Elementary Particle Physics I

Exercise Set 6: Scattering and Particle Detection Techniques

1 Discussion Exercises

1.1 Mott- and Rutherford-Scattering

In the lecture, the matrix element for the electron scattering on a spin-1/2 target was given:

$$< |\mathcal{M}|^2 >= \frac{8e^4}{(p_1-p_3)^4}[(p_1\cdot p_2)(p_3\cdot p_4)+(p_1\cdot p_4)(p_2\cdot p_3)-(p_1\cdot p_3)M^2-(p_2\cdot p_4)m^2+2(mM)^2] \quad (1)$$

where p_1, p_3 are the four-momenta of the electron before and after the scattering, p_2, p_4 are the momenta of the target before and after the scattering, m and M are respectively the electron and the target masses. The average over initial state spins and the sum over the final state spins has been already carried out. Consider the case $M \gg m$, so that it is possible to neglect the target recoil, and assume that the target is at rest in lab system (Mott scattering).

1. Write down the differential cross section $d\sigma/d\Omega$ in terms of the matrix element (remember the Lorentz-invariant phase space we introduced in Exercise 5).
2. Using the given matrix element, write the differential cross section in terms of the incident electron momentum and the scattering angle. Do you expect the cross section to depend on the target mass ?

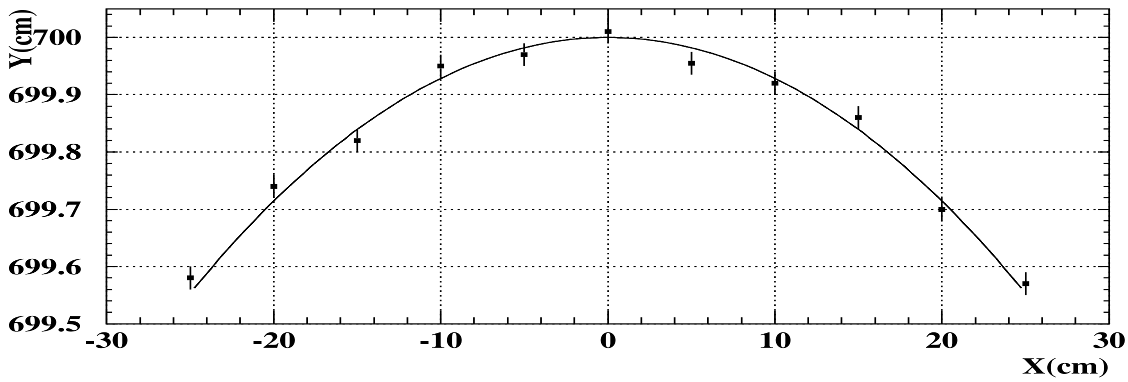
   item Derive the non-relativistic limit for $d\sigma/d\Omega$. Which formula do you obtain ?
3. Why is the scattering angle $\theta = \pi$ forbidden in the ultra-relativistic limit ?

1.2 Design of a Time-of-Flight Detector

For the purpose of particle identification you plan to build a time-of-flight detector using scintillator counters. The scintillators are to be installed 1 m from the beam axis outside of the central drift chamber and inside the barrel calorimeter. By putting a big effort at great expense into the electronic readout of the photomultiplier tubes, you manage to reach a time resolution of 100 ps.

1. What is the maximum momentum at which you can distinguish pions from kaons with 3–σ significance.
2. Up to what momentum can you distinguish kaons from protons?
3. Calculate the fraction of pions and kaons of this maximum momentum which decay before they reach the TOF detector.

subsectionMomentum Measurements with Drift Chambers Consider the trajectory of a charged particle in a big central drift chamber. The particle trajectory is curved because of the magnetic field and over a distance of $L = 50$ cm it is measured in 11 drift chamber cells. We wish to determine the momentum of the particle and also the accuracy with which we measure the momentum, given a certain spatial resolution for each of the 11 points. We work in the approximation $R \gg L$.



1. The figure shows the trajectory in a coordinate system where the point (0,0) is the center of the circle: $R^2 = x^2 + y^2$. Our approximation allows us to approximate the curve as a second-order polynomial: $y(x) = a_0 + a_1 \cdot x + a_2 \cdot x^2$. Find a_2 as a function of R. We will refer to the variable $c = -2 \cdot a_2$ as the "curvature."

