

# Muon Beam Dynamics and Spin Dynamics in the g-2 Storage Ring

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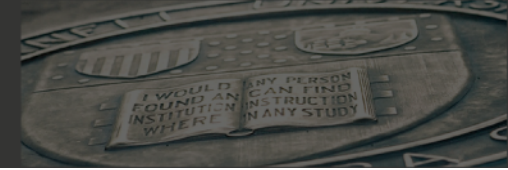
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W. Wu, University of Mississippi

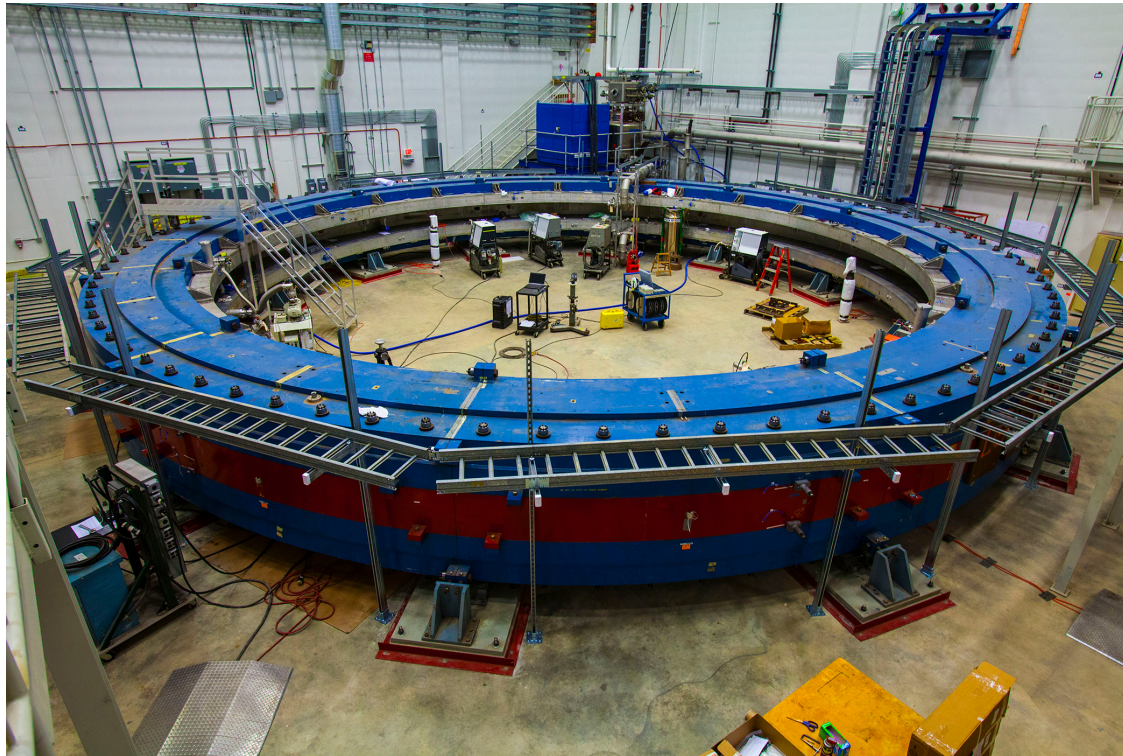




- Goal
  - measure the anomalous magnetic moment of the muon (g-2) with 0.14 ppm uncertainty (*BNL 0.54 ppm*)
- Method
  - Circulate polarized muons at the ‘magic’ momentum, in uniform magnetic field and measure precession frequency
- Polarized muons
  - select of highest energy muons in  $\pi^+ \rightarrow \mu^+ \nu_\mu$
- Measure muon polarization
  - Correlation with energy of decay positrons



- Commissioning run summer 2017
- Production run is just beginning
- Statistics in excess of BNL by summer 2018



- *Upgrades of several systems planned for summer 2018*



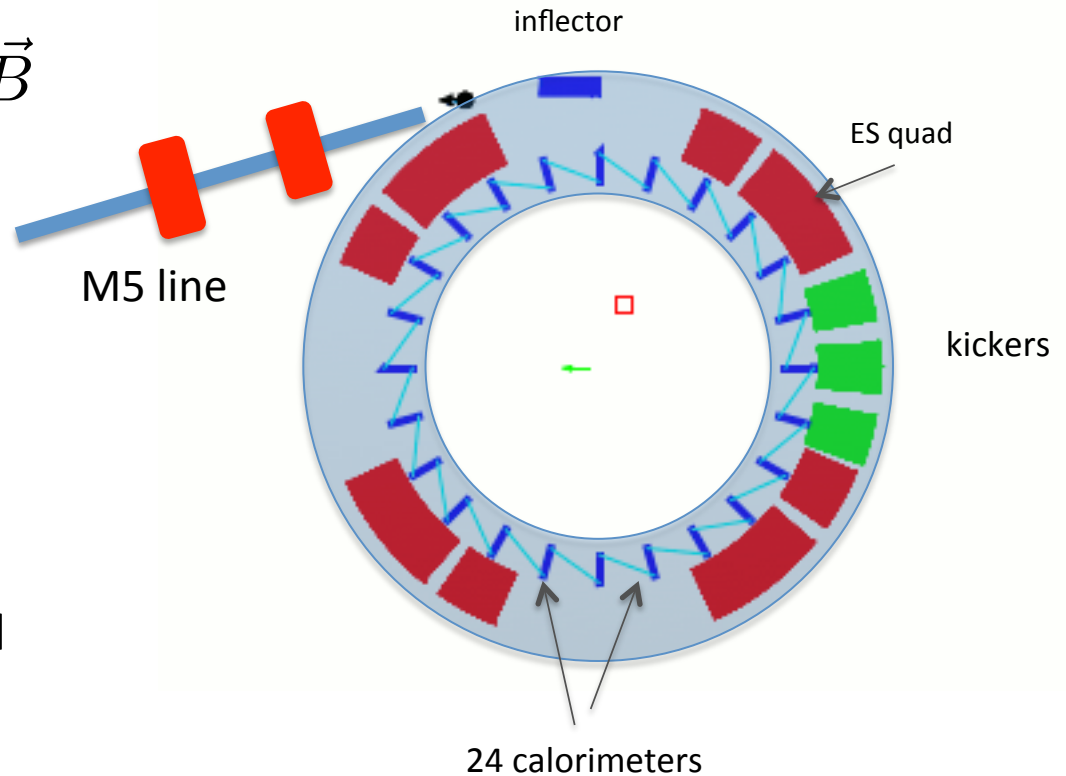
# Muon storage ring

$$a_\mu = \frac{g - 2}{2}$$

$$\vec{\omega}_a = \vec{\omega}_S - \vec{\omega}_C = -\frac{q}{m} a_\mu \vec{B}$$

(ideal field and trajectory)

Requirement of uniform B-field  
=> electrostatic focusing



Circumference = 44.69 m





# Muon storage ring

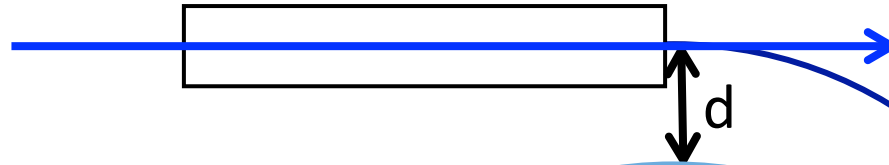


- Circumference = 44.69 m
- Revolution period = 149.1 ns
- $B_y = 1.45$  T
- $P_\mu = 3.09$  GeV/c
- $\gamma = 29.3 = \left(\frac{1}{a_\mu} - 1\right)^{1/2}$
- Aperture = 9 cm
- $\tau_\mu = 64.4$   $\mu$ s (at 3.09 GeV/c) = 432 turns



# Electrostatic quadrupoles

inflexor



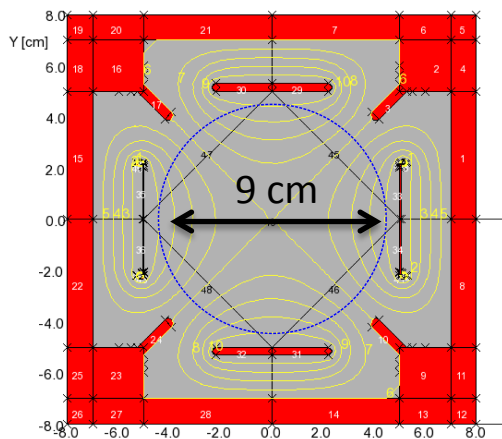
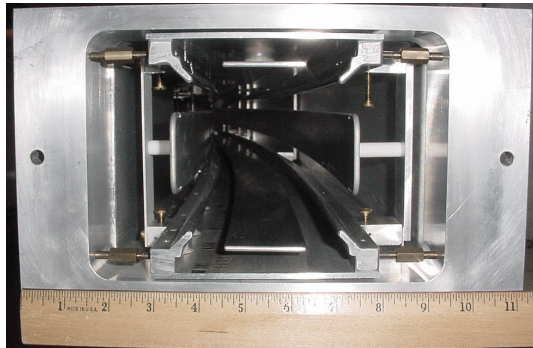
quadrupoles

711.2 cm

kickers

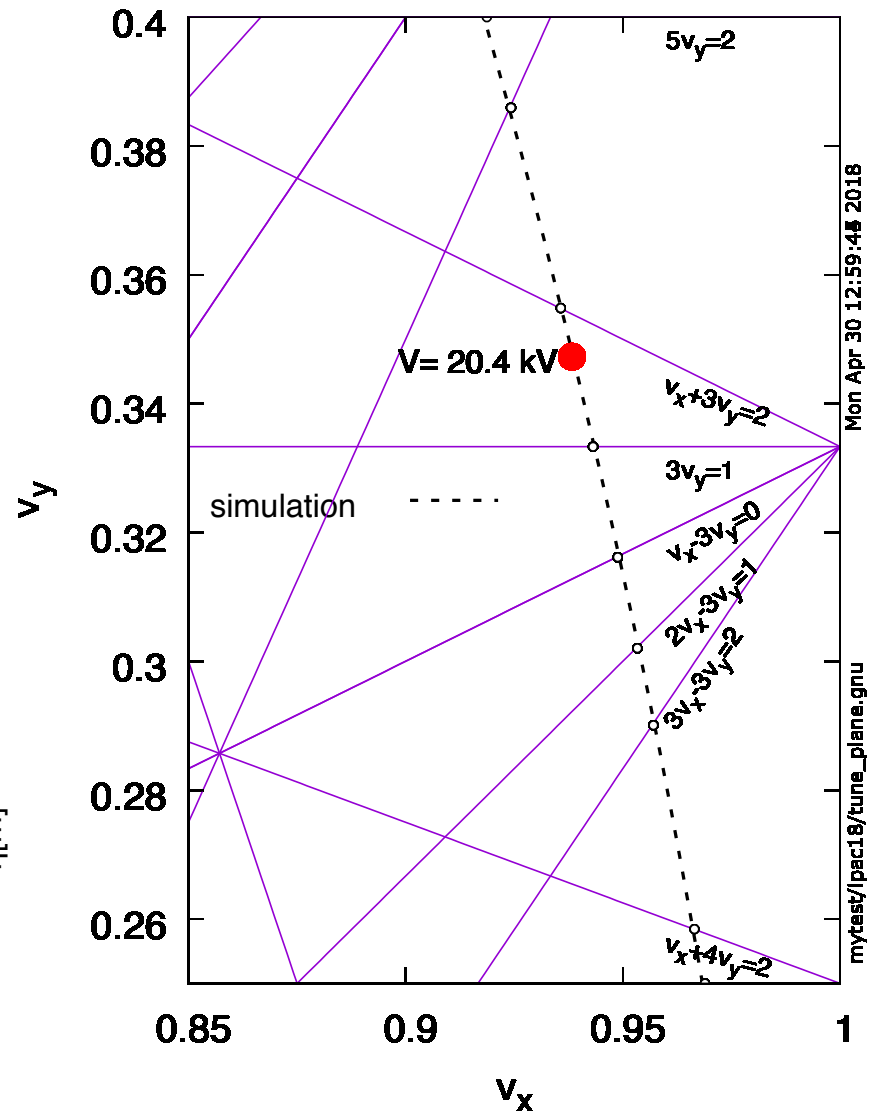
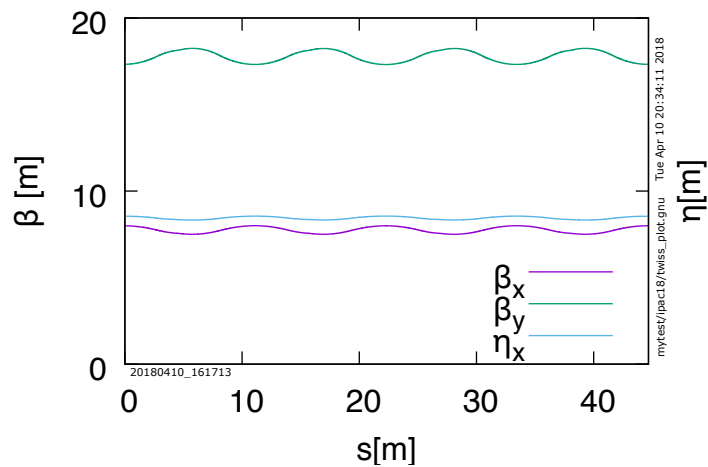
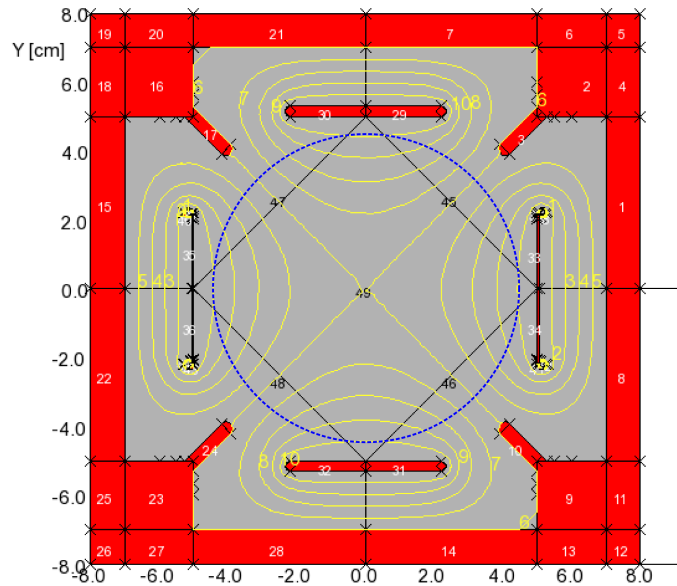
10.8 mrad

