\
(\omega_1, \omega_2, \omega_3, \omega_4) &= (\omega_a, -\omega_a, \omega_b, -\omega_b)
\end{aligned} \tag{16}
$$

and for z_b

$$
\widehat{z}_b(n) = \sum_{j,k} \widehat{z}_{b,jk}\; e^{i(\omega_j+\omega_k)\,n} = \sum_{j,k} \frac{A_j\, A_k\, T_{z,3jk}\; e^{i\,(\omega_j+\omega_k)\,n}}{e^{i(\omega_j+\omega_k)} - e^{i\,\omega_b}} \tag{17}
$$

The solutions for $\widehat{z}_a$ and $\widehat{z}_b$ show that, along with a DC orbit shift, there are oscillations $\omega_j + \omega_k$ at all the sum and difference frequencies $\pm 2\omega_a$, $\pm 2\omega_b$, $\pm(\omega_a + \omega_b)$ and $\pm(\omega_a - \omega_b)$. Excluding the DC terms, For each term $\widehat{z}_{a,jk}$ which oscillates at $\omega_j + \omega_k$ there is a mirror term $\widehat{z}_{a,\widetilde{j}\widetilde{k}}$ that oscillates at $-\omega_j - \omega_k$ with

$$
\widetilde{1} = 2, \quad \widetilde{2} = 1, \quad \widetilde{3} = 4, \quad \widetilde{4} = 3 \tag{18}
$$

Excluding the DC terms, the four terms $\widehat{z}_{a,jk}$, $\widehat{z}_{a,\widetilde{j}\widetilde{k}}$, $\widehat{z}_{b,jk}$, $\widehat{z}_{b,\widetilde{j}\widetilde{k}}$ all contribute to the oscillations seen at a given resonance frequency at a BPM. Combining Eqs. (5), (8), (15), (17) gives for a given $\omega_j + \omega_k$ resonance frequency

$$
\begin{aligned}
x(n) = \frac{1}{2} \Big(& V_{11}(\widehat{z}_{a,jk} + \widehat{z}^*_{a,\widetilde{jk}}) + i\, V_{12}(\widehat{z}_{a,jk} - \widehat{z}^*_{a,\widetilde{jk}}) + \\
& V_{13}(\widehat{z}_{b,jk} + \widehat{z}^*_{b,\widetilde{jk}}) + i\, V_{14}(\widehat{z}_{b,jk} - \widehat{z}^*_{b,\widetilde{jk}}) \Big) \; e^{i\,(\omega_j + \omega_k)\,n} + \mathrm{CC} \\
y(n) = \frac{1}{2} \Big(& V_{31}(\widehat{z}_{a,jk} + \widehat{z}^*_{a,\widetilde{jk}}) + i\, V_{32}(\widehat{z}_{a,jk} - \widehat{z}^*_{a,\widetilde{jk}}) + \\
& V_{33}(\widehat{z}_{b,jk} + \widehat{z}^*_{b,\widetilde{jk}}) + i\, V_{34}(\widehat{z}_{b,jk} - \widehat{z}^*_{b,\widetilde{jk}}) \Big) \; e^{i\,(\omega_j + \omega_k)\,n} + \mathrm{CC}
\end{aligned} \tag{19}
$$

If the lattice is decoupled, and all the sextupoles are not tilted, the first two lines appear only with the a-mode (horizontal) motion and the latter two appear only with the b-mode (vertical) motion.

2 Overview

The idea is to measure the phase and amplitude of the horizontal and vertical resonance lines at all BPM's while resonantly shaking at both the horizontal and vertical betatron resonance frequencies. This information allows the calculation of sextupole strengths and locations.

This analysis uses first order perturbation theory. That is, only effects that are first order in the sextupole strength are considered. A more general analysis can be obtained using Normal Form analysis[1]. While not as general as Normal Form analysis, the following derivation has the advantage of simplicity and makes a clear connection between the sextupole kicks and and the beam response.

