G-2 Storage Ring

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1 Introduction

The magnetic moment of the muon is determined in the g-2 experiment by measuring its precession frequency as it circulates in a uniform magnetic field. The radius of curvature of the 3.096 GeV/c muons in the 1.45T field is 7.112 m. Vertically focusing electrostatic quadrupoles stabilize vertical as well as horizontal motion. The muon beam is injected into the ring through a hole in the backleg iron and then a septum magnet (inflector). The injected muons enter the ring vacuum chamber displaced radially outward 77 mm from the closed orbit. A single turn fast kicker magnet steers the muons onto the storage ring closed orbit. Characteristics of the injection and ring optics are reviewed.

2 Ring lattice

The layout of the ring optical elements in shown in Figure **??**. The four pairs of electrostatic quadrupoles[**?**] provide vertical focusing. The short and long quad in each pair have arc lengths of 13 deg. and 26 deg respectively. The nominal voltages of +31.5kV on the top and bottom plates and −31.5kV on the inner and outer plates corresponds to an electric field gradient $\nabla_x E_x = 26$ kV/m^2 and focusing strength $k = 8.3 \times 10^{-6}$ m^{-2}. The focal length of each quad pair is 24.9 km. The twiss parameters of the closed ring are shown in Figure **??**. The horizontal tune is just below unity, $Q_x = 0.9038$ and $Q_y = 0.4327$.

3 Injection channel

Approximately 1.2m downstream from the final focus quadrupole in the M5 line, beam enters the hole in the backleg iron of the g-2 magnet. The residual magnetic field in the hole is a couple of hundred gauss. The beam emerges from the hole between the outer coil cryostat and into the fringe field region. The vertical component of the field increases from near zero at the inner edge of the iron to the full field of 1.45T along the trajectory of the beam. Part way through the fringe field region the beam enters the inflector with vertical field equal but opposite that of the main magnet. The magnetic fields along the muon trajectory through iron, fringe and inflector are shown in Figures **??** and sum of inflector and main magnet in **??**. From the vantage point of the beam the fringe field has an effective horizontally defocusing gradient. Muons nearer the closed orbit experience a larger field than those at larger radii. In addition, there is a non-negligible gradient in the inflector field that is also defocusing. The optics of the injection channel can thus be modeled as the beamline shown in **??**

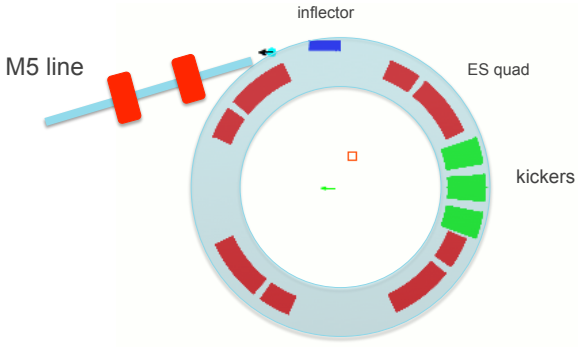


Figure 1: stuff

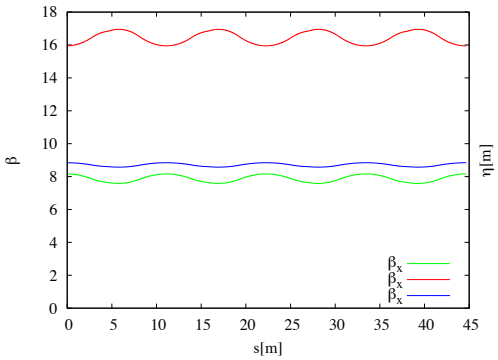
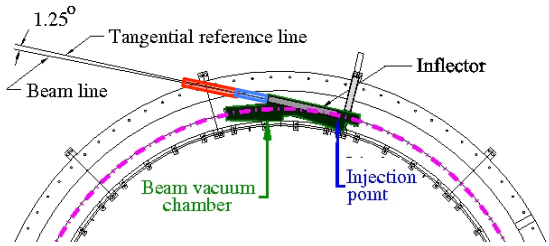


Figure 2:



