Experiment 6

Transverse Coupling

If there is coupling of horizontal and vertical motion in the ring then the one turn transfer matrix at some point s can be written

$$T = \begin{pmatrix} M & m \\ n & N \end{pmatrix} \tag{1}$$

where M, m, n, and N are 2X2 matrices. We can identify normal modes by the similarity transformation

$$\begin{aligned} \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} &= V^{-1}TV \\ &= \begin{pmatrix} \gamma & C \\ C^\dagger & \gamma \end{pmatrix} \begin{pmatrix} M & m \\ n & N \end{pmatrix} \begin{pmatrix} \gamma & -C \\ -C^\dagger & \gamma \end{pmatrix} \end{aligned}$$

where $\gamma^2 = 1 - \mid C \mid$. Then after some matrix algebra we find

$$\begin{aligned} \mathrm{tr}(A-B) &= \sqrt{(\mathrm{tr}(M-N))^2 + 4 \mid m + n^\dagger \mid} \\ \cos\phi_A - \cos\phi_B &= \sqrt{(\cos\phi_M - \cos\phi_N) + \mid m + n^\dagger \mid} \end{aligned}$$

ϕ_A and ϕ_B are the one turn phase advances for each of the two normal modes. ϕ_M and ϕ_M are the phase advances for the lattice in the absence of the coupling. $\mid m + n^\dagger \mid$ is a measure of the coupling strength.

$$\sin\phi \sin\Delta\phi = -\frac{1}{2}\sqrt{(\cos\phi_M - \cos\phi_N) + \mid m + n^\dagger \mid}$$

where $\phi = \frac{1}{2}(\phi_A + \phi_B)$ and $\Delta\phi = \frac{1}{2}(\phi_A - \phi_B)$. We can vary horizontal and vertical tune by adjustment of arc quadrupoles, (QTUNEING5 for vertical tune, QTUNEING 6 for horizontal and QTUNEING 8 for increasing vertical and decreasing horizontal simultaneously.) The minimum tune split, (minimum $\Delta\phi$) occurs when $\phi_M = \phi_N$ and then

$$\sin\phi \sin\Delta\phi = -\frac{1}{2}\sqrt{\mid m + n^\dagger \mid}$$

Measurement of ϕ and $\Delta\phi$ yields the coupling strength.

Skew Quad

Now suppose that we begin with an uncoupled lattice represented by full turn matrix

$$T = \begin{pmatrix} M & 0 \\ 0 & N \end{pmatrix}$$

And we introduce coupling via a skew quad with matrix

$$S = \begin{pmatrix} I & F \\ F & I \end{pmatrix} \tag{2}$$

where I is the 2X2 identity and

$$F = \begin{pmatrix} 0 & 0 \\ k & 0 \end{pmatrix} \tag{3}$$

Then the new full turn matrix

$$T' = TS = \begin{pmatrix} M & MF \\ NF & N \end{pmatrix}$$

$1/k$ is the focal length of the 45° quadrupole. If the coupling is weak then the normal mode splitting $\Delta\phi$ is small and the minimum splitting

$$\begin{aligned} \Delta\phi = 2\pi\Delta\nu &= \frac{1}{2}\sqrt{|\ m + n^\dagger\ |} \\ &= \frac{1}{2}\sqrt{|\ Mk + (Nk)^\dagger\ |} \\ &= \frac{1}{2}\sqrt{|\ \begin{pmatrix} kM_{12} & 0 \\ kM_{22} & 0 \end{pmatrix} - \begin{pmatrix} kN_{12} & 0 \\ kN_{22} & 0 \end{pmatrix}^\dagger\ |} \\ &= \frac{1}{2}\sqrt{k^2 M_{12} N_{12}} \\ &= \frac{1}{2} k \sin 2\pi\nu \sqrt{\beta_x \beta_y} \end{aligned}$$

where $\nu_x = \nu_y = \nu$. Then

$$\Delta\nu = \frac{1}{4\pi} k \sin 2\pi\nu \sqrt{\beta_x \beta_y} \tag{4}$$

A measurement of the minimum tune split, and knowledge of the lattice parameters at the skew quad allows a determination of the strength of the rotated quadrupole.

Procedure

If there were zero transverse coupling in CESR, then we might begin by setting horizontal and vertical tunes equal, and therefore the normal modes as well. But there is likely to be some residual coupling in the machine. Begin by measuring the minimum splitting of normal modes as a measure of that residual coupling. Then we will try to eliminate the residual coupling by adjustment of skew quads, and/or orbit. Finally, we can measure normal mode splitting vs skew quad setting and determine the calibration.

1. Fill a single bunch of positrons to about 3mA

2. Turn off electrostatic separators

3. Measure and if necessary, flatten the orbit. Vertical displacement in sextupole magnets is a major source of coupling.

4. Identify horizontal and vertical tunes on spectrum analyzer. Typically, $f_h \sim$ 220kHz, and $f_v \sim$ 240kHz. To be sure you are looking at the appropriate peak, dial a hundred units or so of QTUNEING 5 and 6 (GROUP TUNEING $\rightarrow$ QTUNEING) and see if what you think are the peaks for vertical and horizontal tunes move in the right directions.

5. Use QTUNEING 8 (GROUP TUNEING $\rightarrow$ QTUNEING 8) to move horizontal and vertical tunes in opposite directions

6. And minimize the tune difference

7. If there is residual coupling, then the exchange of emittance will be enhanced as the difference in the frequencies of the horizontal and vertical oscillations is reduced, and the vertical beam size will increase. Keep one eye on the synchrotron light TV monitor.

8. Assuming that the minimum tune split is not zero, then there is residual coupling. Try to minimize the coupling by adjusting one or more of the skew quads in CESR. (RING MAGNETS $\rightarrow$ CSR SKEWQUAD $\rightarrow$)

9. Now calibrate skew quads. Measure the normal mode splitting vs skew quad strength.

10. The β-functions at the skew quad in question can be found using CESRV with the command, "SHO ELEMENT SK_Q04W" for example, or using ANALYZER and the same command.

11. Compare your measurements with the skew quad calibration that is tabulated in the file CESR29::[CESR.CONSTANTS]SKEWQUADRUPOLE.CAL.