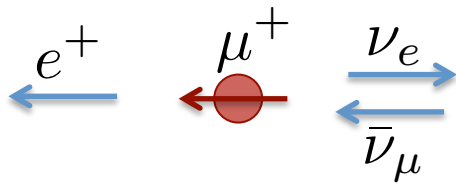
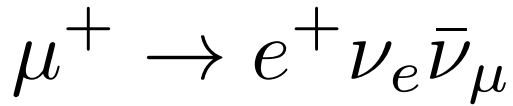
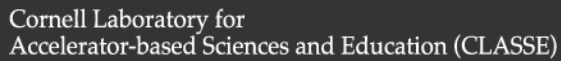
Quads span  $156^\circ$  of arc





# Weak focusing lattice



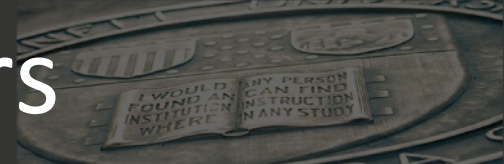


=> Highest energy positron

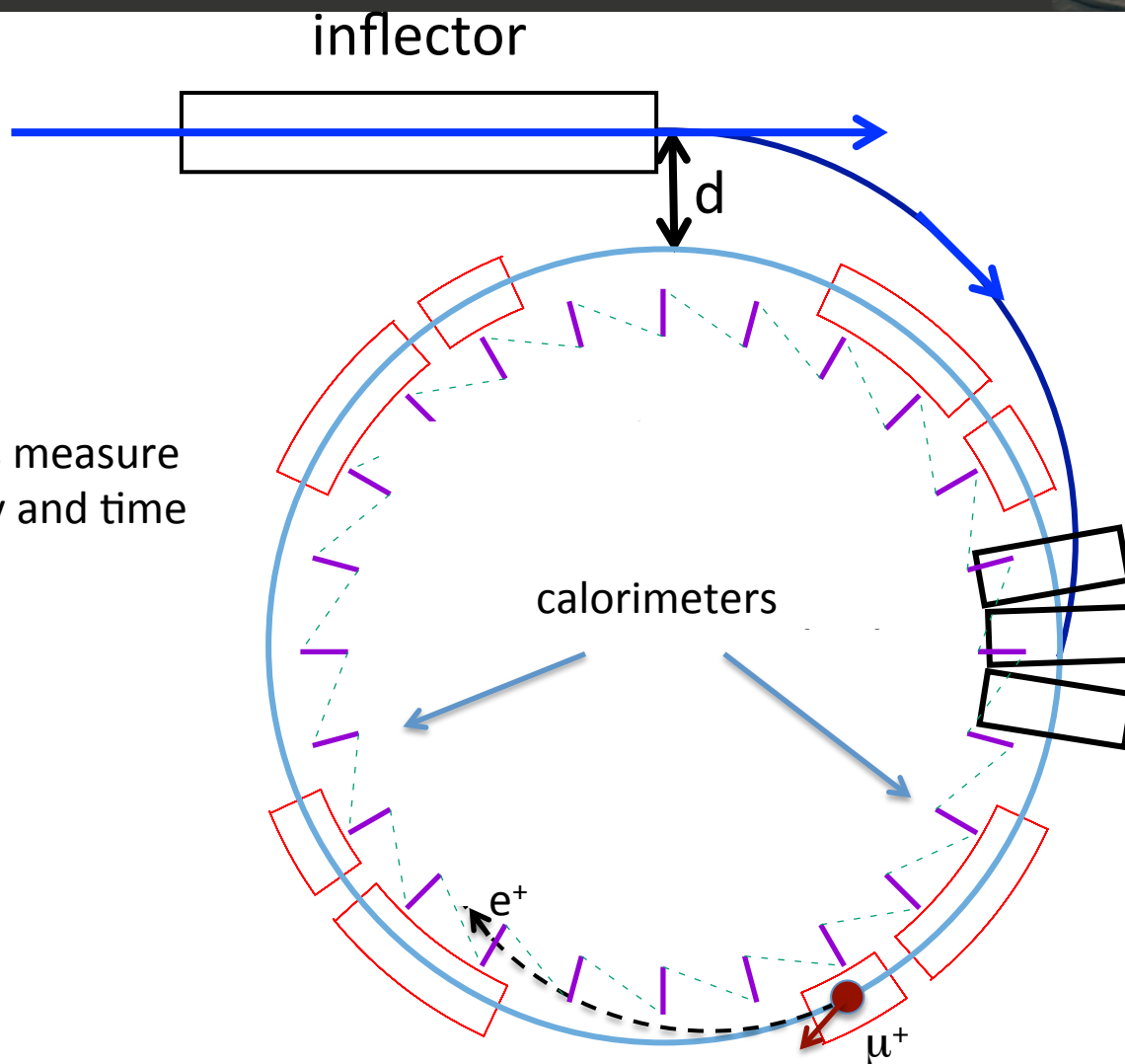
Correlation of positron energy with muon spin is measure of muon polarization



# Calorimeters



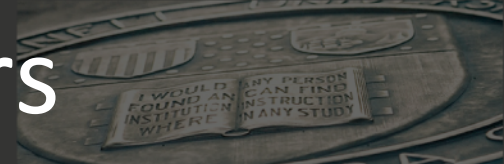
24 calorimeters measure  
positron energy and time



