FRA - orlov interpretation
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Attempt to explain the correction to the frequency distribution for the information missing from t_0 to t_s. Let's make it very simple. Suppose there is a single frequency ω_0 corresponding to revolution period T. The single is $S(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$, a comb with the $n = \pm 1, 0$ teeth missing Suppose $n = \pm 1$ are missing. Then

$$F(\omega) = \int_{-\infty}^{\infty} S(t) e^{i\omega t} dt = \sum_{n \geq 1} \cos(n\omega T)$$

Now the question is what is missing in the range $\omega_- \leq \omega \leq \omega^+$ due to the missing teeth, where $\omega_\pm$ are the frequencies that fit within the vacuum chamber. The missing bit is

$$\Delta F(\omega) = \int_{-\infty}^{\infty} \Delta S(t) e^{i\omega t} dt = \sum_{n \leq 1} \cos(n\omega T)$$

It is clear that

$$S(t) = \int_{-\infty}^{\infty} F(\omega) e^{-i\omega t} d\omega$$

just gets us back where we started. But what is the significance of

$$\begin{aligned}
\tilde{S}(t) &= \int_{\omega_-}^{\omega_+} F(\omega) e^{-i\omega t} d\omega \\
&= \int_{\omega_-}^{\omega_+} e^{-i\omega t} \sum_{n \geq 1} \cos(n\omega T) d\omega \\
&= \int_{\omega_-}^{\omega_+} \sum_{n > 1} e^{-i\omega(t - nT)} + e^{-i\omega(t + nT)} d\omega \\
&= \sum_{n > 1} \left(\frac{e^{-i\omega_+(t-nT)} - e^{-i\omega_-(t-nT)}}{-i(t - nT)} \right) + \left(\frac{e^{-i\omega_+(t+nT)} - e^{-i\omega_-(t+nT)}}{-i(t + nT)} \right) \\
&= \sum_{n > 1} \left(\frac{e^{-i\omega_0(t-nT)} 2 \sin \Delta\omega(t - nT)}{(t - nT)} \right) + \left(\frac{e^{-i\omega_0(t+nT)} 2 \sin \Delta\omega(t + nT)}{t + nT} \right)
\end{aligned}$$

The last step is to compute

$$\int_{t_0}^{t_s} \tilde{S}(t) e^{i\omega t} dt = \sum_{n > 1}$$

Now I don't know if this is justified but. We will integrate from t_0 to t_s, so t is always less than nT where $n > 1$. Then for large n both terms contribute about the same amount. Let's approximate

the first term as

$$\begin{aligned}
\int_{t_0}^{t_s} \tilde{S}(t) e^{i\omega t} dt &\sim \sum_{n>1} \int_{t_0}^{t_s} \left(\frac{e^{-i\omega_0 (t-nT)} 2 \sin \Delta\omega (t-nT) e^{i\omega t}}{nT} \right) dt \\
&\sim \sum_{n>1} \int_{t_0}^{t_s} \left(\frac{e^{-i\omega_0 (t-nT)} (e^{i\Delta\omega (t-nT)} - e^{-i\Delta\omega (t-nT)}) e^{i\omega t}}{nT} \right) dt \\
&\sim \sum_{n>1} \int_{t_0}^{t_s} \left(\frac{e^{-i(\omega_0 - \Delta\omega - \omega)(t-nT)} - e^{-i(\omega_0 + \Delta\omega - \omega)(t-nT)}}{nT} \right) dt \\
&\sim \sum_{n>1} \left(\frac{e^{-i(\omega_0 - \Delta\omega - \omega)(t-nT)}}{(\omega_0 - \Delta\omega - \omega) nT} \right) - \left(\frac{e^{-i(\omega_0 - \Delta\omega - \omega)(t-nT)}}{(\omega_0 + \Delta\omega - \omega) nT} \right) |_{t_0}^{t_s}
\end{aligned}$$