Elementary Particle Physics I

Exercise Set 2: Mesons and Baryons

1 Discussion Exercises

1.1 Natural Units

1. In high energy accelerators particles travel in individual bunches. Assume a pion accelerator with a bunch charge of 2×10^{-11} C. How big is the weight of a bunch in SI-units?

2. The full decay width of a particle Γ is related to its lifetime τ through the simple relation $\tau = 1/\Gamma$. What is the lifetime of a ρ meson (in units of seconds). You can find the ρ meson in the PDG booklet. Its mass is 770 MeV. It is built from u- and d-quarks only and therefore belongs to the 'light unflavored mesons'.

1.2 PDG Booklets

Find the following information in the PDG booklet:

1. The mass of the muon.
2. The branching ratio of the τ lepton into $K^- \nu_\tau$.
3. The value of the fine structure constant.
4. The lifetime of the neutron.
5. What is the spin of the B^0 meson?
6. Which quarks is the K^- made of?
7. What will be the beam energies of the Large Hadron Collider (LHC)?
8. Can you find the definition of the three Mandelstam variables? They have to do with the kinematics of two-body reactions. You are already familiar with one of them (s) as being the center-of-mass energy of the reaction.

1.3 Elementary Particles and Forces

Try to recall as many of these answers as possible from memory.

1. How many elementary fermions do you remember from the lecture? Give their basic properties: spin, mass, charge.
2. Do you remember the four fundamental forces and their mediating bosons?
3. What are the charges associated with these forces? (For the weak force we will have to wait for a complete answer until later in the lecture series.)
4. Which fermions are affected by which forces?
5. Mesons are bound states formed from a quark and an anti-quark. Try to build a neutral meson with exactly one strange (anti-)quark. They are called (neutral) kaons.
6. What can you say about the spin and parity of the neutral kaons?

1.4 The Wavelength of a Particle

To describe a free particle traveling through the vacuum we use planar waves with a de Broglie wavelength of

$$\lambda = \frac{2\,\pi}{p}$$

(You didn't expect to find a factor $\hbar$ there, did you?)

1. How much kinetic energy must an electron have to resolve structures of molecular dimensions ($\approx 10^{-10}$ m)? What would be the energy of a photon with identical wavelength?
2. How much energy does one need to resolve the structure of a proton (dimensions $\approx 10^{-15}$ m) with an electron beam. Imagine a table tennis ball (mass 2.5 g) with the same kinetic energy. What is its wavelength and its speed?

1.5 The Discovery of the Positron

The positron was discovered in 1932 by Carl Anderson. He took photographs of cosmic rays passing through a cloud chamber. The figure shows one of his early pictures. The chamber is placed in a homogenous magnetic field pointing into the plane of the figure. The horizontal bar in the center is a 6-mm-thick lead plate. The rest of the chamber is filled with air at STP.

1. Use the track curvature to relate the charge of particle to its flight direction. If the particle in the picture is indeed positively charged, from which direction did it enter the chamber, from the top traveling downwards or from the bottom traveling upwards?

2. How could Anderson be sure that this is not an electron?
   Tip: What effect do you expect the lead plate to have on the particle?

3. From the picture you can determine the radius r of the positron track in either of the two halves of the chamber. Calculate its momentum in terms of r and the magnetic field B. Estimate the momenta above and below the lead plate using the magnitude of the magnetic field (15 kG) and your estimate of r.

1.6 The Pseudoscalar Meson Octet

This octet is made up of nine (*sic*) light spinless mesons. Three of them (π^+, π^0, π^-) have masses of about 140 MeV and have very similar behavior under the strong interaction. Four of them ($\mathrm{K}^+, \mathrm{K}^-, \mathrm{K}^0, \bar{\mathrm{K}}^0$) have masses of about 500 MeV and are characterized by their long lifetimes, i.e. they do not decay as easily as the pions. These kaons are called *strange* mesons, and carry nonzero values of the strangeness quantum number. Rounding out this octet are the neutral η^0(550 MeV) and $\eta^{0\prime}(960 MeV)$.

- How can one build these mesons using u, d, and s quarks??

- Why do the masses of the mesons differ from the sums of the quark masses?

Quarks	Masses
u	3 MeV
d	6 MeV
s	480 MeV
c	1650 MeV
b	4700 MeV
t	180000 MeV

2 Homework Exercises

2.1 The Fine Structure Constant

1. Calculate the fine structure constant α in SI-units:

$$\alpha = \frac{e^2}{4\pi\, \epsilon_0\, \hbar c}$$

   Why is this a "senseless" exercise?
   What is the magnitude of the electron charge in natural units? (1 point)

2.2 The Baryon Decuplet

1. Most of the members of the baryon decuplet (spin=3/2) decay in a time of approximately $10^{-23}\,s$ into a baryon from the baryon octet (spin=1/2) and a meson from the pseudoscaler meson octet, for example $\Delta^{++} \to p + \pi^+$. Give all the decay modes of this form (baryon (spin $\frac{3}{2}$) $\to$ baryon (spin $\frac{1}{2}$) + pseudoscalar meson) for the Δ^-, Σ^{*+} and Ξ^{*-}. Don't forget that strangeness must be conserved, since these are strong decays. (3 points)

2. Which of the above decays are kinematically forbidden? (2 points)

3. What are the dynamically allowed decays of the above type (baryon (spin $\frac{3}{2}$) $\to$ baryon (spin $\frac{1}{2}$) + pseudoscalar meson) for the Ω^- baryon? It's lifetime is Ω^- ist $8.22 \pm 0.12 \times 10^{-11}\,s$. Why does it live so much longer than the other members of the baryon decuplet? (1 point)

2.3 Collider Kinematics

1. **Large Electron-Positron Collider (LEP)**
   An electron collides head on with a positron each with an energy of 45.6 GeV. What is the center of mass energy of the collision? Calculate energy and momentum of the muons produced in the reaction (1 point)

2. **Hadron-Elektron-Ring-Anlage (HERA)**
   Calculate the center of mass energy of the electron proton collider HERA. The beam energies are 27.5 GeV for the electron beam and 820 GeV for the proton beam (The electron mass is negligible and the proton mass is $m_p = 938\,\text{MeV}$).

   Imagine you try to reach the same center of mass energy in a fixed target experiment, colliding an electron beam onto protons at rest. What energy would the electron beam have to have? (2 points)