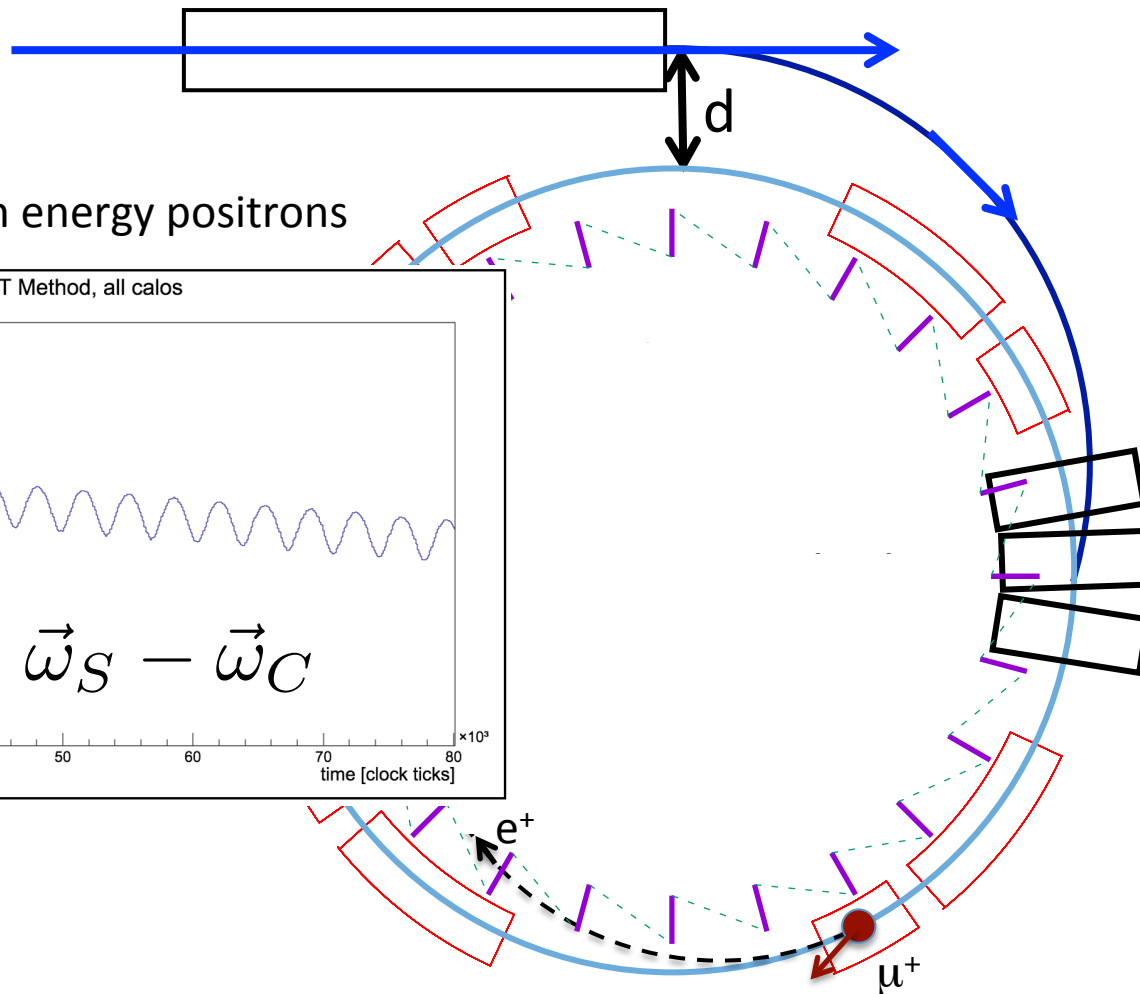




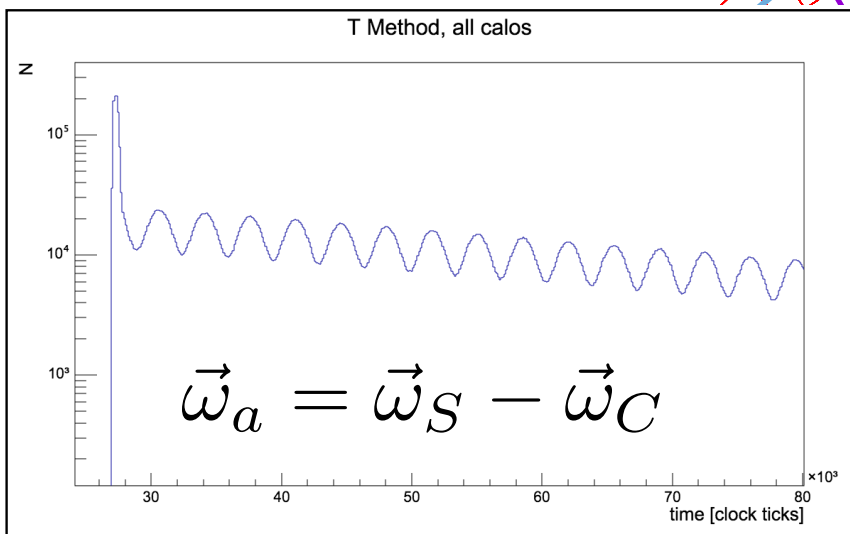
# Calorimeters



inflexor

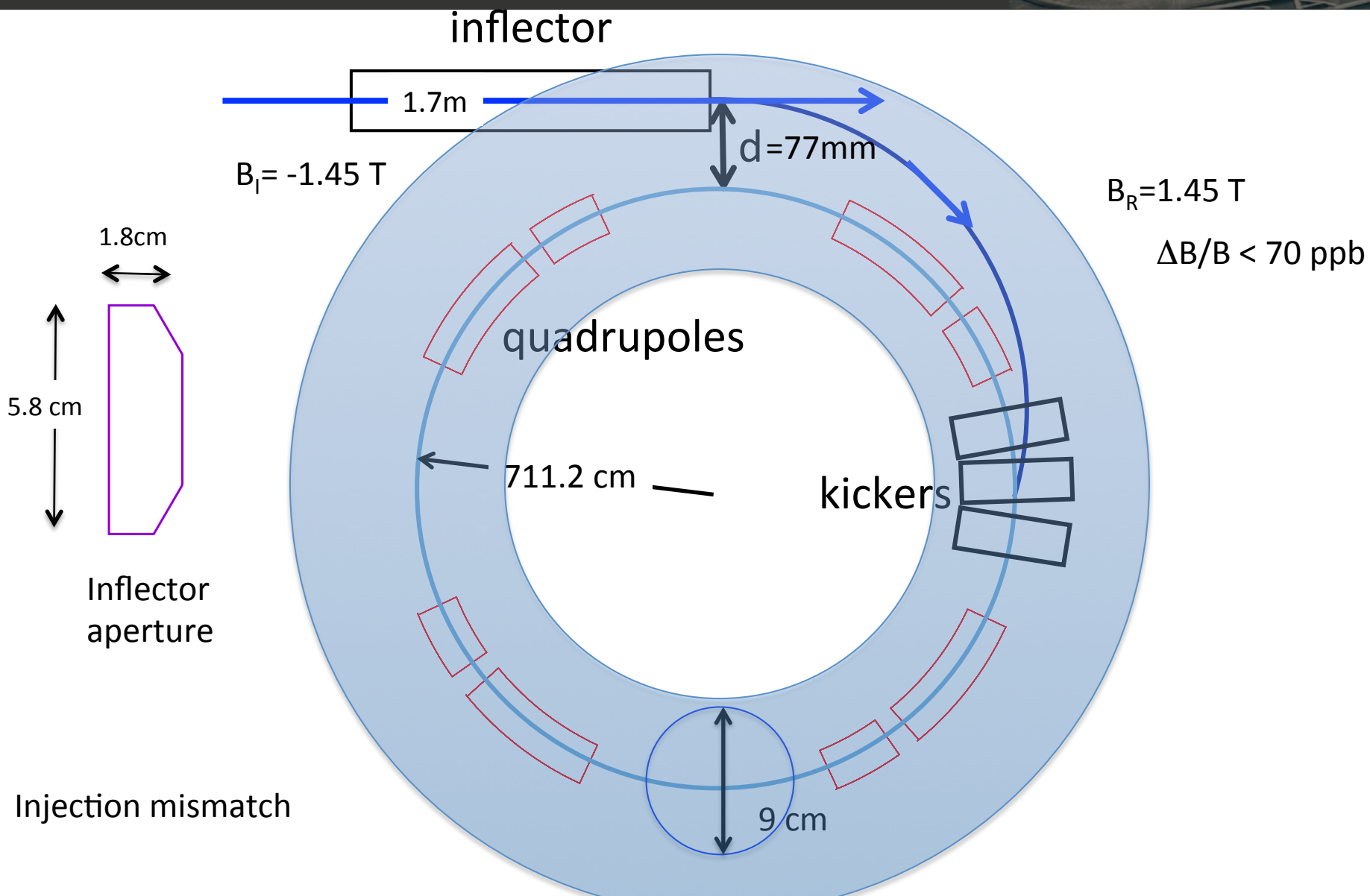
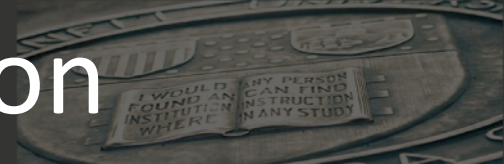


Histogram of high energy positrons



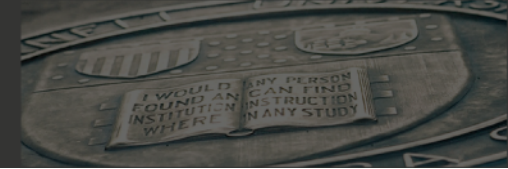


# Injection





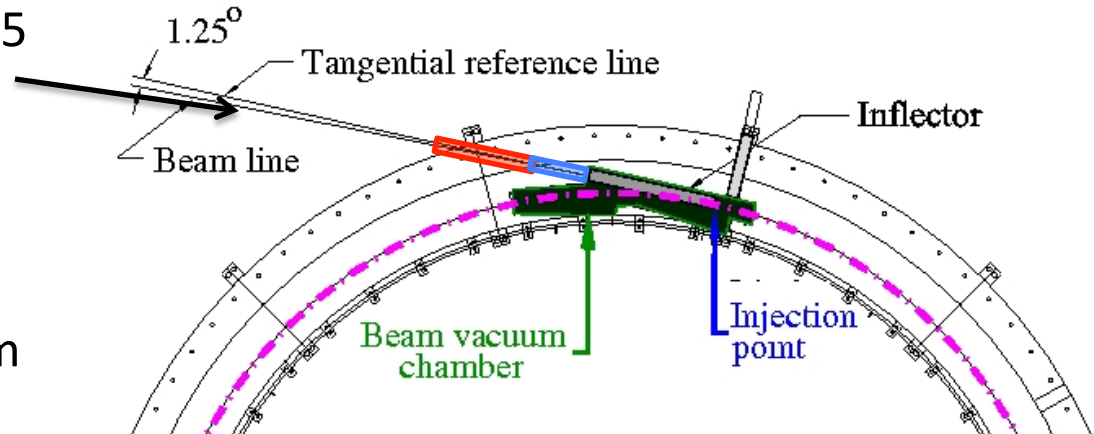
# Injection



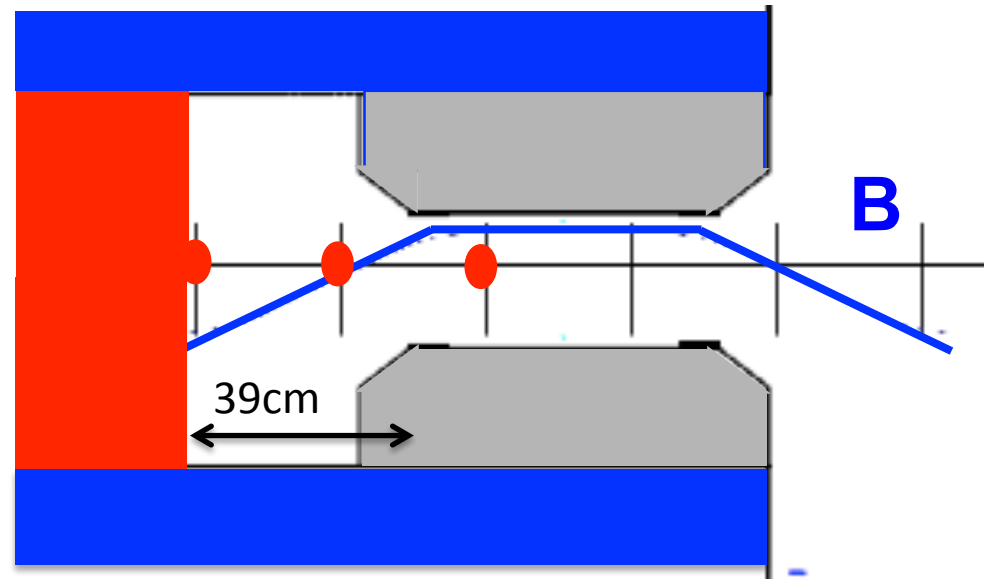
Muons come to the end of the M5 line and then propagate through:

- Hole in magnet yoke
- Dipole fringe field
- Inflector

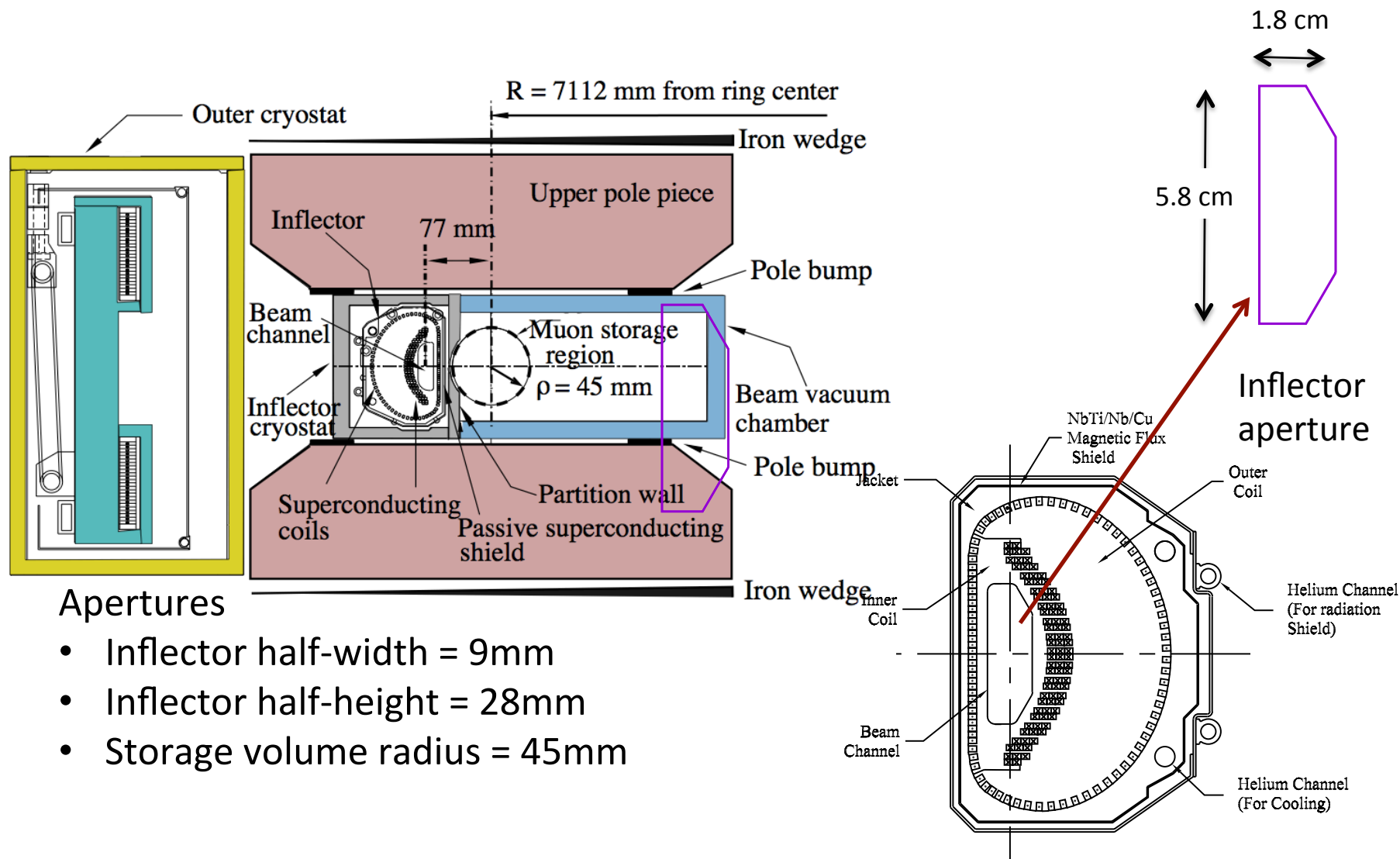
And exit the inflector 77 mm from the center of the dipole aperture



The magnetic field is near zero at the inner surface of the yoke, and rises to 1.45T between the magnet poles, over a distance of ~39cm



# Inflector



## Apertures

- Inflector half-width = 9mm
- Inflector half-height = 28mm
- Storage volume radius = 45mm

Fig. 7. Inflector cross-section.



