Elementary Particle Physics I

Exercise Set 11: Electroweak Unification and Strong Interactions

1 Discussion Exercises

1.1 The Cabibbo-angle

The weak interaction induces transitions within the doublets of left-handed leptons and quarks by radiating or absorbing W-bosons:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \begin{pmatrix} u \\ d \end{pmatrix}_L \tag{1}$$

- Draw the Feynman diagram for the decay $K^+ \to \mu^+\nu_\mu$. Is it possible to understand this decay within the model of eq. 1?

In order to save the doublet structure, Cabibbo suggested in 1963 that the eigenstates of the weak interaction might be different from the mass eigenstates. In this case, the mixed state d' is observed in weak interactions:

$$\begin{pmatrix} u \\ d \end{pmatrix} \longrightarrow \begin{pmatrix} u \\ d' \end{pmatrix} \tag{2}$$

where $d' = d \cdot \cos\theta_C + s \cdot \sin\theta_C$. This parameter θ_C is called *Cabibbo angle.*

- How do the decay widths for the following decays behave in this model?

$$\begin{aligned} K^+ &\to \mu^+\nu_\mu \quad (|\Delta S| = 1) && (3)\\ \pi^+ &\to \mu^+\nu_\mu \quad (|\Delta S| = 0) && (4)\\ \mu^- &\to e^-\bar{\nu}_e\nu_\mu && (5) \end{aligned}$$

1.2 The Decay of the Z^0 Boson

1. The couplings of the Z^0 boson to fermion pairs are given by

   $$\gamma^\mu(c_v - c_a\gamma^5) \tag{6}$$

   where

   $$\begin{aligned} c_a &= I_3 \\ c_v &= I_3 - 2Q\sin^2\theta_W \end{aligned}$$

   where I_3 is the 3rd component of the weak isospin of the left-handed fermions and $\sin^2\theta_W = 0.23157 \pm 0.00018$.

   Calculate the branching ratios of the Z^0 boson by neglecting phase space differences. (Don't forget the colour of the quarks!) Why can't it decay to a pair of top-quarks?

2. On the last page you find some event displays from Z^0 decays in the OPAL detector, one of the four experiments which studied Z^0 decays at LEP. Match each event display to one of the branching ratios you calculated above.

1.3 The Weinberg Angle

By using the Higgs mechanism a relationsship between the so obtained vector boson masses and the Weinberg angle θ_W is obtained:

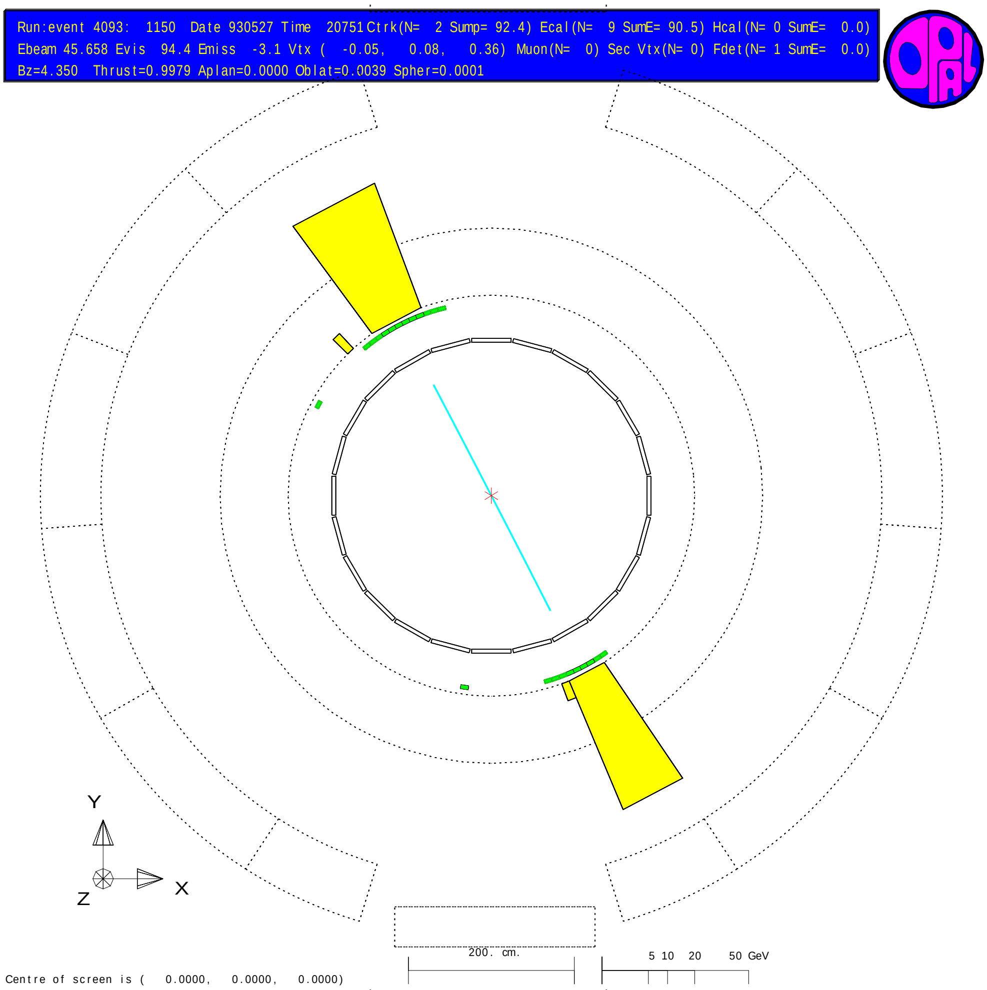
$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

Use this formula to calculate the angle.

1.4 Colors in strong interactions

Discuss the following experimental results under the aspect of color degrees of freedom:

- the existence of the Δ^{++} baryon
- $R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3 \sum_{2m_q < E_{CM}} e_q^2$
- $B(\tau^- \to \nu_\tau e^- \bar{\nu}_e) = 0.1780 \pm 0.0009$
- $\Gamma(\pi^0 \to \gamma\gamma) \approx 9\Gamma(q\bar{q} \to \gamma\gamma)$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X
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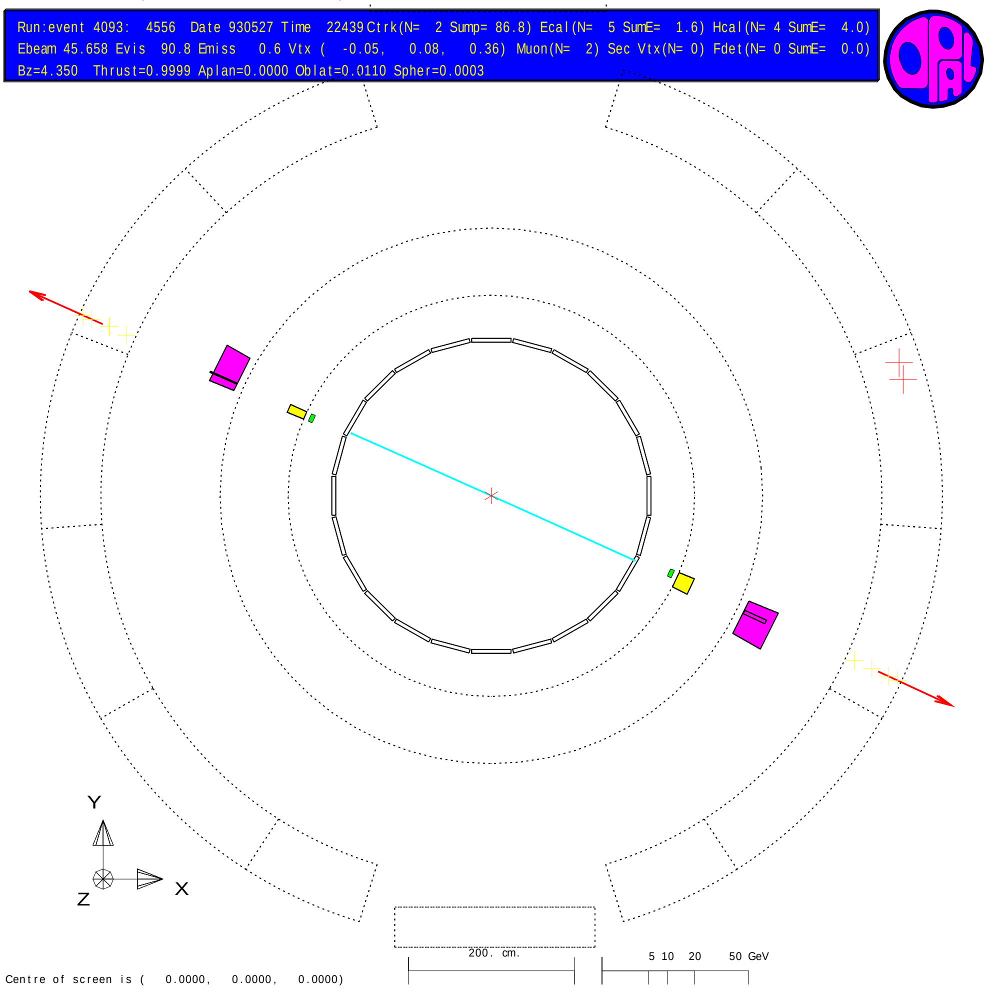
