Elementary Particle Physics I

Exercise Set 8: Symmetries and Conservation Laws

1 Discussion Exercises

1.1 Helicity and Chirality

1. Discuss the following decays with respect to helicity $\frac{2\vec{J}\cdot\vec{P}}{E}$ conservation.
   a) $\pi^0 \rightarrow \nu\bar{\nu}$
   b) $\pi^0 \rightarrow e^+ e^-$

2. What are the chiralities $\frac{1\pm\beta}{2}$ and helicities of the daughter leptons ?

$$\frac{\pi^+ \rightarrow e^+ \nu_e}{\pi^+ \rightarrow \mu^+ \nu_\mu}$$

1.2 Parity

1. Define the parity operation.

2. Why are all interactions with neutrino's fully parity-violating?

3. Which spin-parity states are called scalar, pseudoscalor, vector, and pseudovector ? Give some examples.

1.3 Use of Detailed Balance for Spin and Parity of the Pion

The cross section for the reaction $A + B \rightarrow C + D$ is given by

$$\sigma_{A+B\rightarrow C+D} = k_{CD}\ (2S_C + 1)\ (2S_D + 1)\,|\vec{P}_C|^2\,\overline{|T_{fi}|}^2,$$

where S_C and S_D are the spins of the final-state particles and $\overline{|T_{fi}|}^2$ is the spin-averaged transition amplitude. k_{CD} is a statistical factor. It is $\frac{1}{2}$ if C and D are identical and 1 otherwise.

The principle of detailed balance claims that the transition amplitude for $A + B \rightarrow C + D$ is identical to that of the backward reaction $C + D \rightarrow A + B$ at the same center-of-mass energy.

1. Give the ratio of cross sections for the two reactions $\pi^+ d \rightarrow pp$ and $pp \rightarrow \pi^+ d$ at the same center-of-mass energy.

2. Both reactions were measured by scattering 25 MeV (kinetic energy) pions on a deuteron target and 340 MeV protons on a proton target. The cross sections are $\sigma_{\pi^+ d} = (3.1 \pm 0.3)$ mb and $\sigma_{pp} = (0.17 \pm 0.05)$ mb. Calculate the center-of-mass energy for both reactions.

3. These measurements where used in 1951 to determine the spin of the pion. At that time the spins of the deuteron and the proton were already known. They are 1 and 1/2, respectively. Determine the spin of the pion.

4. The parity of the pion can be determined by stopping negative pions in a deuteron target. They form a pionic atom and the pion cascades down into the atomic S-state before the reaction $\pi^- d \to nn$ takes place.

   What is the total angular momentum J in the initial state? What are possible combinations of the total orbital angular momentum L and total spin S for the final state?

5. The final state consists of two identical fermions. Therefore the wave function must be anti-symmetric under the exchange of the two neutrons. The spin part of the wave function is anti-symmetric for $S = 0$ and symmetric for $S = 1$. Including the orbital part the overall factor is

   $$(-1)^{L+S+1}$$

   Reconsider all possible combinations of L and S for the final state. Which one is correct (anti-symmetric)?

6. The parity of a two-particle state is $(-1)^L$ times the intrinsic parities of the two particles. The neutron and deuteron each has positive parity. Determine the parity of the pion.

2 Homework Exercises

2.1 The Four-vector Potential A^μ

We define the four-vector potential as $A^\mu = (V, \vec{A})$. Then the field tensor can be written as

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu.$$

Gauge invariance allows us to impose the Lorentz condition

$$\partial_\mu A^\mu = 0,$$

and in source-free space we can also choose $V = 0$. We have thus chosen the Coulomb gauge. The wave function for a photon is then

$$A^\mu(x) \propto \epsilon^\mu(p) e^{-\frac{i}{\hbar} p \cdot x}.$$

Use the Lorentz condition to show that photons are massless and transversely polarized. (Hint: Write down the source-free Maxwell equation in case of Lorentz condition.) (2 points)

2.2 Decays to Two Photons

Consider the case of a decay into two photons of a particle with spin $\vec{J}$. The matrix element can depend only on the available observables:

$$M \propto \vec{J} \cdot \vec{V},$$

where $\vec{V}$ can be formed only by a linear combination of the two polarization vectors $\vec{\epsilon}_1$ and $\vec{\epsilon}_2$ and the momentum vector $\vec{k}$ of one of the two photons. There are three such possible vectors:

1. $\vec{V} = \vec{\epsilon}_1 \times \vec{\epsilon}_2$
2. $\vec{V} = (\vec{\epsilon}_1 \cdot \vec{\epsilon}_2)\vec{k}$
3. $\vec{V} = \vec{k} \times (\vec{\epsilon}_1 \times \vec{\epsilon}_2)$

Show that the first two possibilities are forbidden by Bose-Einstein statistics, and that the third must also be zero due to the transverse polarization of the photons. The theorist C.N. Yang used this proof to show that a particle with spin cannot decay to two photons. (3 points)

2.3 Angular Momentum, Spin, and Parity

1. What values for the orbital angular momentum in the final state are allowed in the decay $\Delta^{++} \to p + \pi^+$? (1 point)
2. The decay $n \to p + e$ necessarily violates conservation of angular momentum, since all three particles have spin 1/2. If you were Wolfgang Pauli and wished to postulate an additional particle in the final state, what value would you choose for its spin? (1 point)
3. What symmetries forbid the decay $\rho \to \pi^0\pi^0$? (1 point)
4. How does the helicity operator $\frac{2\vec{J}\cdot\vec{p}}{E}$ transform under the parity operation? What does parity conservation mean for the change in the expectation value of $\vec{J} \cdot \vec{p}$ in a reaction? (1 point)
5. How do the electric and magnetic fields transform under space inversion (parity)? Discuss parity invariance in electromagnetism, using the expression for the Lorentz force: $\vec{F} = q(\vec{E} + (\vec{v}/c) \times \vec{B})$. (1 point)