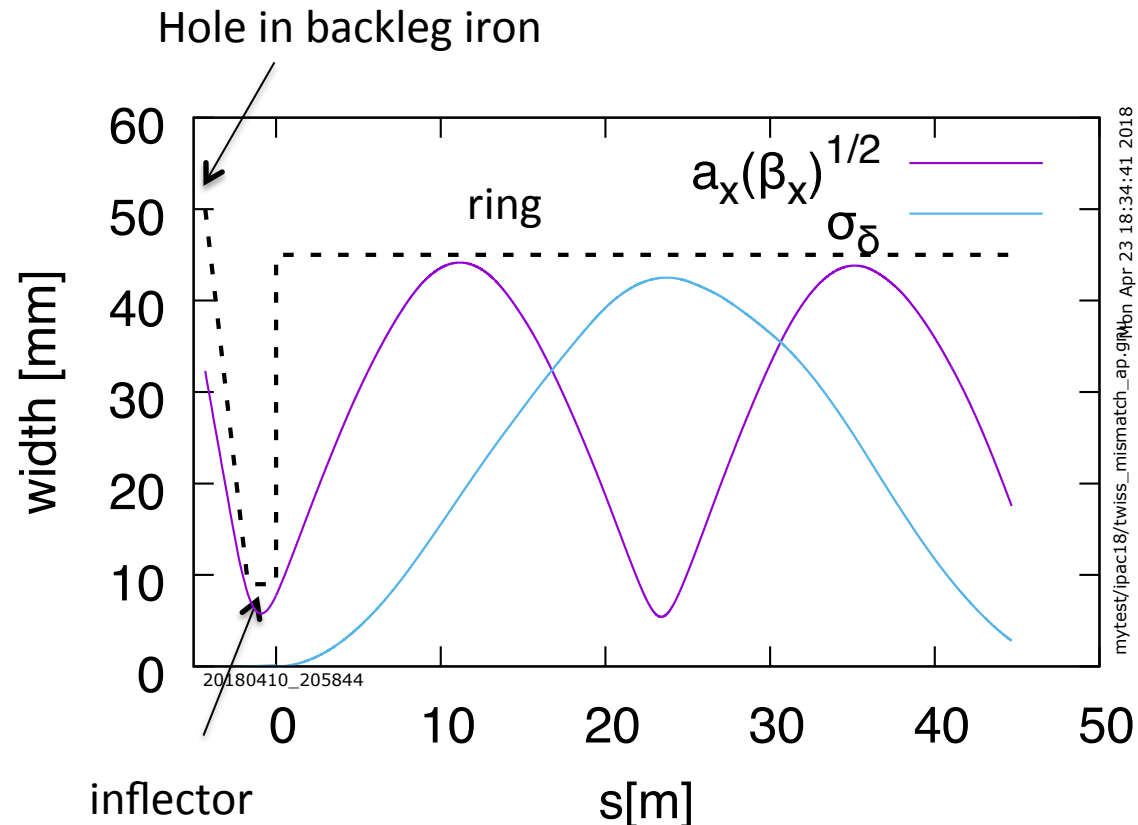
# Injection phase space

## Injected beam

$$\varepsilon = 40 \text{ mm-mrad (95\%)}$$

$$\Delta p/p = 2\%$$

- Beam is focused to a waist to clear inflector aperture
- (Muons are scattered in coil overlapping inflector aperture)
- Dispersion is zero in inflector



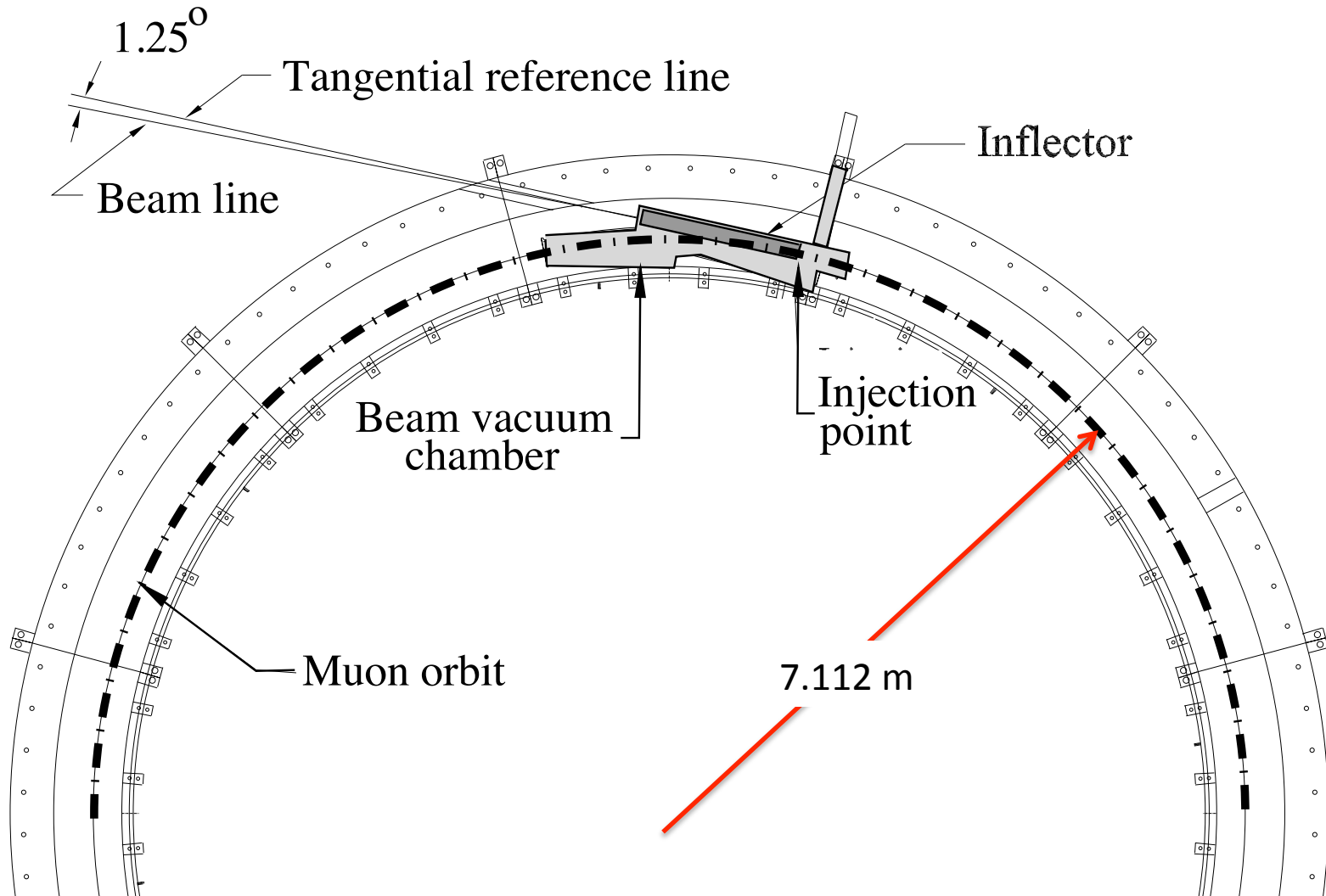
=> Dispersion and phase space mismatch

Ring  
 $\beta_x = 7.8 \text{ m}$   
 $\eta = 8.2 \text{ m}$





# Injection channel



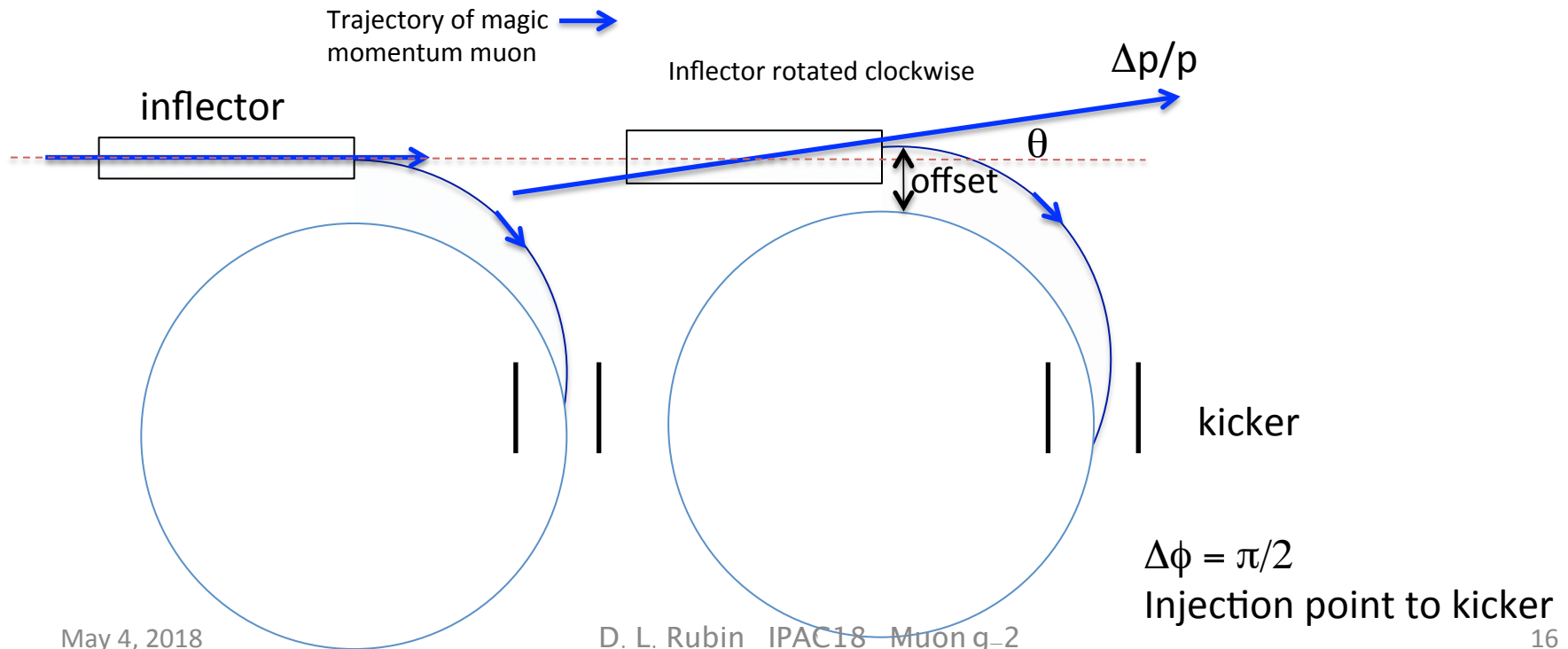


# Injection Dependencies

Injected particles oscillate about the closed orbit with amplitude that depends on; momentum, injection angle, offset, and kick angle