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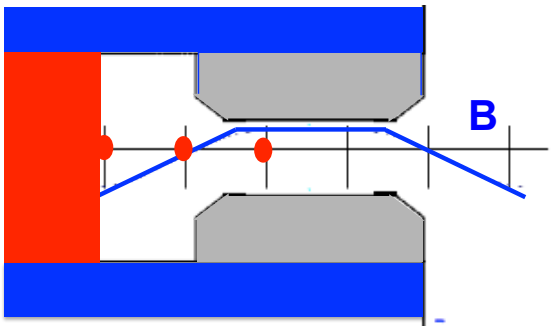


Figure 4:

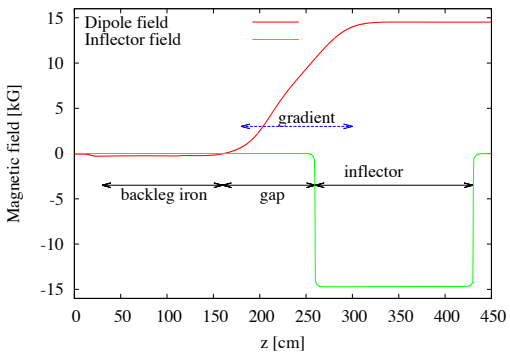
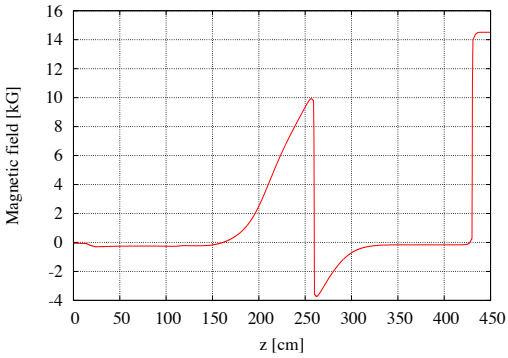


Figure 5: stuff



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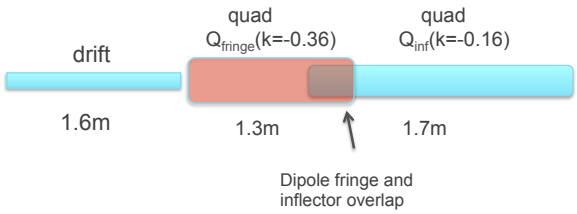

Figure 7: stuff

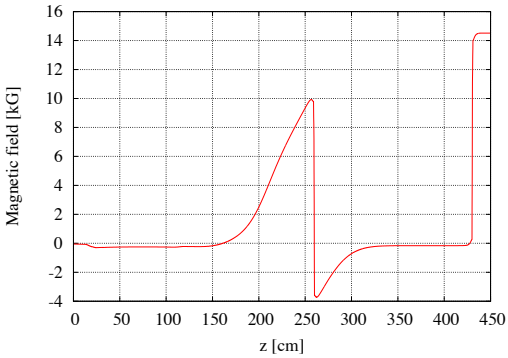

Figure 8:

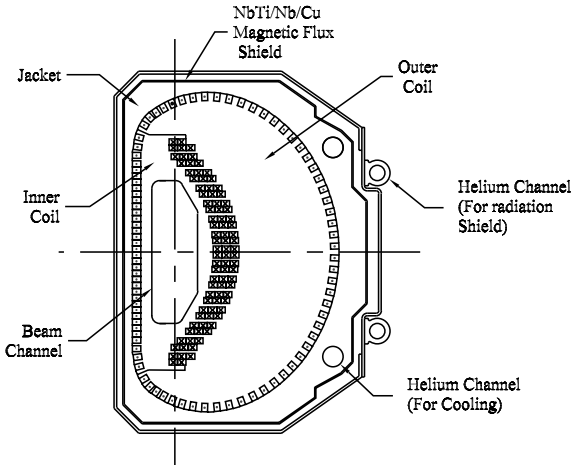

Fig. 7. Inflector cross-section.

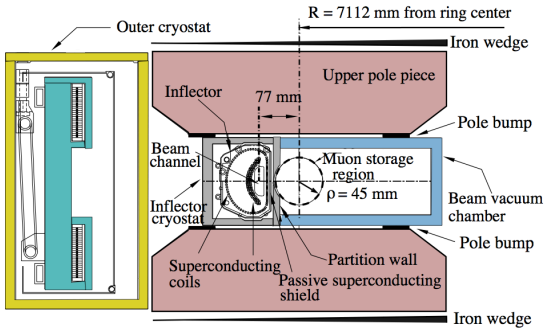

Figure 9: stuff

Figure 10:

4 Final focus

5 Optimization of Capture Efficiency

6 Scattering

7 Mismatch

8 Scraping - lost muons

9 Quad nonlinearity

10 Chromaticity

11 Momentum spread

The energy dependence of the betatron tunes and the revolution frequency, contribute to the evolution and decoherence of betatron motion due to the finite energy spread in the beam. In a cyclotron with electrostatic focusing distributed uniformly around the ring, the tune depends on the focusing index n according to

$$Q_x = \sqrt{1-n}$$

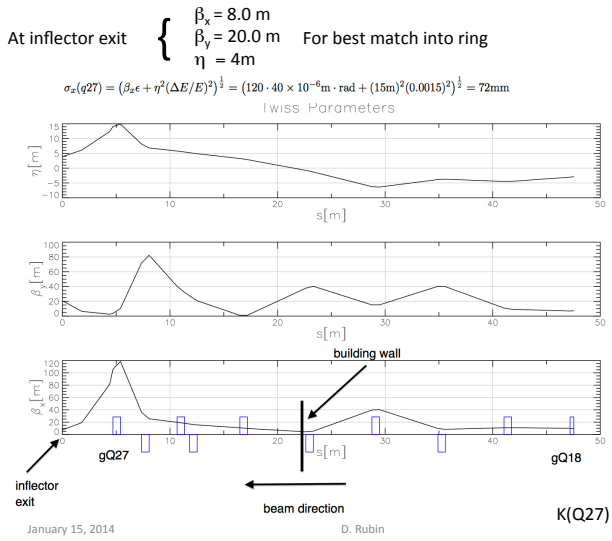

Figure 11: stuff

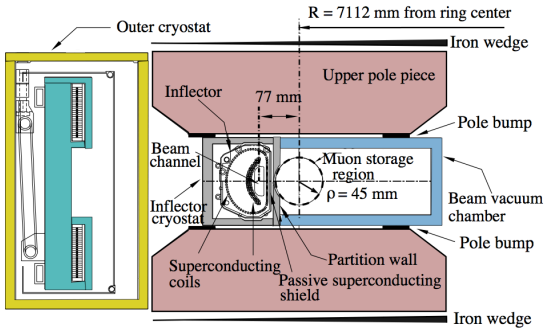

Figure 12:

$$Q_y = \sqrt{n}$$

where

$$n = \left(\frac{r}{v_s B}\right)\frac{\partial E_r}{\partial r}$$

r is the radius of curvature of the on momentum muon in magnetic field B and v_s is the azimuthal velocity, that is $r = \frac{\gamma m v_s}{qB} = \frac{p}{qB}$. The dependence of betatron tune on energy, (chromaticity) follows from

$$n(\delta) = \frac{p(1+\delta)}{q v_s B^2}\frac{\partial E_r}{\partial r}$$

so that

$$\begin{aligned}\frac{\partial Q_x}{\partial \delta} &= -\frac{1}{2}\frac{1}{\sqrt{1-n}}\frac{\partial n}{\partial p} = -\frac{n}{2\sqrt{1-n}} = -0.103\\ \frac{\partial Q_y}{\partial \delta} &= \frac{n}{2\sqrt{n}} = 0.215\end{aligned}$$

evaluated for $n = 0.185$.

Of course in the g-2 ring, the electrostatic quadrupoles do not extend continuously around the circumference with important implications for the chromaticity. Let's revisit the various contributions to the chromaticity. The contribution to the horizontal and vertical tune of the electrostatic quads is

$$Q_{h/v} = \frac{1}{2\pi}\oint \frac{1}{\beta_{h/v}} ds$$

$$Q_{h/v} = \frac{1}{2\pi}\oint \sqrt{K} ds Q_{h/v} = \sqrt{K} R$$

where

$$\begin{aligned} K_x &= \frac{1}{\rho^2} - \frac{q}{mv^2}\frac{\partial E}{\partial r} \\ K_y &= \frac{q}{mv^2}\frac{\partial E}{\partial y} \end{aligned}$$

and in a ring with uniform β, $K = 1/\beta^2$. For convenience define $-J \equiv \frac{q}{mv^2}\frac{\partial E}{\partial r} = n$ and note that $Q_y = \sqrt{J}$ There are three distinct contributions to the chromaticity.