2. The sagitta is a measure of the curvature and comparison with the position resolution (here $\sigma_y = 0.2$ mm) gives the accuracy with which one can determine the curvature, and therefore the momentum. The relationship $A_N = \sigma_s^2/\sigma_y^2$ clearly depends on the number of equidistant measurements, since 1) with less than 3 points one cannot determine the sagitta, and 2) the determination accuracy increases with the number of measurements. For our purposes it suffices to know that $A_N \simeq 0.7$ für $N = 11$. In our approximation we also have $s = L^2/8R$. Express the uncertainty in the curvature σ_c in terms of σ_y, L, and A_N, and calculate the values of σ_c and σ_s.

3. This curvature yields only the component of the momentum perpendicular to the magnetic field, which we call P_t. Use the formula for the Lorentz force (see homework exercise 2.1), in order to give the transverse momentum resolution σ_{Pt} as a function of σ_c. What is the momentum of the particle and with what accuracy to we measure it, given a magnetic field of 1 Tesla?

4. Show that the relative momentum resolution σ_{Pt}/P_t is a linear function of P_t.

1.3 Energy Measurement with Sampling Calorimeters

A sampling calorimeter has, in contrast to a drift chamber, a relative energy resolution which increases with increasing energy of

he particle. The reason for this is the increasing number of charged particles which are in the shower generated by the primary particle, and which provide a measurement of the total absorbed energy. Provided that the samplings of the charged multiplicities in the longitudinal development of a shower, are independent of each other (this can be obtained through a proper choice of materials and geometry), then the energy resolution of the calorimeter is just given by : $\sigma_E/E = N^{-1/2}$. Calculate this relative energy resolution as function of the energy (in GeV), when 50 MeV per particle is absorbed on average.

1.4 Comparison Drift Chambers / Calorimeters

With which resolution will the energy of the particle in Exercise 1.3 be measured with this calorimeter ? (Take the worst case, where $E = P_t$, and neglect the mass). For which value of P_t do the detectors of 1.3 and 1.4 have the same resolution ?

2 Homework Exercises

2.1 Applications of the Lorentz force (4 Points)

The measurement of the deflection of cathode beams in crossed electric and magnetic fields by J.J. Thomson in 1897 is frequently considered as the discovery of the electron since it showed that the beams behave like particles with a specific ratio of charge to mass e/m. Here we discuss the question, of how Thomson designed the experiment.

1. What is the velocity of a charged particle which passes through an electric and a magnetic field ($\vec{E}$, $\vec{B}$, perpendicular to each other and perpendicular to the direction of the particle) without changing its direction ? (1 Point)

2. If the electric field is switched off, the particle follows a circular path with radius R. How can the ratio e/m be calculated ? (1 Point)

3. In the case of the electron, take the values $R = 10$ cm and $|\vec{B}| = 20$ Gauss, and calculate its velocity and the value of the electric field $|\vec{E}|$ necessary to fullfill the condition of point 1. (1 Point)

4. Derive from the Lorentz force the formula $P = 0.3 \cdot B \cdot R$, in units of GeV/c, Tesla and m. (1 Point)

2.2 High-Energy $e^-\mu^-$ Scattering (6 Points)

1. Draw the Feynman diagrams for $e^-\mu^-$ scattering and the process $e^+e^- \to \mu^+\mu^-$ (photon exchange only). Assign four-momenta to the initial and final states. (1 Point)

2. The matrix element for $e^-\mu^-$ scattering is given in the ultra-relativistic limit by

$$< |\mathcal{M}|^2 >= 32\pi^2\alpha^2 \frac{s^2+u^2}{t^2}$$

   with the Mandelstam variables s, u, and t. Calculate them for both processes in terms of the four-momenta. (1 Point)

3. Use crossing symmetry to derive the matrix element for $e^+e^- \to \mu^+\mu^-$. (1 Point)

4. Calculate the differential cross section from Fermi's Golden Rule. For both processes it reads (1 Point):

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\,\pi^2\,s}\frac{|\vec{p}_f|}{|\vec{p}_i|} < |\mathcal{M}|^2 > .$$

5. Express the $e^+e^- \to \mu^+\mu^-$ cross section in terms of the production angle θ of the μ^- with respect to the electron direction. (1 Point)

6. Calculate the total $e^+e^- \to \mu^+\mu^-$ cross section. (1 Point)