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JAC

Calculate statistical error in Tao beam-size calculation
See AS slides 26 September 2022

`See also /home/critten/bss/readme_beam.`

Sextupole 12W

The centroid of the $\boldsymbol{p_x}$ distribution upstream is $\mathbf{8.92 \times 10^{-7}}$ radians and the RMS width $\boldsymbol{\sigma_{p_x}}$ is $\mathbf{1.50 \times 10^{-4}}$ radians.
The centroid of the $\boldsymbol{p_x}$ distribution downstream is $\mathbf{4.28 \times 10^{-7}}$ radians and the RMS width $\boldsymbol{\sigma_{p_x}}$ is again $\mathbf{1.50 \times 10^{-4}}$ radians.

The statistical error in each centroid is $\boldsymbol{\delta(< p_x >) = \sigma_{p_x}/\sqrt{N},}$ where N=2000 positrons.

$$\boldsymbol{\delta(< p_x >) = 1.50 \times 10^{-4}/\sqrt{2000} = 3.35 \times 10^{-6}\ \textbf{radians}} \tag{1}$$

Already we see the problem that the error is much larger than the angle change with $\boldsymbol{\Delta K_2 L}$, so the error in our result for $\boldsymbol{\sigma_x}$ will be larger than its value. Since we get the right answer for the beam size, this means that the values for the phase space coordinates are not randomly distributed. Tao's particle tracking is deterministic. David S. can explain this.

We continue with the statistical error analysis for illustration purposes. Now that we have the values for the statistical uncertainties in the centroid values, we can propagate them to the error in the difference.

$$\boldsymbol{\delta^2(< p_x^{\text{up}} > - < p_x^{\text{down}} >) = \delta^2(< p_x^{\text{up}} >) + \delta^2(< p_x^{\text{down}} >) >} \tag{2}$$

$$\boldsymbol{= 2\ (3.35 \times 10^{-6})^2\ \textbf{radians}^2.} \tag{3}$$

The RMS widths do not change when we change $\boldsymbol{K_2}$, so this error in the centroid difference also does not change. Again we apply the propogation rule for differences and get for the angle change caused by the $\boldsymbol{K_2}$ change

$$\boldsymbol{\Delta p_x =< p_x^{\text{after}} > - < p_x^{\text{before}} >} \tag{4}$$

and

$$\boldsymbol{\delta^2(\Delta p_x) = 4\ (3.35 \times 10^{-6})^2\ \textbf{radians}^2.} \tag{5}$$

Since the beam size squared is

$$\boldsymbol{\sigma_x^2 = -2\ \Delta p_x/\Delta K_2 L} \tag{6}$$

we have

$$\boldsymbol{\delta(\sigma_x^2) = -2\ \delta(\Delta p_x)/\Delta K_2 L.} \tag{7}$$

Now apply the chain rule to get the error $\boldsymbol{\delta(\sigma_x)}$:

$$\boldsymbol{\delta(\sigma_x^2) = 2\ \sigma_x \delta(\sigma_x)} \tag{8}$$

i.e.

$$\boldsymbol{\delta(\sigma_x) = \delta(\sigma_x^2)/(2\sigma_x)} \tag{9}$$

with $\boldsymbol{\sigma_x = 1.59}$ mm. The final result is

$$\boldsymbol{\sigma_x = 1.6 \pm 11.4\ \textbf{mm.}} \tag{10}$$