1. The effective gradient (K) decreases with energy.

$$\begin{aligned} \frac{\partial K_x^{1/2}}{\partial \delta} &= \frac{1}{2}K^{-1/2}\frac{\partial K}{\partial \delta} = -\frac{(\rho^{-2} - \frac{1}{2}J)}{\sqrt{K}} = -\sqrt{K} - \frac{J}{\sqrt{K}} = -Q_x + \frac{Q_y^2}{2Q_x} \\ \frac{\partial K_y^{1/2}}{\partial \delta} &= \frac{1}{2}\sqrt{K} = \frac{1}{2}Q_y \end{aligned}$$

2. The path length increases with energy, $\Delta P = 2\pi\eta\delta$. If the inner and outer quad plates have equal angular length, then a longer path corresponds to longer quads and more focusing.

$$\frac{\partial Q_{h/v}}{\partial \delta} = \frac{1}{4\pi}\oint \beta_{h/v} K(s) 2\pi\eta = -\frac{\eta}{R} Q_{h,v}$$

   where R is the magic radius. If the quad plates have equal linear length (rather than equal angular length), then there is no pathlength dependent focusing.

3. The sextupole component of the quad fields couples to the tune via the dispersion. If the quad plates have no curvature, then the quadrupole symmetry precludes a sextupole moment. But there is curvature in the g-2 quads, and solutions to Laplace's equation in cylindrical coordinates are guaranteed a sextupole component. In cartesian coordinates the quadrupole potential

$$V(x,y) = \frac{1}{2}k(x^2 - y^2)$$

   gives

$$\mathbf{E} = -\nabla V = -kx\hat{\mathbf{x}} + ky\hat{\mathbf{y}}$$

   and $\nabla \cdot \mathbf{E} = 0$. In cylindrical coordinates, a solution to the Laplace equation with lowest order term linear in displacement, all of the nonlinearity in the radial coordinate, and a strictly linear vertical dependence is

$$V = k\left(\frac{1}{2}(r^2 - 1) - r_0 \ln\frac{r}{r_0} - y^2\right)$$

   and

$$\mathbf{E} = \frac{1}{2}k\left((r - \frac{r_0^2}{r})\hat{\mathbf{r}} - 2y\hat{\mathbf{y}}\right)$$

Now $\nabla \cdot \mathbf{E} = 0$ in cylindrical coordinates. With the substitution $r = r_0 + x$, where r_0 is the magic radius,

$$\mathbf{E} = k\left((x - \frac{1}{2r_0}x^2 + \ldots)\hat{\mathbf{r}} - y\hat{\mathbf{y}}\right)$$

The closed orbit of an off energy particle is displaced from $r = r_0$ to $r_0 + \eta\delta$. The radial gradient for an off energy muon becomes

$$E_r = k(1 - \frac{\delta\eta}{2r_0})x.$$

The energy dependence of the horizontal gradient due to the sextupole component will be

$$\frac{\partial K}{\partial \delta} = k(-\frac{\eta}{2r_0})$$

with contribution to the chromaticity

$$\frac{\partial Q_h}{\partial \delta} = -\frac{1}{4\pi}\oint \beta_h \frac{\eta}{2r_0}K(s)ds = \frac{\eta}{2r_0}Q_h$$

In summary, contributions to chromaticity include

1. the energy dependence of the quad gradient vertical tune,
2. energy dependence of path length
3. The nonlinearity associated with the curvature.

Assuming equal angular length of inner and outer plates and strictly linear vertical quad focusing

$$\frac{\partial Q_h}{\partial \delta} = -Q_h\left(1 - \frac{\eta}{r_0} + \frac{\eta}{2r_0}\right)$$

$$\frac{\partial Q_v}{\partial \delta} = -Q_v\left(1 - \frac{\eta}{r_0}\right)$$

Suppose that all of the particles in the initial distribution appear in the ring at the same point in space and time but with a spread in energy. The particles will execute betatron oscillations with a frequency that depends on the energy, namely $Q_x(\delta) = Q'_x\delta$ and $Q_y(\delta) = Q'_y\delta$, where δ is the fractional energy offset. The particles will circulate with cyclotron frequencies $\frac{1}{\omega(\delta)} = \frac{1+\delta}{\omega_0}$. The betatron frequency

$$\begin{aligned} \omega_\beta &= Q\omega \quad \text{becomes} \\ \omega_\beta &\rightarrow (Q + Q'\Delta)\frac{\omega_0}{1+\Delta} \end{aligned}$$

12 Diagnostics

13 Conclusion