Amplitude and envelope of betatron oscillation of stored beam depends on

- injection angle  $\theta$  and
- Kicker angle
- Momentum

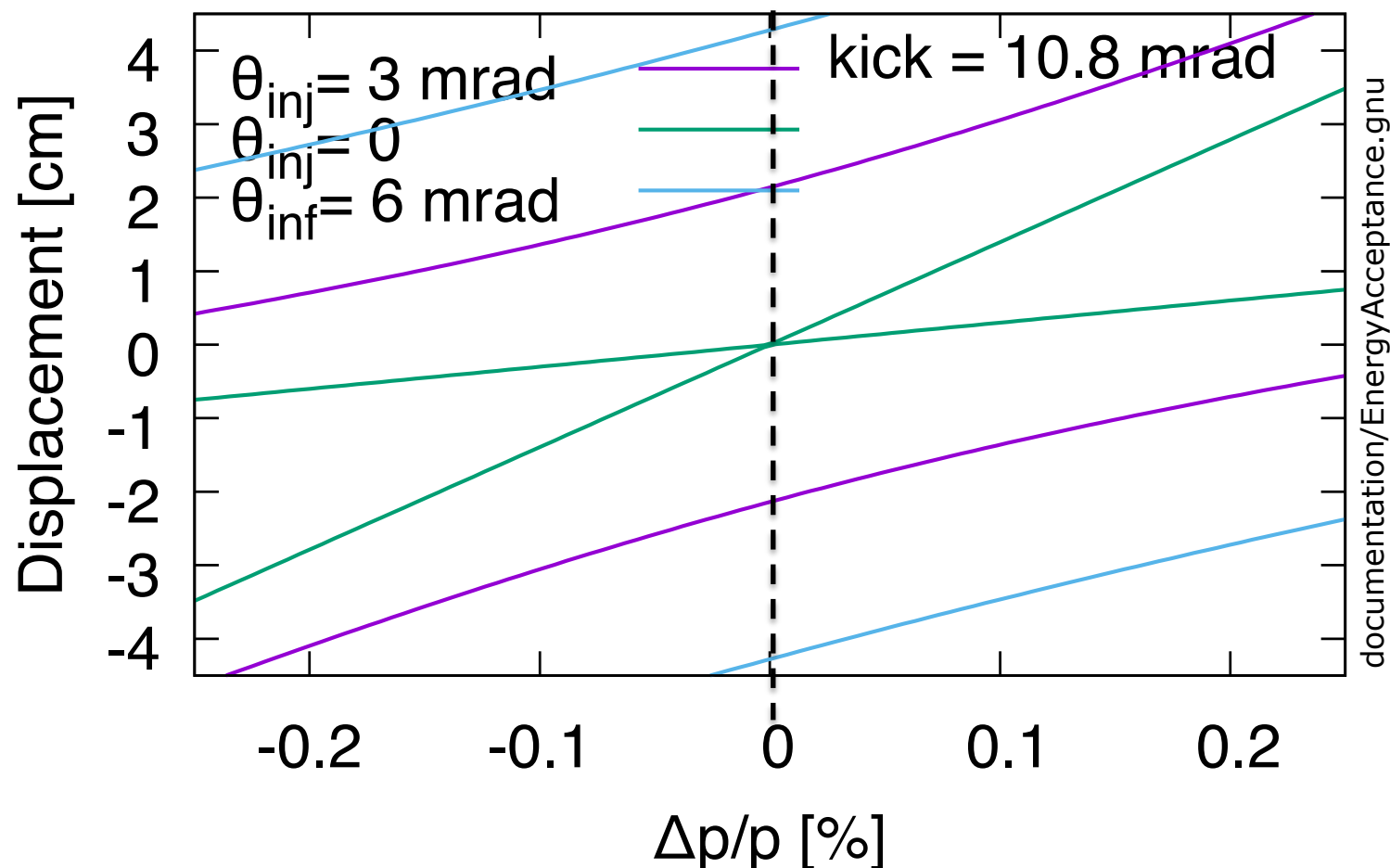




# Muon capture



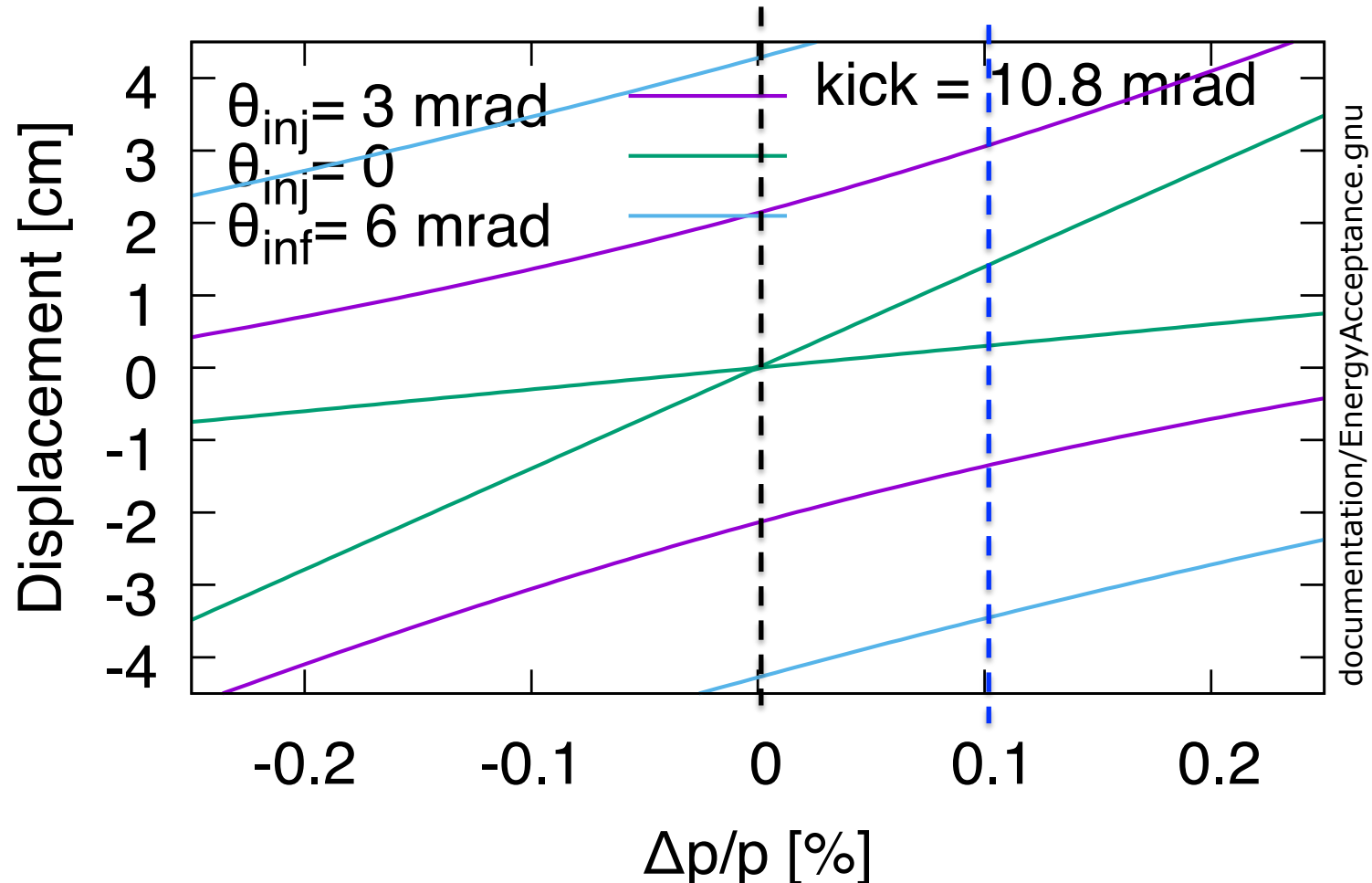
Envelope of betatron motion of stored muon  
(10.8 mrad kick angle puts on momentum muon on axis)





# Muon capture

Envelope of betatron motion of stored muon  
(10.8 mrad kick angle puts on momentum muon on axis)