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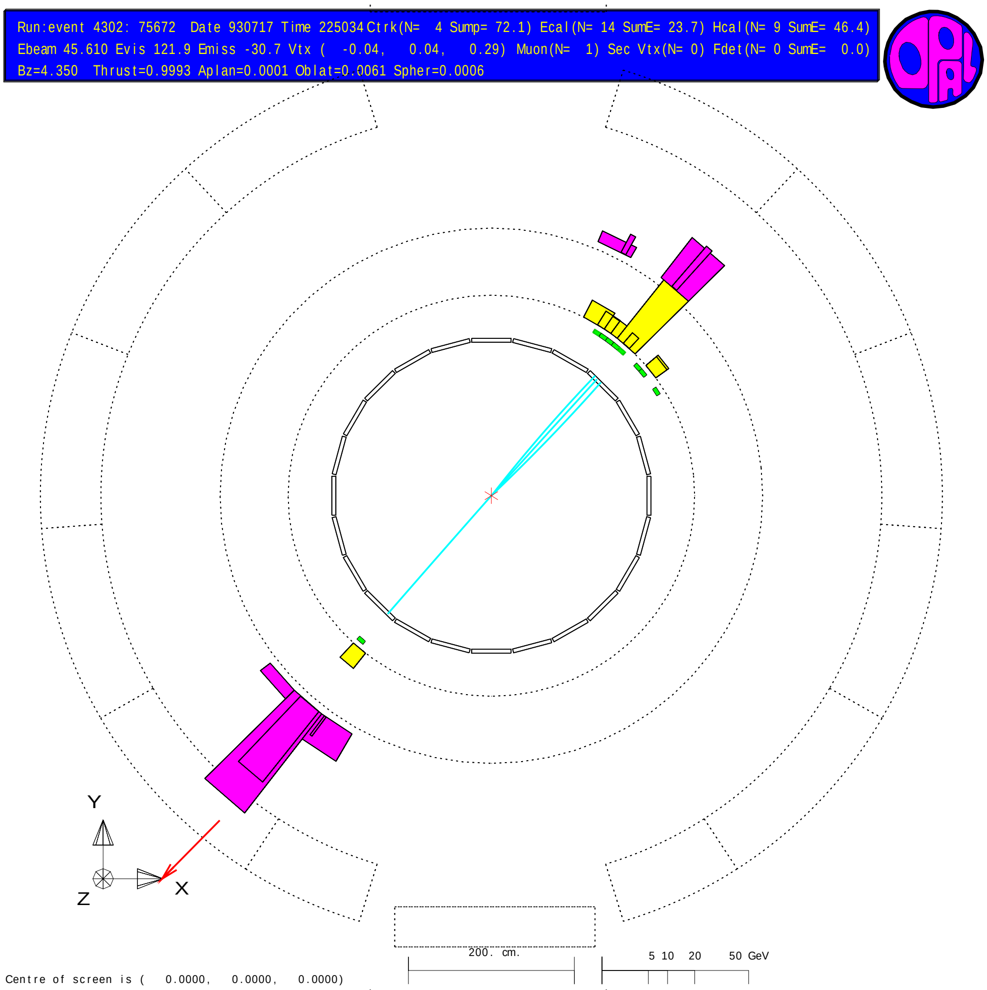
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X
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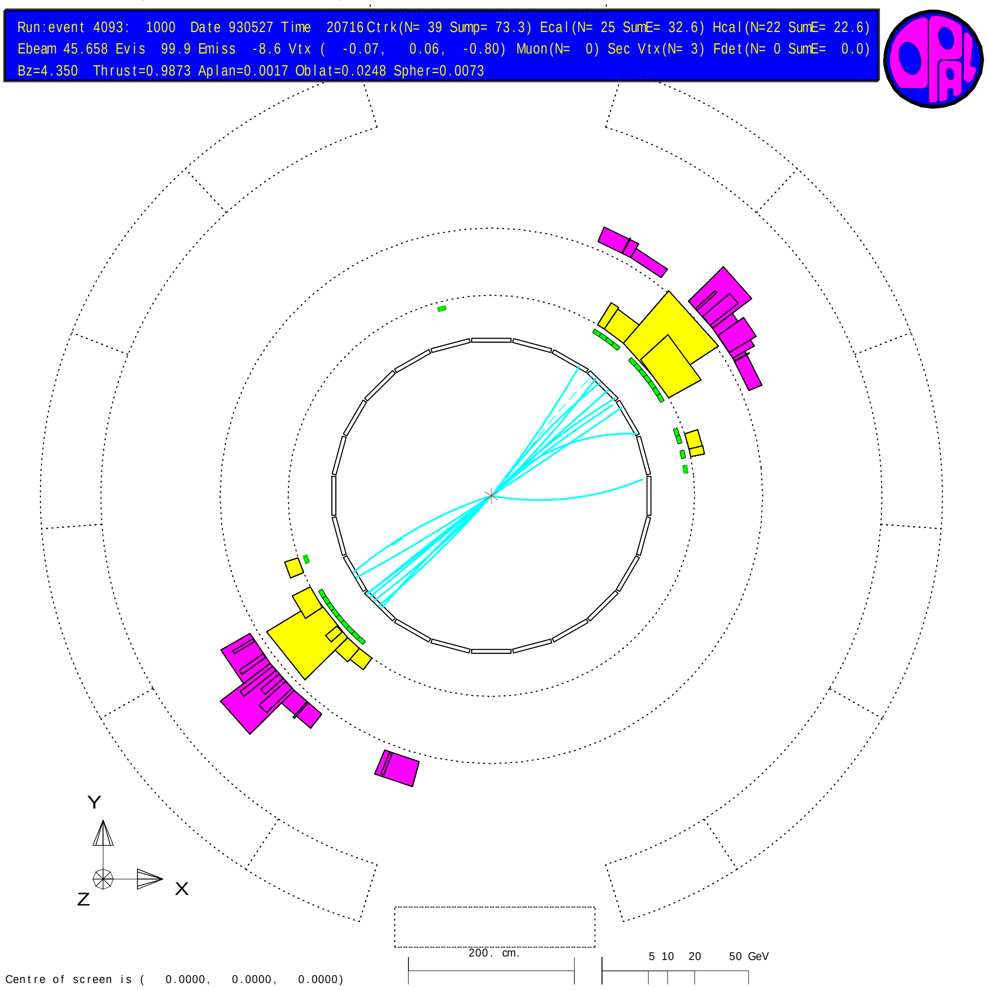
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Z
X
200. cm.
5 10 20 50 GeV
Centre of screen is (0.0000, 0.0000, 0.0000)