Elementary Particle Physics I

Exercise Set 3: Relativistic Kinematics

1 Discussion Exercises

1.1 Relativistic Decay Length of a Muon

Suppose that a muon is generated by a cosmic ray in the atmosphere 10 km above the surface of the earth with a momentum of 10 GeV pointing radially to the surface.

1. What is the probability for the muon to reach the surface of the earth before it decays?

2. Consider the problem in the rest system of the muon, where its lifetime is the proper lifetime of about 2 μs. How, if at all, does the calculation of this probability change?

1.2 Mandelstam Variables

Consider the two-body scattering process $A + B \to C + D$.

- Define the Mandelstam variables s, t and u for this process.

- How do you know that these variables are Lorentz invariant?

- Show that $s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2$.

The Lorentz invariance of the Mandelstam variables makes them very useful for describing such scattering processes. In order to use them, however, their relationship to the final-state measured quantities must be established.

- Consider the elastic scattering of two particles of the same mass ($A + B \to A' + B'$). Express the Mandelstam variables as functions of the momentum and scattering angle in the center-of-mass system.

1.3 Threshold Energies for Particle Production

1. Consider the collision of particles A and B in the rest system of B. N particles (C_1, ..., C_N) are produced. The minimum energy of the incident particle A necessary to produce the final-state particles is called the "threshold energy", E_A. This minimum energy is thus the energy for which the final-state particles are produced with no kinetic energy. Calculate E_A as a function of the produced particle masses for this reaction.

2. Use the above result to calculate the threshold energies for the following reactions. (The left-most particle designates the beam particle.)

   (a) $p + p \to p + p + p + \overline{p}$

   (b) $p + p \to p + p + \pi^- + \pi^+$

(c) $\pi^- + p \to K^0 + \Sigma^0$

(d) $p + p \to p + \Sigma^+ + K^0$

(e) $e^- + p \to J/\psi + e^- + p$

2 Homework Exercises

2.1 Collider Kinematics at the PEP-II "B Factory"

Last year, a new e^+e^- collider began operation at the Stanford Linear Accelerator Center (SLAC) in California. It is an asymmetric "factory" of B mesons, called PEP-II. It will run at exactly the energy necessary to create a pair of particles called the B^0 mesons. Their masses are 5.28 GeV.

The collider is called asymmetric because the electron and positron beams have different energies. The electron beam will have an energy of 9 GeV. What is the energy of the positron beam? (You may neglect the mass of the electron in all calculations.) (1 point)

Write down the 4-momentum vectors of the B^0 mesons. Assume opposite directions for the electron and positron momenta and assume the electron beam to point in the positive z-direction. (1 point)

What is the average decay length of the B^0 mesons given that the product of the proper lifetime with the speed of light is $c\,\tau_{B^0} = 468\,\mu\text{m}$? (1 point)

2.2 Kinematics of Particle Decays

Consider the two-body decay of particle A to particles B und C in its rest frame.

(a) Calculate the energies of particles B und C as functions of the masses m_A, m_B und m_C. (2 point)

(b) Show that the magnitudes of the momenta p_B und p_C bear the following relationship to the masses (2 point):

$$|p_B| = |p_C| = \frac{\sqrt{\lambda(m_A^2, m_B^2, m_C^2)}}{2\,m_A},$$

where λ is the triangle function:

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz.$$

(c) Use the above result to calculate the energies of the following decay products in the rest system of the decay particle (3 points):

i. $\pi^- \to \mu^- + \overline{\nu}_\mu$

ii. $\pi^0 \to \gamma + \gamma$

iii. $K^0 \to \pi^+ + \pi^-$

iv. $\Lambda \to p + \pi^-$

v. $\Omega^- \to \Lambda + K^-$

vi. $J/\psi \to D^0 + \bar{D}^0$