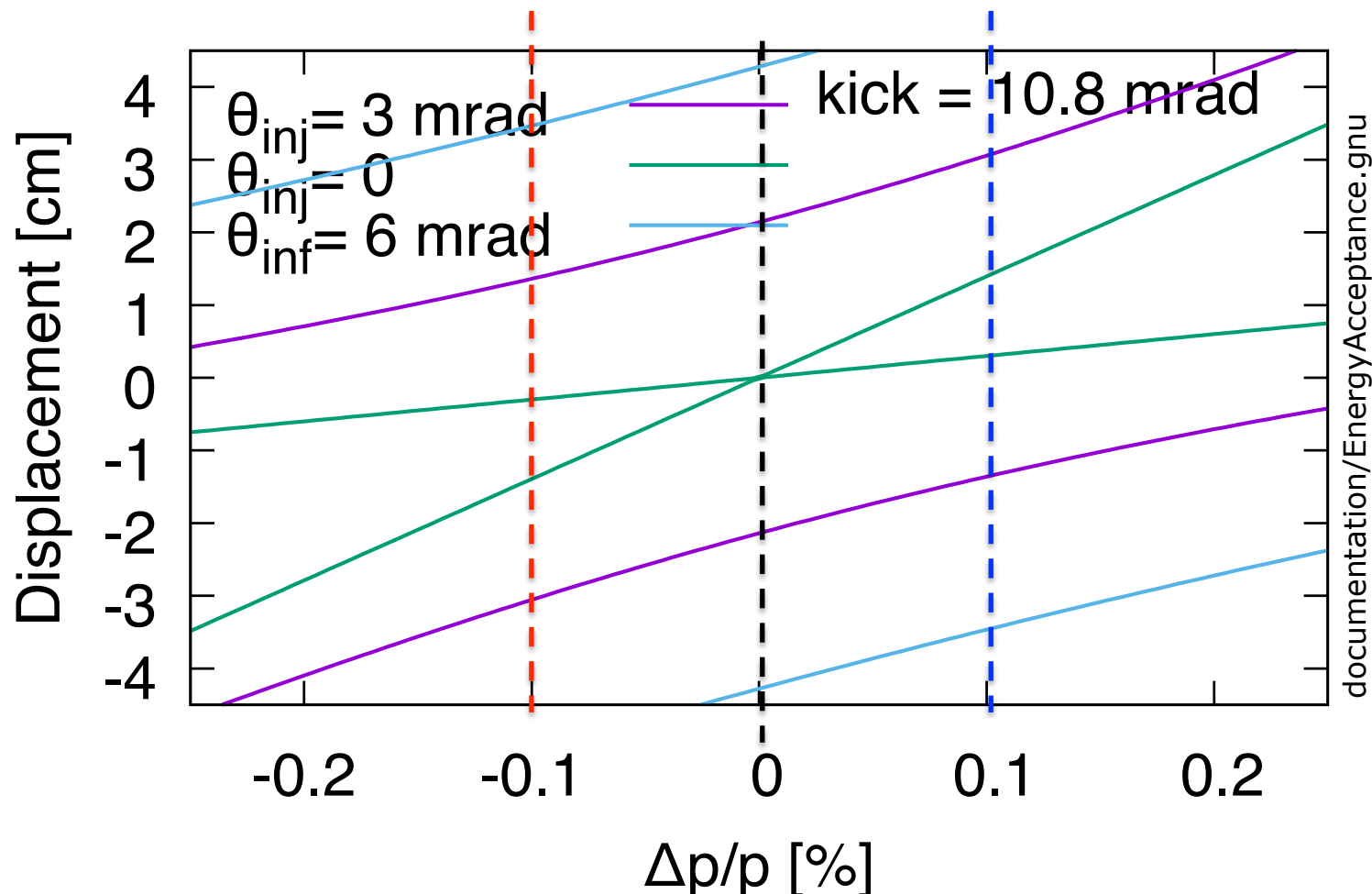




# Muon capture



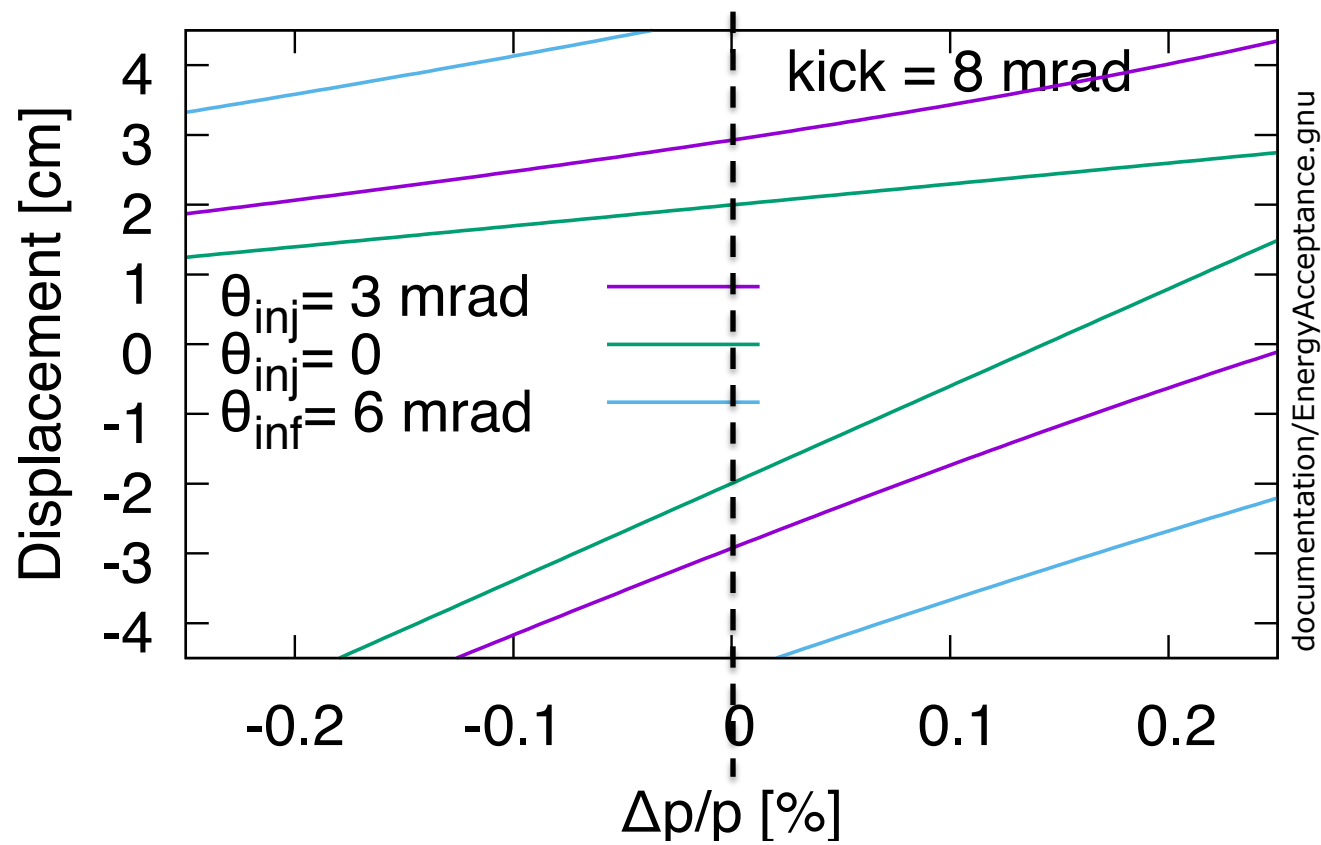
## Envelope of betatron motion of stored muon







## Envelope of betatron motion of stored muon (8 mrad kick = 75% nominal – under kicking)

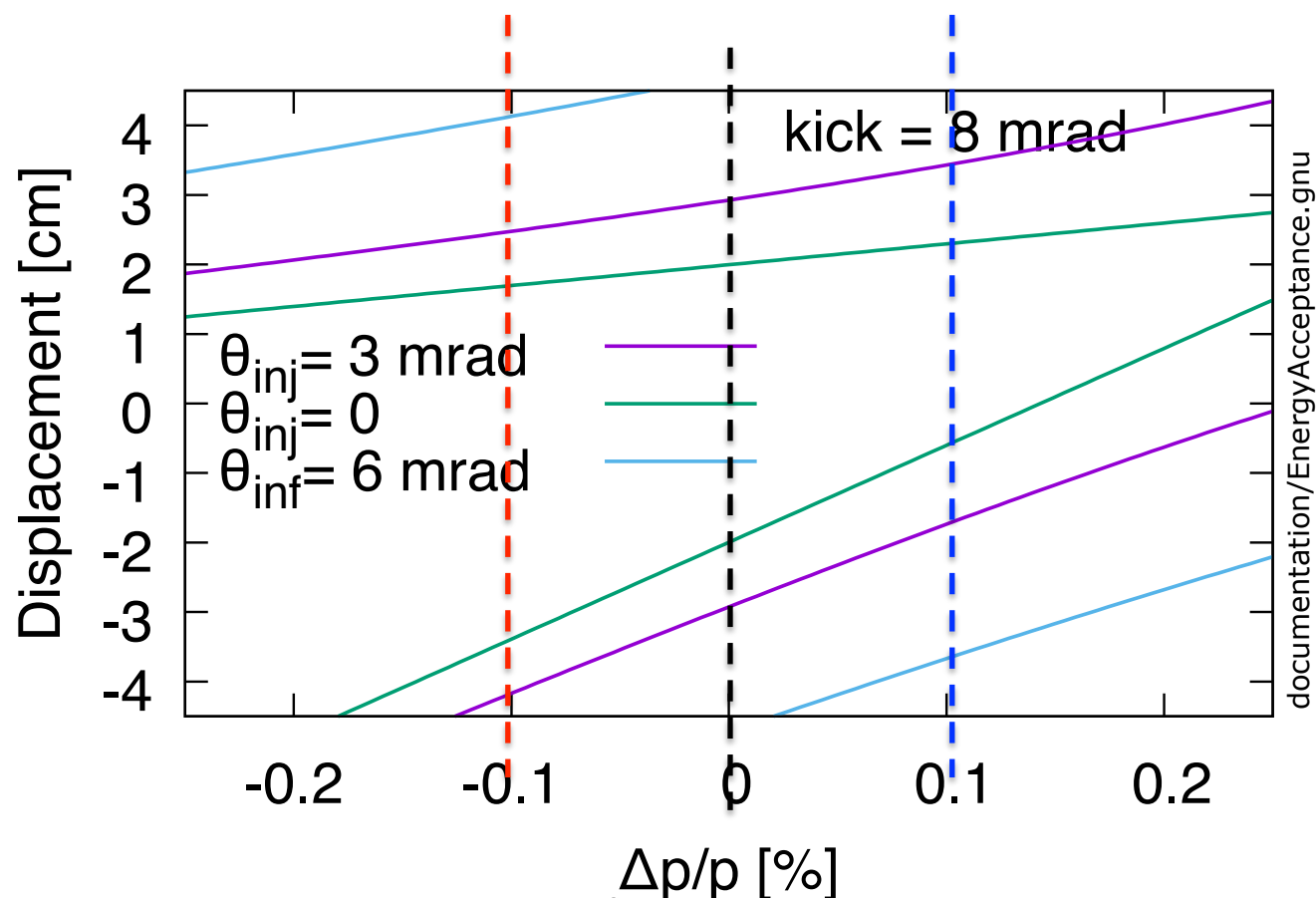




# Muon capture

In the event of *under kicking*

- Acceptance is skewed to higher momenta





## Systematics

$$\vec{\omega}_a = -\frac{q}{m} \left[ a_\mu \vec{B} - a_\mu \left( \frac{\gamma}{\gamma + 1} \right) (\vec{\beta} \cdot \vec{B}) - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

In general

- Finite momentum distribution
- Transverse motion
- Non zero E-field



# Pitch correction

$$\vec{\omega}_a = -\frac{q}{m} \left[ a_\mu \vec{B} - a_\mu \left( \frac{\gamma}{\gamma + 1} \right) (\vec{\beta} \cdot \vec{B}) - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

Pitch correction:

- Contribution depends on vertical phase space distribution

$$\beta_y = p_y / (\gamma m c)$$

- Correction requires measurement of

*vertical phase space*



# E-field correction

$$\vec{\omega}_a = -\frac{q}{m} \left[ a_\mu \vec{B} - a_\mu \left( \frac{\gamma}{\gamma + 1} \right) (\vec{\beta} \cdot \vec{B}) - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

E-field correction:

- Choose momentum.

$$\text{At } \vec{p}_{\text{magic}} = 29.3 \text{ GeV}/c \rightarrow a_\mu = \frac{1}{\gamma^2 - 1}$$

Correcting for contribution from non-magic momentum muons requires

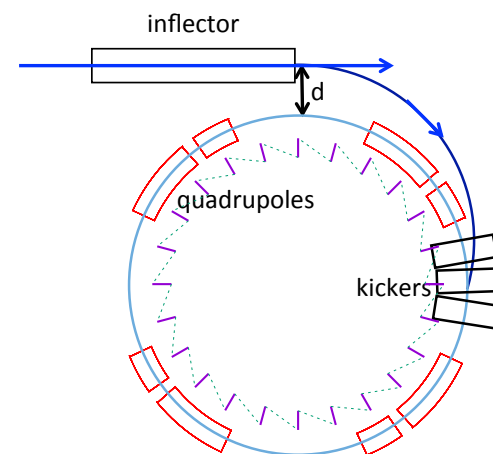
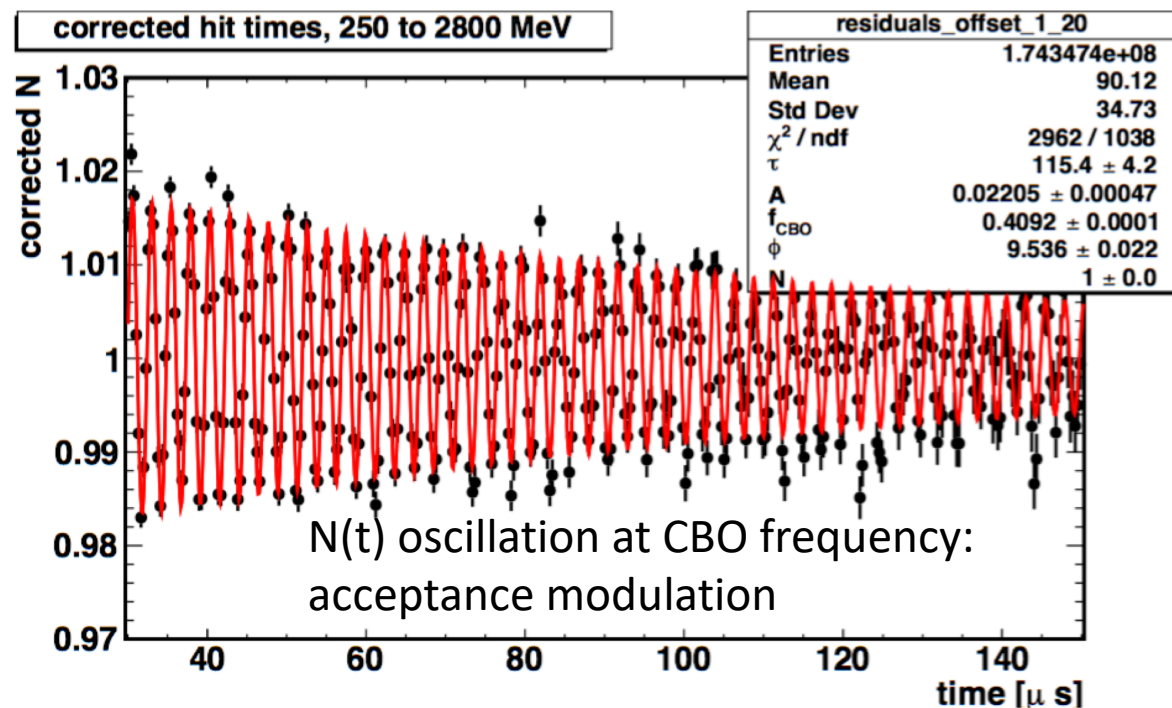
- Measurement of *momentum distribution*
- Measurement of *radial distribution*  $\vec{E} = k(x\hat{i} - y\hat{j})$



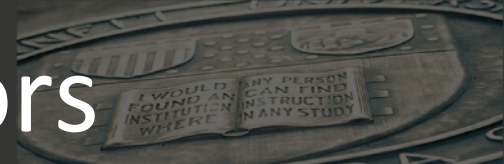
# Acceptance correction

The acceptance of the calorimeters depends on the radial offset of the decay muon

- Radial offset varies with modulation of beam width
- And with oscillation of the centroid