First order perturbation theory consists of using the unperturbed oscillations in the kick equations to calculate the oscillations at the various resonant frequencies. The unperturbed oscillations of the beam are given by

$$x_0(s,n) = \sqrt{A_x\,\beta_x(s)}\,\cos(\omega_x\,n + \phi_x(s) + \phi_{x0}) \tag{20}$$

$$y_0(s,n) = \sqrt{A_y\,\beta_y(s)}\,\cos(\omega_y\,n + \phi_y(s) + \phi_{y0}) \tag{21}$$

The kick (dx', dy') given by a sextupole of strength k_2 and length L is

$$(dx', dy') = k_2\,L\,(\frac{1}{2}\,(x^2 - y^2), x\,y) \tag{22}$$

With first order perturbation theory, the effect of a collection of sextupoles is simply the sum of the effects of each one so the problem may be simplified by considering a single sextupole.

The result will be that there are four resonance lines that can be seen in the position data. Each resonance line is comprised of two resonance frequencies that are opposite in sign but have different amplitudes. At any one BPM, it is not possible to distinguish these resonant lines apart. Using multiple BPMs however, breaks the symmetry and allows one to differentiate the two.

In the horizontal plane, the resonant lines correspond to the $\pm 2\omega_x$ and $\pm 2\omega_y$ frequencies. In the vertical plane the resonant lines correspond to $\pm(\omega_x + \omega_y)$ and $\pm(\omega_x - \omega_y)$. Each resonance line represents an oscillation in either the horizontal or vertical plane. That is, none of the resonance lines couple horizontal and vertical motions.

3 Horizontal Resonance Analysis

The horizontal response is written in normalized coordinate $(\tilde{x}, \tilde{x}')$ given by

$$\begin{pmatrix} \tilde{x} \\ \tilde{x}' \end{pmatrix} = \begin{pmatrix} 1/\sqrt{\beta_x} & 0 \\ \alpha/\sqrt{\beta_x} & \sqrt{\beta_x} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix} \tag{23}$$

To first order in the sextupole strength, the horizontal kick $d\tilde{x}'$ due to a sextupole at position $\bar{s}$ is

$$
\begin{aligned}
d\tilde{x}'(\bar{s},n) &= \sqrt{\beta_x(\bar{s})}\,dx' = \frac{\sqrt{\beta_x(\bar{s})}}{2}\,k_2\,L\,(x_0^2(s,n) - y_0^2(s,n)) \\
&= K_{x2}\,\cos(2\,\bar{\omega}_{xn}) - K_{y2}\,\cos(2\,\bar{\omega}_{yn}) + (K_{x2} - K_{y2})
\end{aligned} \tag{24}
$$

where

$$
\bar{\omega}_{xn} \equiv \omega_x\,n + \phi_x(\bar{s}) + \phi_{x0} \tag{25}
$$

$$
\bar{\omega}_{yn} \equiv \omega_y\,n + \phi_y(\bar{s}) + \phi_{y0} \tag{26}
$$

and

$$
K_{x2} \equiv \frac{1}{4}\,k_2\,L\,A_x\,\beta_x^{3/2}(\bar{s}) \tag{27}
$$

$$
K_{y2} \equiv \frac{1}{4}\,k_2\,L\,A_y\,\beta_x^{1/2}(\bar{s})\,\beta_y(\bar{s}) \tag{28}
$$

At the sextupole, the position on the nth turn is due to the kicks on all the previous turns

$$
\tilde{x}(\bar{s},n) = \sum_{k=-\infty}^{n} d\tilde{x}'(\bar{s},k)\,R_{x12}(n-k) \tag{29}
$$

where $R_{x12}(m)$ is the response to a kick after n turns

$$
R_{x12}(m) = \sin(\omega_x\,m) \tag{30}
$$

Similarly, the angle of the beam just after the sextupole is given by

$$
\tilde{x}'(\bar{s}^+,n) = \sum_{k=-\infty}^{n} d\tilde{x}'(\bar{s},k)\,\cos(\omega_x\,(n-k))
$$

where $\bar{s}^+$ denotes the position just after the sextupole.

