Fitting Closed Orbit

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1 Introduction

Ideally we measure the closed orbit and the magnetic field over the volume of the storage region. Suppose we write the muon distribution $M(r, \theta, \phi)$ where r is the distance from the magic radius, ϕ is the azimuthal angle around the ring, and θ is the angle in the plane at fixed ϕ. Similary, the magnetic field is given as $B = B(r, \theta, \phi)$. The average field is

$$\langle B \rangle = \int BMdv$$

If we know the distribution and the magnetic field everywhere in space, we can compute the average field.

The magnetic field is measured with the trolley, with respect to the rails. The rails are aligned to the magic radius and the midplane of the magnet gap with some precision ? The muon distribution can be described in terms of the closed orbit, or the centroid of the distribution, and the width. We will assume that except for azimuthal variation of the closed orbit, that the width and height scale as the root of the local beta-functions.

If the closed orbit is measured precisely then we know the magnetic field $\vec{B}(\vec{r} = \vec{r}_0)$ where $\vec{r}_0$ is the closed orbit. A measure of $\beta(s)$ gives the gradient along the closed orbit

$$\frac{d\vec{B}}{dx}, \frac{d\vec{B}}{dy}.$$

Also

$$\Delta Q_{x,y} = \frac{1}{4\pi} \frac{1}{B\rho} \int \frac{1}{\beta(s)} \frac{dB_{x,y}(s)}{d(x,y)} ds$$

In the PR article about E821 we find that the quadrupole moment is

$$\frac{dB_y}{dx} \sim \frac{0.06 \times 10^{-6}(1.45 \text{ T})}{0.045 \text{ m}} \times \frac{0.3}{3.094} \sim 0.193 \times 10^{-6} \text{ m}^{-2}$$