Cornell University
Laboratory for Elementary-Particle Physics

Heuristic Study of the Effect of Chromatic Nonlinearities on BBU Instability Thresholds (II)

Initial investigation with the toy lattice used in Hoffstaetter and Bazarov, PRST-AB 7, 054401 (2004)

$$X_i^{(N+1)} = R_{ij} X_j^{(N)} + M_{ijk} X_j^{(N)} X_k^{(N)} + M_{ijkl} X_j^{(N)} X_k^{(N)} X_l^{(N)} \qquad i, j, k=1\text{-}6$$

Introduce nonzero M_{1266} to see how particle energy distributions affect the instability threshold calculation

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Introducing the Chromatic Terms

$$X_{out} \sim X'_{in} \sin (\Delta\psi + \xi \delta)$$

1) expand the sine and cosine terms

2) account for $X'_{in} \sim 1/(1+\delta)$

3) keep terms up to second order in δ (odd power terms average to zero)

to obtain

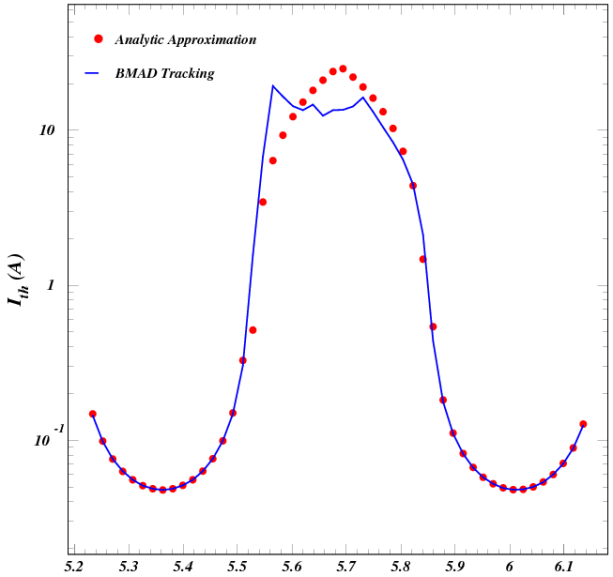
$$X_{out} \sim (1 + (1\text{-}0.5\ \xi^2)\ \delta^2) \sin(\Delta\psi) - \xi\ \delta^2 \cos(\Delta\psi)$$

NB: $\sin(\Delta\psi) = M_{12}/(\beta_1\beta_2)^{1/2} \approx -5\times10^3/(\beta_1\beta_2)^{1/2}$ in the toy model

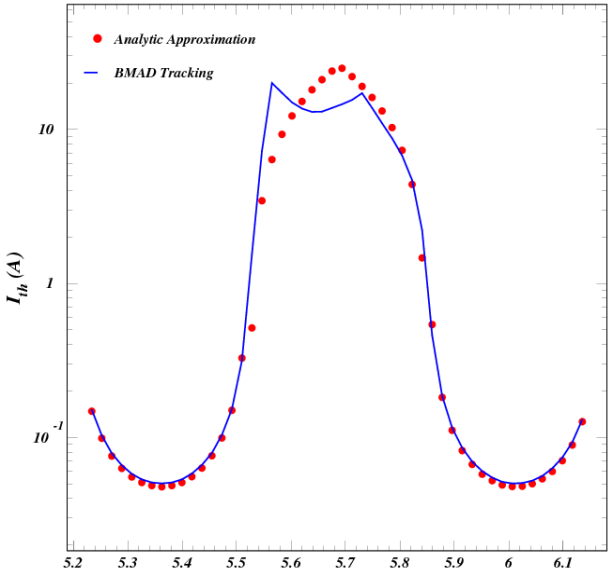
Quantitative estimate of BBU threshold current dependence on ξ in the toy model

Toy lattice of 2004 PRST-AB article with 1000 beam particles in a Gaussian δ =Δp/p distribution of rms 0.6e-3 improves minimum instability threshold current from 48 to 50 mA for ξ = ± 500.

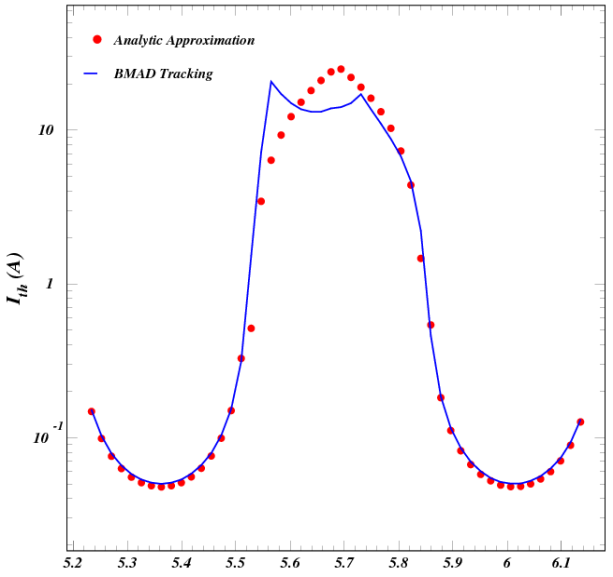
ξ = 0



ξ = −500



ξ = 500



Ratio of Return Time to Bunch Spacing