The beam response is divided up into three parts corresponding to the three terms on the RHS of Eq. (24). The first term gives the response at the $\pm 2\omega_x$ resonance line. Using Eqs. (29) and (31), this position response is

$$
\tilde{x}(\bar{s},n) = \frac{K_{x2}}{4}\left[\frac{\cos(2\bar{\omega}_{xn} + 3\omega_x/2)}{\sin(3\omega_x/2)} - \frac{\cos(2\bar{\omega}_{xn} + \omega_x/2)}{\sin(\omega_x/2)}\right] \tag{31}
$$

Similarly, the angle of the beam just after the sextupole at $\bar{s}^+$ is given by

$$
\tilde{x}'(\bar{s}^+,n) = \frac{K_{x2}}{4}\left[\frac{\sin(2\bar{\omega}_{xn} + 3\omega_x/2)}{\sin(3\omega_x/2)} + \frac{\sin(2\bar{\omega}_{xn} + \omega_x/2)}{\sin(\omega_x/2)}\right] \tag{32}
$$

Notice that $\tilde{x}'$, unlike $\tilde{x}$, is not continuous across the sextupole. Starting from $\tilde{x}$ and $\tilde{x}'$ at the sextupole, the response at a given location $s > \bar{s}$ is

$$
\tilde{x}(s,n) = \tilde{x}(\bar{s},n)\,\cos(\phi_x(s) - \phi_x(\bar{s})) + \tilde{x}'(\bar{s}^+,n)\,\sin(\phi_x(s) - \phi_x(\bar{s})) \tag{33}
$$

The general solution for all s is

$$
\tilde{x}(s,n) = \begin{cases} \dfrac{K_{x2}}{4}\left[\dfrac{\cos(2\bar{\omega}_{xn} - 3\omega_x/2 - d\phi_x(s))}{\sin(3\omega_x/2)} - \dfrac{\cos(2\bar{\omega}_{xn} - \omega_x/2 + d\phi_x(s))}{\sin(\omega_x/2)}\right] & s < \bar{s} \\[2ex] \dfrac{K_{x2}}{4}\left[\dfrac{\cos(2\bar{\omega}_{xn} + 3\omega_x/2 - d\phi_x(s))}{\sin(3\omega_x/2)} - \dfrac{\cos(2\bar{\omega}_{xn} + \omega_x/2 + d\phi_x(s))}{\sin(\omega_x/2)}\right] & s > \bar{s} \end{cases} \tag{34}
$$

where

$$d\phi_x(s) = \phi_x(s) - \phi_x(\bar{s}) \tag{35}$$

The first term on the RHS of Eq. (34), which comes from the first term on the RHS of Eqs. (31) and (32), represents an oscillation at a frequency of $-2\omega_x$. The second terms on the RHS in these three equations represent an oscillation at $2\omega_x$. In the Normal Form analysis, the $-2\omega_x$ oscillations are driven by the h_{3000} term and the $2\omega_x$ oscillations are driven by the h_{1200} term[1].

The position in Eq. (32) at a given s position can be considered as the sum of two phasers of amplitude $A_- = K_{x2}/(4\,\sin(3\omega_x/2))$ and $A_+ = K_{x2}/(4\,\sin(\omega_x/2))$. Each phaser rotates as $2\omega_x$ with an s-dependent phase difference. The amplitude of oscillation, $|\tilde{x}|$, at a given s position will thus vary between $|A_- - A_+|$ when the phasers are out-of-phase and $|A_- + A_+|$ when the phasers are in-phase. Since there are two phasers involved here, the phase of the $\tilde{x}$ oscillations at a given s-position is not a simple function of the phase advance $\phi_x(s)$.

The second term on the RHS of Eq. (24) gives the $\pm 2\omega_y$ resonance line. The analysis of the $\pm 2\omega_y$ line is similar to the $\pm 2\omega_x$ analysis. The result is that at any given location the position response is

$$\tilde{x}(s,n) = \begin{cases} \dfrac{-K_{y2}}{4}\left[\dfrac{\cos(2\bar{\omega}_{yn} - \omega_y - \omega_x/2 - d\phi_x(s))}{\sin(\omega_y + \omega_x/2)} - \dfrac{\cos(2\bar{\omega}_{yn} - \omega_y + \omega_x/2 + d\phi_x(s))}{\sin(\omega_y - \omega_x/2)}\right] & s < \bar{s} \\ \dfrac{-K_{y2}}{4}\left[\dfrac{\cos(2\bar{\omega}_{yn} + \omega_y + \omega_x/2 - d\phi_x(s))}{\sin(\omega_y + \omega_x/2)} - \dfrac{\cos(2\bar{\omega}_{yn} + \omega_y - \omega_x/2 + d\phi_x(s))}{\sin(\omega_y - \omega_x/2)}\right] & s > \bar{s} \end{cases} \tag{36}$$

