University of Bonn Summer Semester 2000

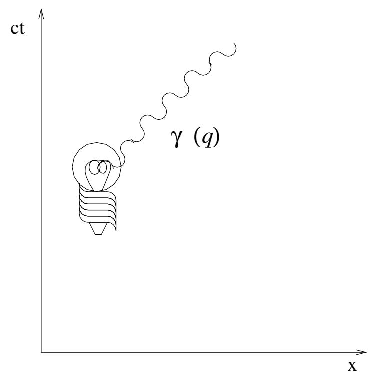
Elementary Particle Physics I

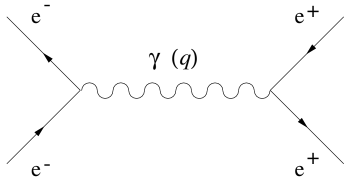
Exercise Set 4: Scattering Processes

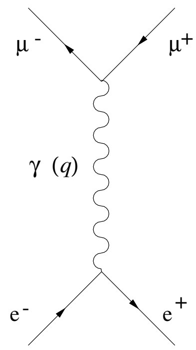
1 Discussion Exercises

1.1 Real and virtual photons

Calculate the squared four-momentum of the photon in the following three cases :







Label the diagrams with the terms "timelike", "lightlike" and "spacelike".

1.2 The cross section for neutrinos

A neutrino beam is produced with decays of pions and kaons. Of course, it can't be focussed because the neutrino has no electrical charge. To absorb the muons in the beam one needs massive shielding. That leads to a big distance between the neutrino detector and the decay vertices. That's why neutrino beams have a huge transverse spread. We calculate with the following values : A cylindrical beam with 2 m diameter and a flux of 10^5 $\mathrm{cm}^{-2}\mathrm{s}^{-1}$.

The cross section for neutrinos on an iron target is $5\ fb \times E$ per nucleon. Where E is the energy of the neutrinos in GeV and fb are femtobarn (femto=10^{-15}). (Is the theory of weak interaction really not renormalisable ?) The energy spectrum of the decay neutrinos is also very broad, but for the purpose of this exercise we assume a monochromatic beam with $E_\nu = 100\,GeV$.

1. Calculate the absorption length in iron.
2. Assume a detector made out of iron blocks and thin scintillator layers. What total depth of iron do you need to achieve an interaction rate of 1Hz ?
3. What is the probability that a neutrino interacts in this detector ?

4. Compare these values with those of the extracted proton beam in the Fermi National Accelerator Laboratory, which produces 10^{13} 800 GeV protons per second. The cross section in iron is 13 millibarn per nucleon and depends very little on the energy. How long must an iron target be, so that 10% of the protons interact ? And what is the interaction rate ?

2 Homework Exercises

2.1 Cross Sections

A beam of cross section A and current j hits a target with density n_0 and thickness d. A certain reaction i happens at a rate $\dot{N}_i$.

1. Write down the cross section σ_i in terms of these quantities. (1 point)
2. Consider the scattering of a point particle from hard spheres of radius R. Calculate the scattering angle θ as a function of the impact parameter. Then calculate the differential cross section. (2 points)
3. Integrate the differential cross section to the total cross section. Did you expect this result? (1 point)

2.2 Luminosity

The reaction rate $R = \dot{N}$ for a certain process at a collider is proportional to the cross section σ. The constant of proportionality is called the luminosity $\mathcal{L}$:

$$R = \sigma \mathcal{L}.$$

It characterizes the quality of the accelerator. It can be calculated for a particle anti-particle collider as

$$\mathcal{L} = f \, n_b \, \frac{n_1 \, n_2}{4\pi \sigma_x \sigma_y}$$

where f is the revolution frequency of the beams, n_b the number of bunches in each beam, n_1 the number of particles in the bunches of the first beam and n_2 is the corresponding number for the second beam, and σ_x, σ_y the vertical and horizontal width of both beams at the interaction point.

1. The LEP collider is a 27 km long e^+e^- collider with a center of mass energy of 91.2 GeV (that should be enough information to calculate the revolution frequency). There are 4 bunches in each beam with 7.5×10^{10} particles each. The beams are focused down to $\sigma_x = 100\ \mu$m and $\sigma_y = 20\ \mu$m at the interaction point.

   Calculate the luminosity. (1 point)
2. The cross section for the production of Z^0 bosons at this energy is roughly 60 nb. How many Z^0 bosons do you get in a typical year of running (10^7 seconds). (1 point)

3. Experimentally the luminosity is not calculated from the beam parameters ($\sigma_{x/y}$ is to inaccurate). It is derived from the rate of bhabha scattering ($e^+e^- \rightarrow \mathrm{e}^+e^-$). The cross section is well known. It is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2}{s} \frac{1}{\sin^4 \frac{\theta}{2}}.$$

   Thereby is $\alpha = 1/137$ the fine-structure constant and s the squared center of mass energy. The OPAL detector has two special calorimeter (called the luminosity monitors) to count the bhabhas. Each is located 2.5 m downstream of the interaction point and detects any bhabha at a radius between 77 mm and 127 mm from the beam. What is the cross section for bhabha scattering in the angular region of this detector? (2 points)

4. During the course of another year 800000 bhabha events are collected and at the same time 19031 events of the type $e^+e^- \rightarrow \mu^+\mu^-$. What is the cross section for this process? (1 point)

5. Assume you know the inner radius of the luminosity monitors only to a precision of 1 mm. Does that have any impact on the previous measurement? (1 point)