Phase space matching through injection channel

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$B^y_{\text{tot}}$ vs. $z$ ("injec_fld.dat"+"inf_field_alone.dat")

- $\mu^-$ curl inward
- $\mu^-$ curl outward
- Injection point

$B^y_{\text{tot}}$ [Tesla]

$z$ [meters]
Wuzeng Through Iron + Square Inflector

Magnetic Field [G]

Z [cm]
Track through iron, cryostat, inflector
Construct transfer matrix about trajectory

\[ \text{transverse offset [cm]} \]

\[ \Delta x \quad \Delta y \]

~ 0.86 deg
Transfer matrix through iron, cryostat, inflector
Total length = 4.39m

Transfer Matrix : Kick  [Matrix symplectic error:  1.185E-02]

\[
\begin{pmatrix}
1.61951 & 5.68287 & -0.10265 & -0.12099 & 0.00032 & 0.02890 & : -2.820E-02 \\
0.29750 & 1.65985 & -0.03583 & -0.05500 & 0.00025 & 0.00047 & : -1.318E-02 \\
-0.08510 & -0.05582 & 0.44649 & 3.19115 & -0.00014 & 0.00884 & : -9.206E-03 \\
-0.02370 & -0.04136 & -0.25378 & 0.40416 & -0.00010 & -0.00031 & : -5.138E-03 \\
0.00776 & 0.04529 & -0.00296 & 0.00323 & 1.00000 & 0.00046 & : -4.947E-13 \\
0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 1.00000 & : 0.000E+00
\end{pmatrix}
\]

The 6X6 transfer matrix propagates the 6-d phase space vector

\[
x \rightarrow \begin{pmatrix}
x \\
x' \\
y \\
y' \\
z \\
\delta
\end{pmatrix}
\]
E 821
V line D5 to g-2 ring

DQQ Q QOD QQQQ
focusing in HV H VH HHVH

(SHIELDING Ref. B14-1618-C-5)
40 mm-mrad beam clears inflector if at exit
\( \beta_x = 2.45, \alpha_x = -0.41, \beta_y = 19.1, \alpha_y = 0.045 \)

\( \sigma_E/E = 0.15\% \) clears inflector if at exit
\( \eta = 7.96, \eta' = 0.057 \)

Propagate backwards from downstream end of inflector through backleg iron and through beam line quadrupoles
Propagate forward through beam line quadrupoles, backleg iron, inflector and into ring
\[ \sigma = [\beta \varepsilon + (\eta \delta)^2]^{1/2} \]
\[ \varepsilon = 40 \text{ mm-mrad}, \quad \delta = 0.5\% \]

Propagate forward through beam line quadrupoles, backleg iron, inflector and into ring

Q1 requires large aperture
How do twiss parameters propagate through iron, cryostat, inflector into ring

(1) hole in back leg, (2) storage ring fringe field, (3) inflector channel
Effective gradient \quad G \approx \frac{1+\cos(63.9)}{2} \left( \frac{\Delta B_y}{\Delta R} \right)
Twiss parameters (A-matrix) is chosen to be single valued in ring.
$\beta, \alpha$ at $s=0$ are chosen so that 40mm-mrad beam fits through inflector aperture. Quads upstream of iron (F-quads and D-quads) optimized to achieve reasonable values at 120m.

(Energy aperture has not been optimized)
Conclusion
- There will significant horizontal defocusing in dipole fringe
- Better estimate of effective gradient will be determined from OPERA calculation of B-field along injection channel
- Place focusing quadrupole in the injection line as close to the iron as possible in order to most effectively compensate fringe defocusing
Suppose we choose $\beta_x$, $\beta_y$ at upstream end of inflector so that 40 mm-mrad so that most of the beam fits through the inflector aperture

(Assuming *ideal* inflector: zero field, opened ends)
Cross Section of Storage Ring and Magnetic Field
Propagate backwards once around ring (cc) and then through fringe with gradient 3.625 T/m
The effective focusing is very strong.
In the other direction:
Use the same $\beta_x \beta_y$ that got beam through *ideal* inflector but this time include focusing of fringe.