Correction requires measurement of  
*evolution of radial distribution*



Fiber Harps  
Traceback trackers  
*Calorimeters*

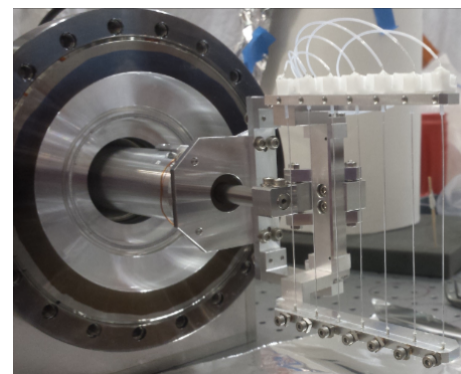
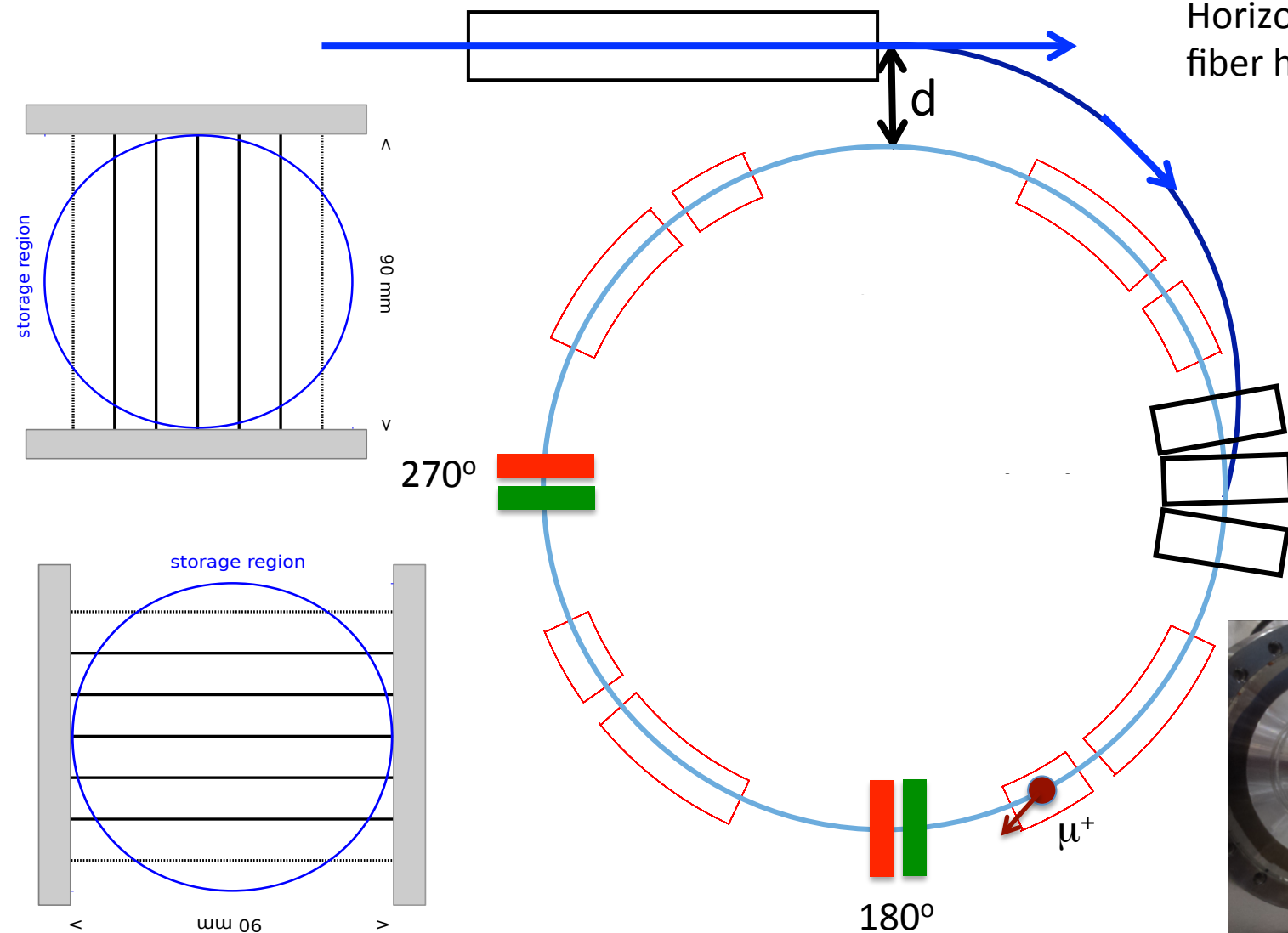


# Fiber Harps



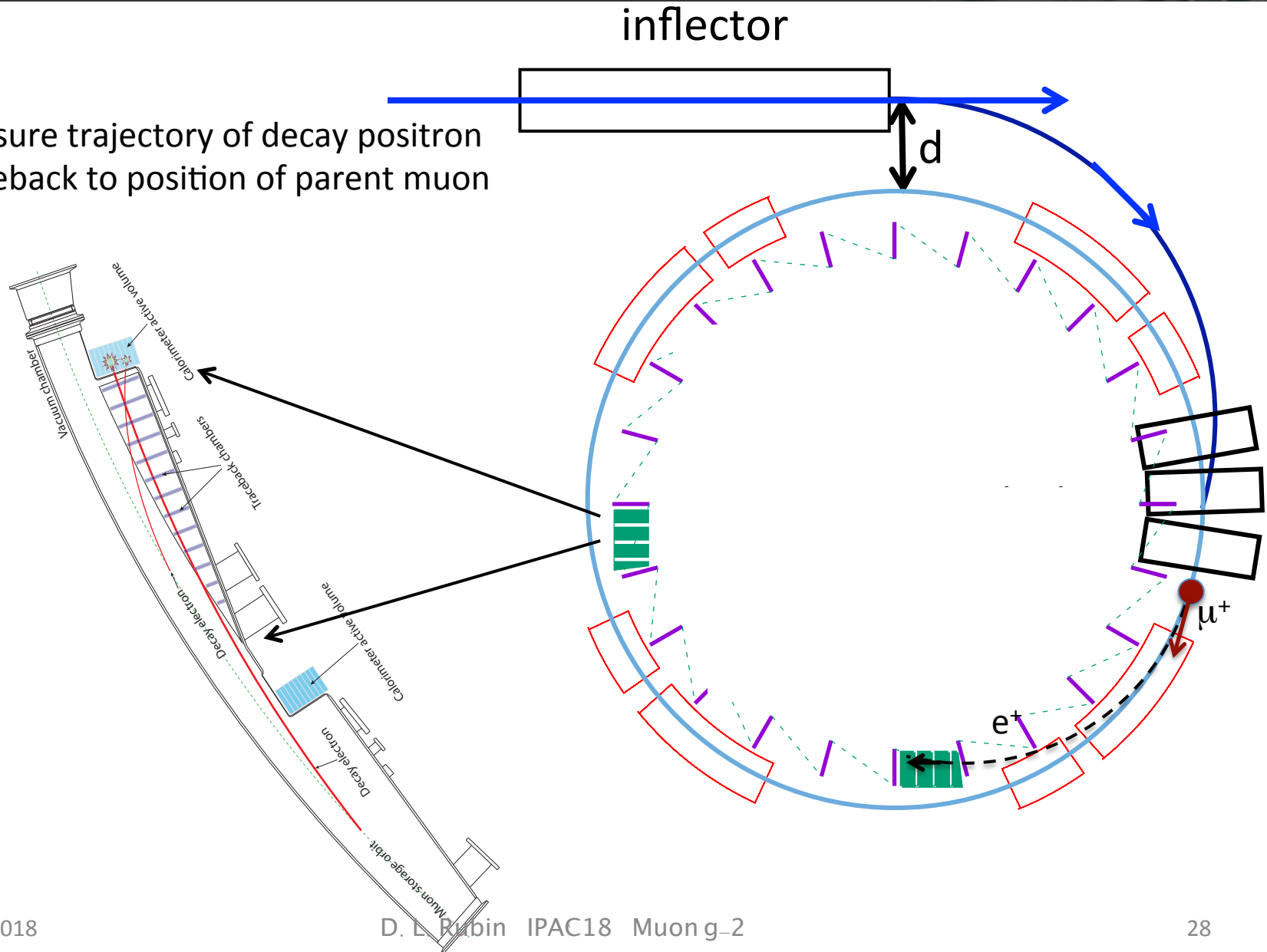
inflexor

Horizontal and vertical  
fiber harps at 180 and 270



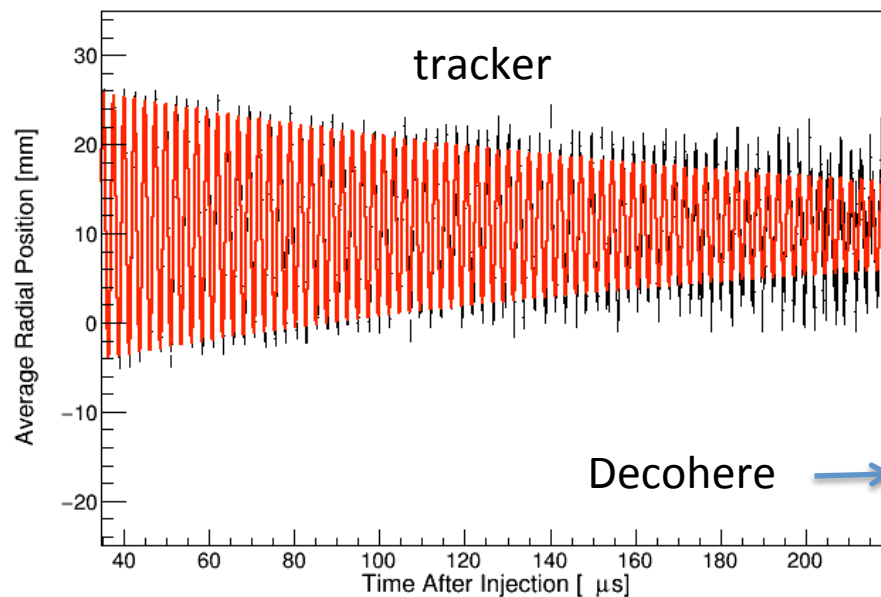
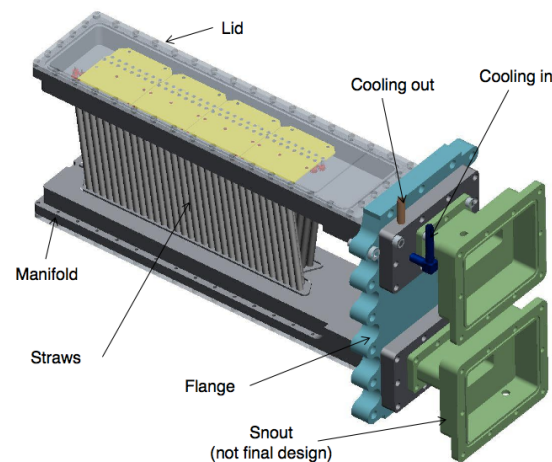
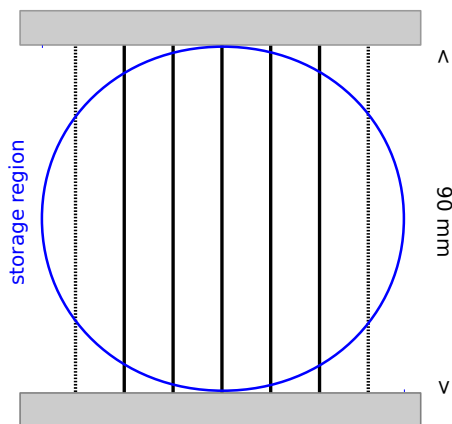
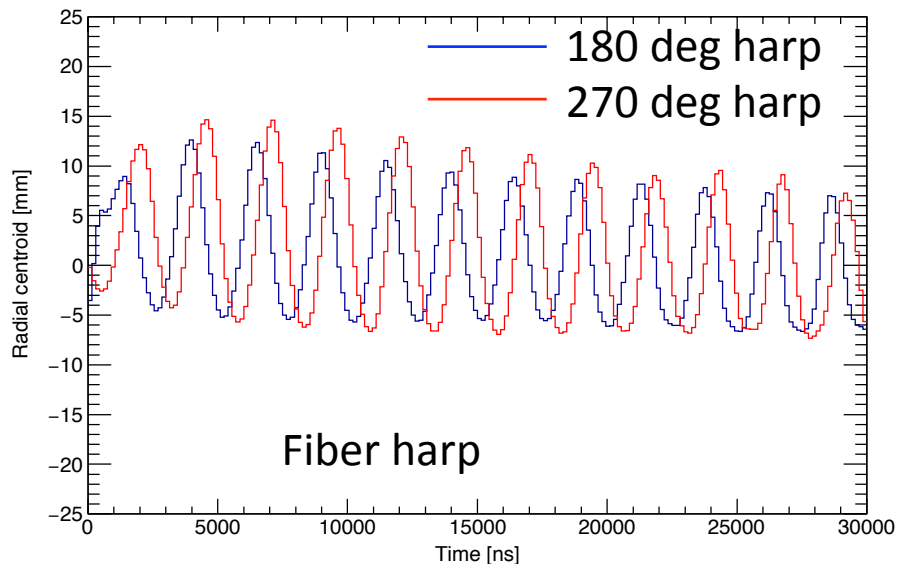
# Traceback trackers