The third term on the RHS of Eq. (24) is constant and thus results in an orbit distortion. The result is the standard closed orbit due to a kick

$$\tilde{x}(s) = \frac{(K_{x2} - K_{y2})\,\cos(\omega_x/2 - |d\phi_x(s)|)}{2\,\sin(\omega_x/2)} \tag{37}$$

4 Vertical Resonance Analysis

The vertical motion is analyzed like the horizontal motion. The vertical kick is given by

$$\begin{aligned} d\tilde{y}'(\bar{s},n) &= \sqrt{\beta_y(\bar{s})}\,dy' = \sqrt{\beta_y(\bar{s})}\,k_2\,L\,x_0(s,n)\;y_0(s,n) \\ &= K_{xy}\;[\cos(2\bar{\omega}_{xyn+}) - \cos(2\bar{\omega}_{xyn-})] \end{aligned} \tag{38}$$

where

$$\bar{\omega}_{xyn\pm} = \frac{1}{2}\,[(\omega_x \pm \omega_y)\,n + (\phi_x(\bar{s}) \pm \phi_y(\bar{s})) + (\phi_{x0} \pm \phi_{y0})] \tag{39}$$

and

$$K_{xy} \equiv \frac{1}{4}\,K_2\,L\,\sqrt{A_x\,A_y}\,\beta_x^{1/2}(\bar{s})\,\beta_y(\bar{s}) \tag{40}$$

The $\pm(\omega_x + \omega_y)$ resonance line corresponds to the first term on the RHS of Eq. (38). The analysis is the same as the horizontal resonances. The result is

$$\tilde{y}(s,n) = \begin{cases} \dfrac{K_{xy}}{4}\left[\dfrac{\cos(2\bar{\omega}_{xyn+} - \omega_{xy+} - \omega_y/2 - d\phi_y(s))}{\sin(\omega_{xy+} + \omega_y/2)} - \dfrac{\cos(2\bar{\omega}_{xyn+} - \omega_{xy+} + \omega_y/2 + d\phi_y(s))}{\sin(\omega_{xy+} - \omega_y/2)}\right] & s < \bar{s} \\ \dfrac{K_{xy}}{4}\left[\dfrac{\cos(2\bar{\omega}_{xyn+} + \omega_{xy+} + \omega_y/2 - d\phi_y(s))}{\sin(\omega_{xy+} + \omega_y/2)} - \dfrac{\cos(2\bar{\omega}_{xyn+} + \omega_{xy+} - \omega_y/2 + d\phi_y(s))}{\sin(\omega_{xy+} - \omega_y/2)}\right] & s > \bar{s} \end{cases} \tag{41}$$

where

$$\omega_{xy+} = \frac{1}{2}\,(\omega_x + \omega_y) \tag{42}$$

and

$$d\phi_y(s) = \phi_y(s) - \phi_y(\bar{s}) \tag{43}$$

Finally, The oscillations of the $\pm(\omega_x - \omega_y)$ resonance line is

$$\tilde{y}(s,n) = \begin{cases} \dfrac{K_{xy}}{4} \left[\dfrac{\cos(2\bar{\omega}_{xyn-} - \omega_{xy-} - \omega_y/2 - d\phi_y(s))}{\sin(\omega_{xy-} + \omega_y/2)} - \dfrac{\cos(2\bar{\omega}_{xyn-} - \omega_{xy-} + \omega_y/2 + d\phi_y(s))}{\sin(\omega_{xy-} - \omega_y/2)} \right] & s < \bar{s} \\ \dfrac{K_{xy}}{4} \left[\dfrac{\cos(2\bar{\omega}_{xyn-} + \omega_{xy-} + \omega_y/2 - d\phi_y(s))}{\sin(\omega_{xy-} + \omega_y/2)} - \dfrac{\cos(2\bar{\omega}_{xyn-} + \omega_{xy-} - \omega_y/2 + d\phi_y(s))}{\sin(\omega_{xy-} - \omega_y/2)} \right] & s > \bar{s} \end{cases} \tag{44}$$

