Elementary Particle Physics I

Exercise Set 9: Isospin, Parity, and Charge Conjugation

1 Discussion Exercises

1.1 The Decay of the Charged Pion

The charged pion has the decay modes: $\pi^- \to e^- \overline{\nu}_e$ and $\pi^- \to \mu^- \overline{\nu}_\mu$. The decay rate is given by Fermi's Golden Rule:

$$\Gamma = \frac{|\vec{p}|}{8\pi m_\pi^2} \times |\overline{\mathcal{M}}|^2 \times S,$$

where $|\vec{p}|$ is the momentum of each of the final-state particles and S is the spin factor of the final-state. (Assume massless neutrinos. Use the rest frame of the pion wherever you have to explicitly use a reference frame.)

1. Comparing the two decay modes, what would you expect for the ratio Γ_e/Γ_μ from phase space considerations?
2. The matrix element averaged over the spins of the final-state particles can be calculated from the Feynman rules. The result is:

$$|\overline{\mathcal{M}}|^2 = 2G_F^2 f_\pi^2 \cos^2\theta_C m_\ell^2 \left(m_\pi^2 - m_\ell^2\right)$$

   where G_F is Fermi's constant, $\cos\theta_C$ the cosine of the Cabibbo angle (to be discussed later in the lectures), f_π is a constant describing the coupling of the pion to the W with all the corrections due to the strong interaction included, and ℓ is an abbreviation for a charged lepton with mass m_ℓ (e or μ).
   Insert the matrix element into Fermi's Golden Rule to get an expression for the decay widths.
3. Calculate $R = \frac{\Gamma_e}{\Gamma_\mu}$. Compare with the PDG values and your answer to the first question.

1.2 An Experimental Limit on the Mass of the Muon Neutrino

The figure on the next page shows the experimental setup of an experiment searching for a mass of the muon neutrino (They did not find one). (reference: Physics Letters 146B (1984) 431)

They study the decay $\pi^+ \to \mu^+ \nu_\mu$ at rest.

1. Discuss the experimental setup.

2. Give the formula for the muon momentum for arbitrary neutrino mass.

3. The measurement gives a muon momentum of $p_\mu = (29.79139 \pm 0.00083)$ MeV. Calculate m_ν^2. How can such a result occur? ($m_\mu = (105.658389 \pm 0.000034)$ MeV, $m_\pi = (139.5679 \pm 0.0007)$ MeV)

4. Calculate the statistical error on m_ν^2 and derive the 90 % confidence level upper limit on m_ν^2. (Adjust the normalization of the gaussian, so that the total probability to find any physical value is 1.)

5. What is the impact of the uncertainties on m_π and m_μ?

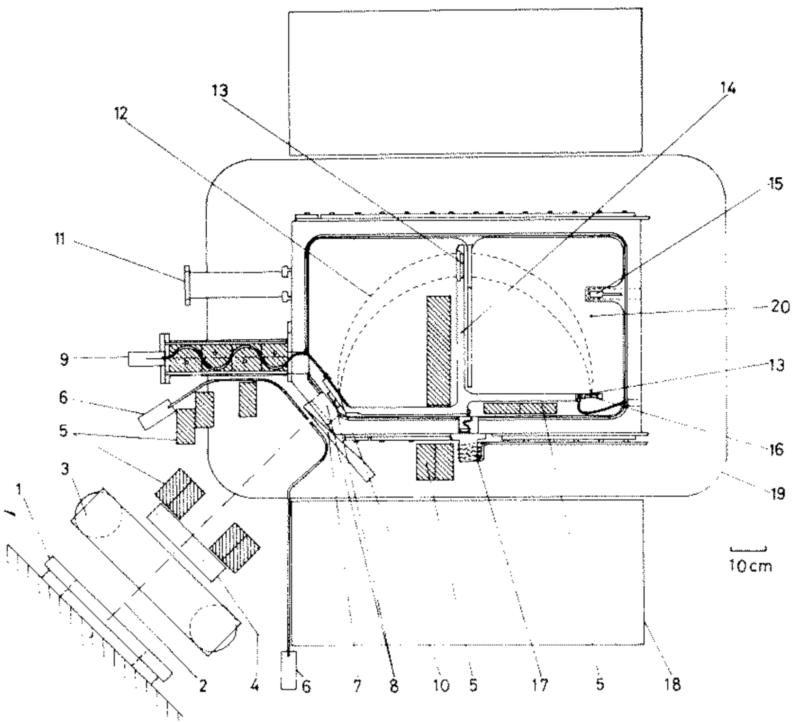

Fig. 1. Experimental set-up for the muon momentum measurement: (1) Exit vacuum window of the pion channel, (2) central trajectory of πE1 pion channel, (3) horizontal beam steering magnet, (4) multiwire proportional chamber for beam profile monitoring, (5) lead bricks, (6) light guides of the two scintillation counters used for beam steering, (7) remotely controlled pion degrader of variable thickness (graphite), (8) fixed graphite degraders, (9) S1 pion-stop scintillation counter light guide, (10) scintillator S1 (NE102A), (11) port for vacuum pump, (12) accepted muon trajectories, (13) copper collimators, (14) aluminium T-arm supporting scintillator, collimators and silicium counter, removable for distance measurements, (15) nuclear magnetic resonance probe for magnetic field measurements and stabilization, placed at "reference point", (16) silicium surface barrier detector (Si), (17) cooling spiral for temperature control of vacuum chamber and T-arm, (18) yoke of the magnet, (19) magnet coils, (20) ^{241}Am α source for calibration of the silicium detector.

2 Homework Exercises

2.1 Parity and Neutrinos

Is the neutrino an eigenstate of the parity transformation? (1 point)

2.2 Helicity Suppression

Reconsider the decay of the charged pion from the discussion exercise Nr 1.1:

1. In the rest frame of the pion, determine the angular momentum between the charged lepton and its neutrino and the orientation of their spins. (1 point)

2. What is the branching ratio of the pion into a fictitious massless charged lepton ℓ and its neutrino $\overline{\nu}_\ell$? (1 point)

2.3 States of Isospin 1

Which of the following states can have total isosopin 1? If not, why not? (5 points)

1. $\pi^0\pi^0$

2. $\pi^+\pi^-$

3. $\pi^+\pi^+$

4. $\Sigma^0\pi^0$

5. $\Lambda\pi^0$

2.4 Beta Decay and CP Symmetry

Apply the parity operator to the process $n \to p + e^- + \overline{\nu_e}$. Does this process occur in nature? Now apply the charge conjugation operator. What process is obtained? Is CP conserved in beta decay? (1 point)

2.5 Multipion CP Eigenstates

Deterine the CP eigenvalue of an N-pion state with I_3^{total}=0 and orbital angular momentum L=0. (1 point)