where

$$\omega_{xy-} = \frac{1}{2}(\omega_x - \omega_y) \tag{45}$$

5 Data Analysis

Two ways to approach the data are discussed here. The first approach involves fitting the measured data using a software model of the lattice. From the oscillations at the fundamental betatron frequencies, the Twiss parameters at the BPMs can be measured. The first step in the fit involves fitting the model quadrupole strengths to the measured Twiss parameters. Once the model quadrupoles have been fit, the beta function and betatron phase at the sextupoles can be calculated from the model. Using this, the only unknowns in the appropriate equation for $\tilde{z}$ (which is $\tilde{x}$ or $\tilde{y}$ depending upon which resonance is being analyzed) developed above (one of Eq. (34), (36), (41), or (44)) are the sextupole strengths k_2. A fit between the measured $\tilde{z}$ at a resonance line and the $\tilde{z}$ from the model using the strengths of the sextupoles may now be done. This can be done by decomposing the oscillations at a BPM for a given resonance line into two components

$$\tilde{z}(s,n) = A_c \cos(2\bar{\omega}_{rn}) + A_s \sin(2\bar{\omega}_{rn}) \tag{46}$$

Where $\bar{\omega}_{rn}$ is the resonance frequency of the line (one of $\bar{\omega}_{xn}$, $\bar{\omega}_{yn}$, $\bar{\omega}_{xyn+}$, or $\bar{\omega}_{xyn-}$). Comparing this to the appropriate equation for $\tilde{z}$, equations for A_c and A_s may readily be derived.

Once a set of strengths are calculated, the model k_2 values can be compared to the k_2 values as calculated from the calibration value (which is, say, calculated from the currents going through the sextupoles). This is a check that nothing is mis-wired, etc. If the calibrations are reasonably accurate (but not exact) the sextupole strengths may be corrected to achieve some desired distribution. Calling $k_2(model)$ the strengths from the fit, and $k_2(design)$ the strengths that are desired, then the change in k_2 needed is

$$dk_2 = k_2(design) - k_2(model) \tag{47}$$

The second approach involves trying to find where sextupole error are originating. Assume for for the sake of illustration that there is no error in the sextupole calibrations so that k_2 of all the sextupoles is known. The difference $d\tilde{z} \equiv \tilde{z}(measured) - \tilde{z}(model)$ for any line should now be zero but will, in general, not be due to various imperfections in the lattice (say a vertical steering has an associated sextupole component). The task at hand is to locate and calculate the strength of such errors.

It is assumed that any errors are "isolated". That is, there are error free regions around any given error. An error free region can be fit to a freely propagating orbit "wave". The fit is of the form

$$d\tilde{z}(s,n) = A_{c+} \cos(\omega_r n + \phi_z(s)) + A_{s+} \sin(\omega_r n + \phi_z(s)) + A_{c-} \cos(\omega_r n - \phi_z(s)) + A_{s-} \sin(\omega_r n - \phi_z(s)) \tag{48}$$

where ω_r is the resonance frequency under consideration (one of $2\omega_x$, $2\omega_y$, $\omega_x + \omega_y$, or $\omega_x - \omega_y$). The fit parameters are A_{c+}, A_{s+}, A_{c-}, and A_{s-}. From the fits to two regions surrounding a putative error, the amplitude and placement of the error may be determined[2].

By varying the strength of a given sextupole and looking at the change in the resonance lines, this second approach may also be used to calibrate sextupoles.

References

[1] R. Bartolini and F. Schmidt, “Normal Form via Tracking or Beam Data”. Part. Accel. 59, pp 93–106 (1998). LHC Project Report 132, (1997).

[2] D. Sagan, “Betatron phase and coupling correction at the Cornell Electron/Positron Storage Ring”, Phys. Rev. ST Accel. Beams 3, 102801 (2000).

[3] D. Sagan and D. Rubin, “Linear Analysis of Coupled Lattices,” Phys. Rev. Special Topics — Acc. and Beams. **2**, 074001 (1999).

[4] E. Forest, *Beam dynamics, A New attitude and framework.*, Harwood Academic, (1998).