- Measure trajectory of decay positron
- Traceback to position of parent muon



# Beam centroid

## Measurement of evolution of centroid



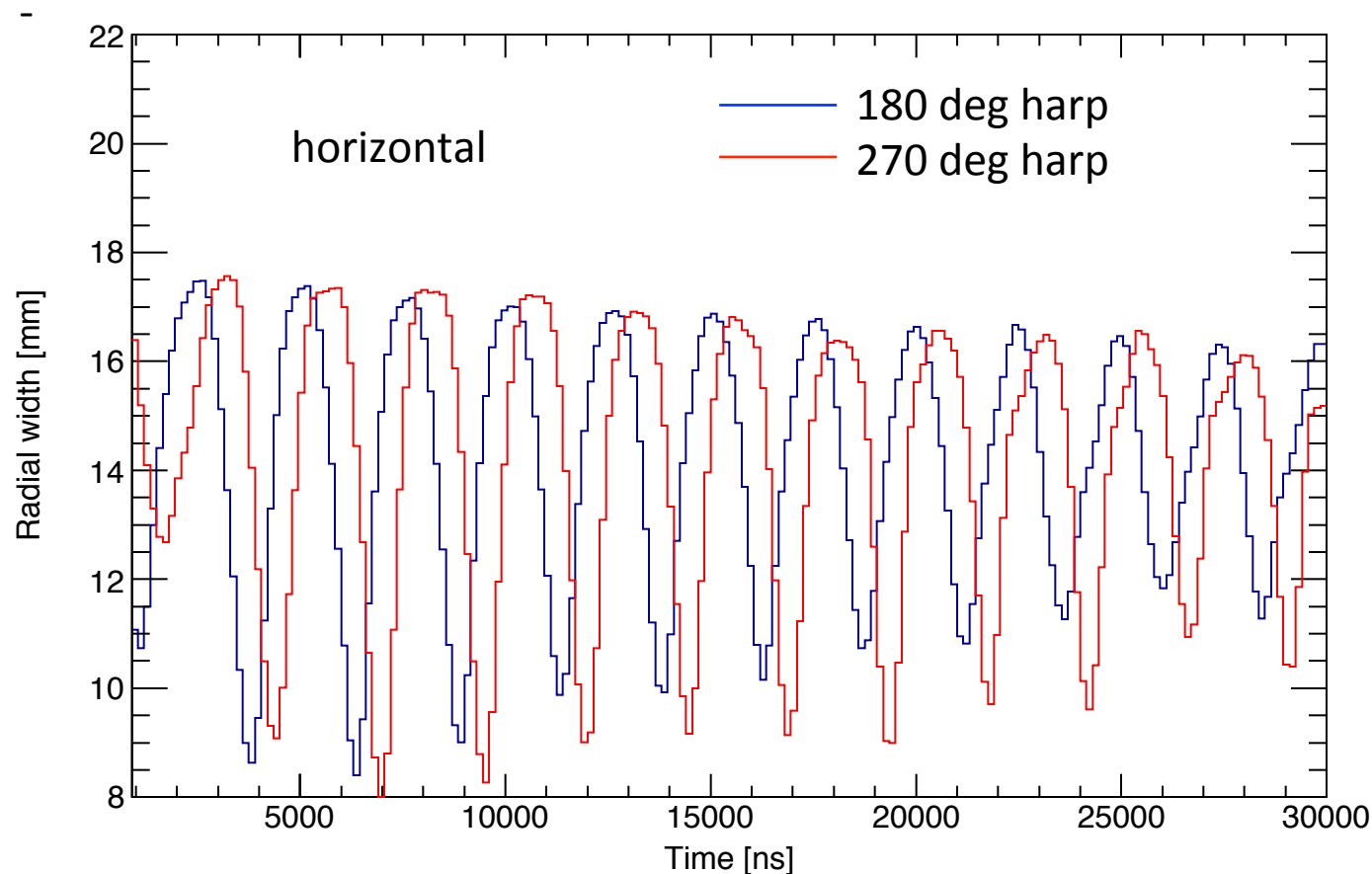


# Fiber harp



Radial width / 150 ns

- Phase space and dispersion mismatch at injection
- Width modulated at  $Q_x$  and  $2Q_x \Rightarrow$  dispersive and betatron components



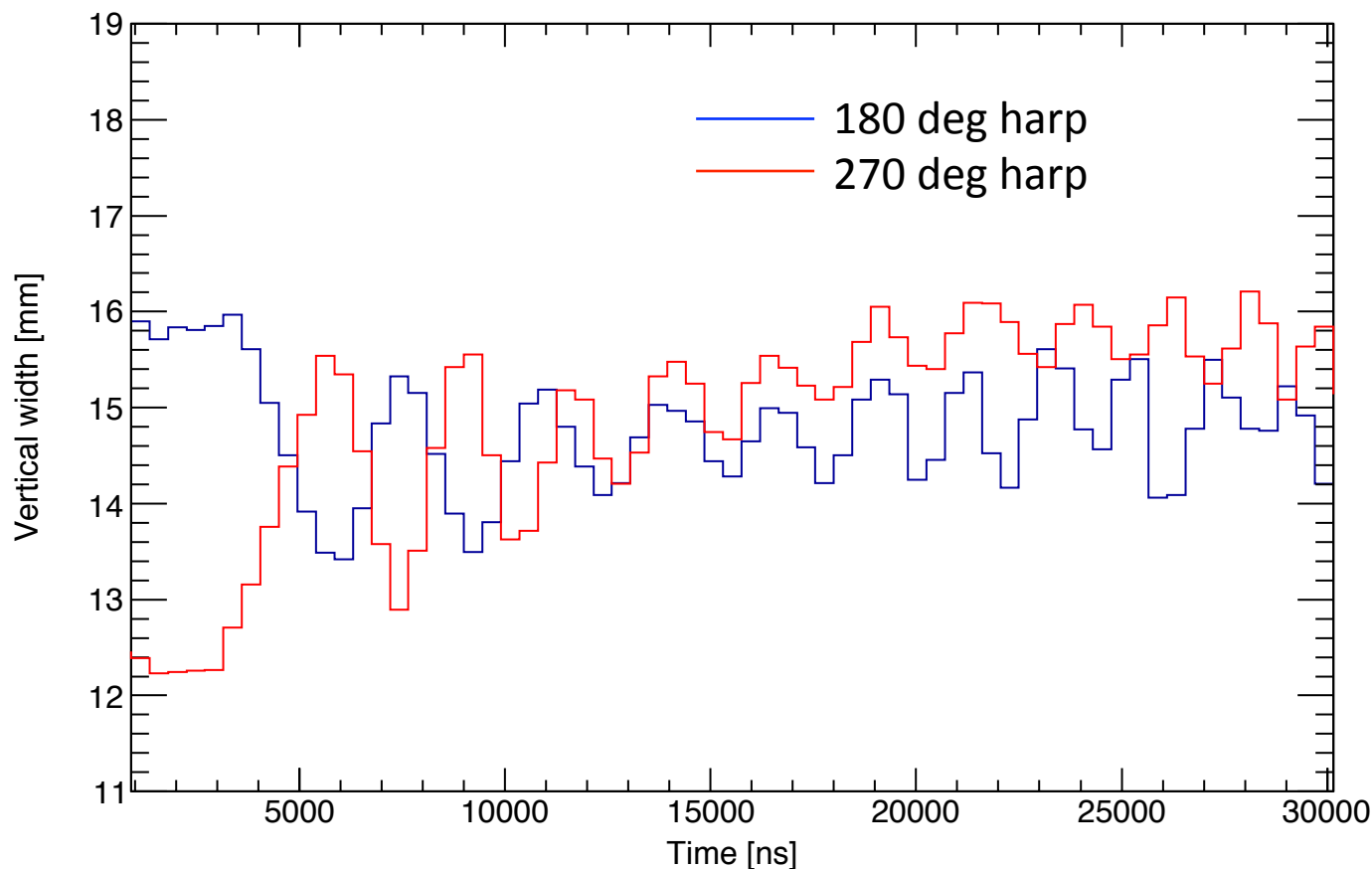


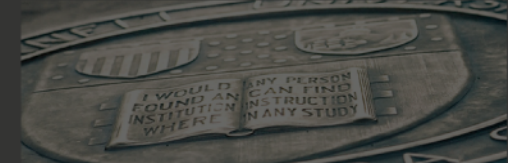
# Fiber Harp



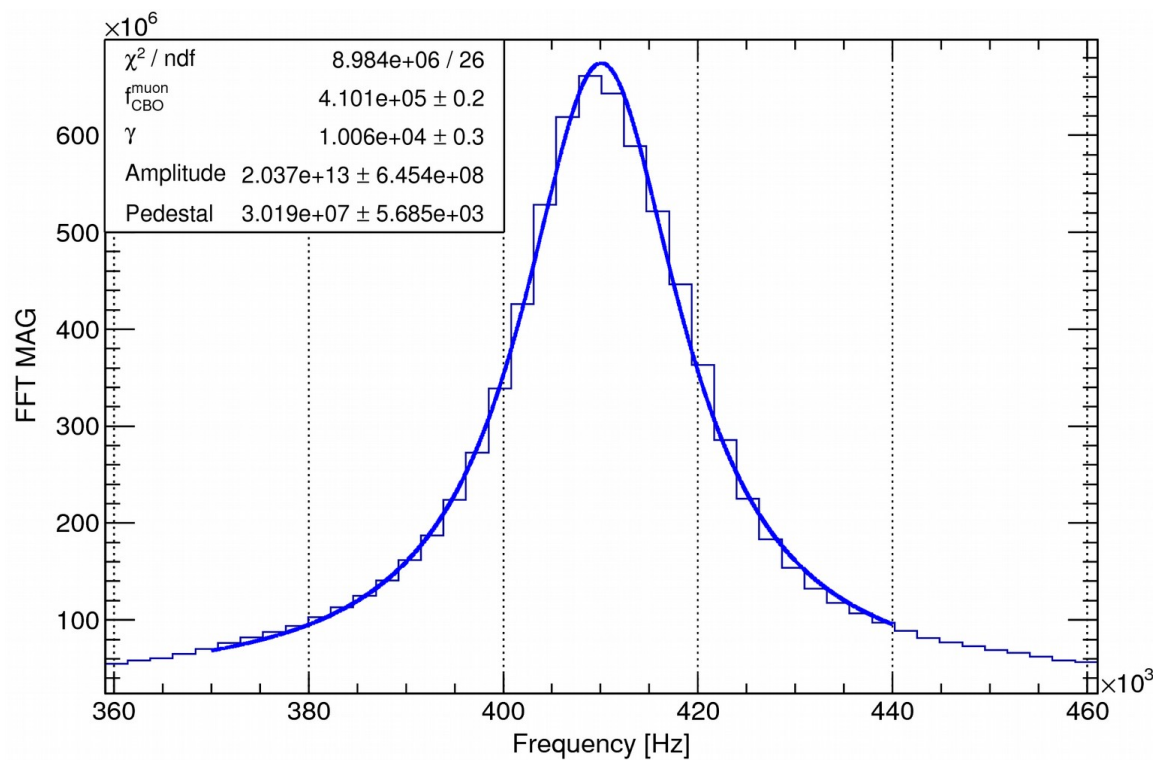
## Vertical height / 150 ns

- Modulation at  $2Q_y \Rightarrow$  phase space mismatch at injection
- Informs pitch correction





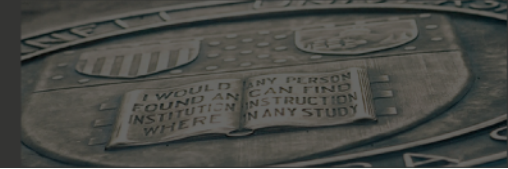
## FFT of fiber harp centroid



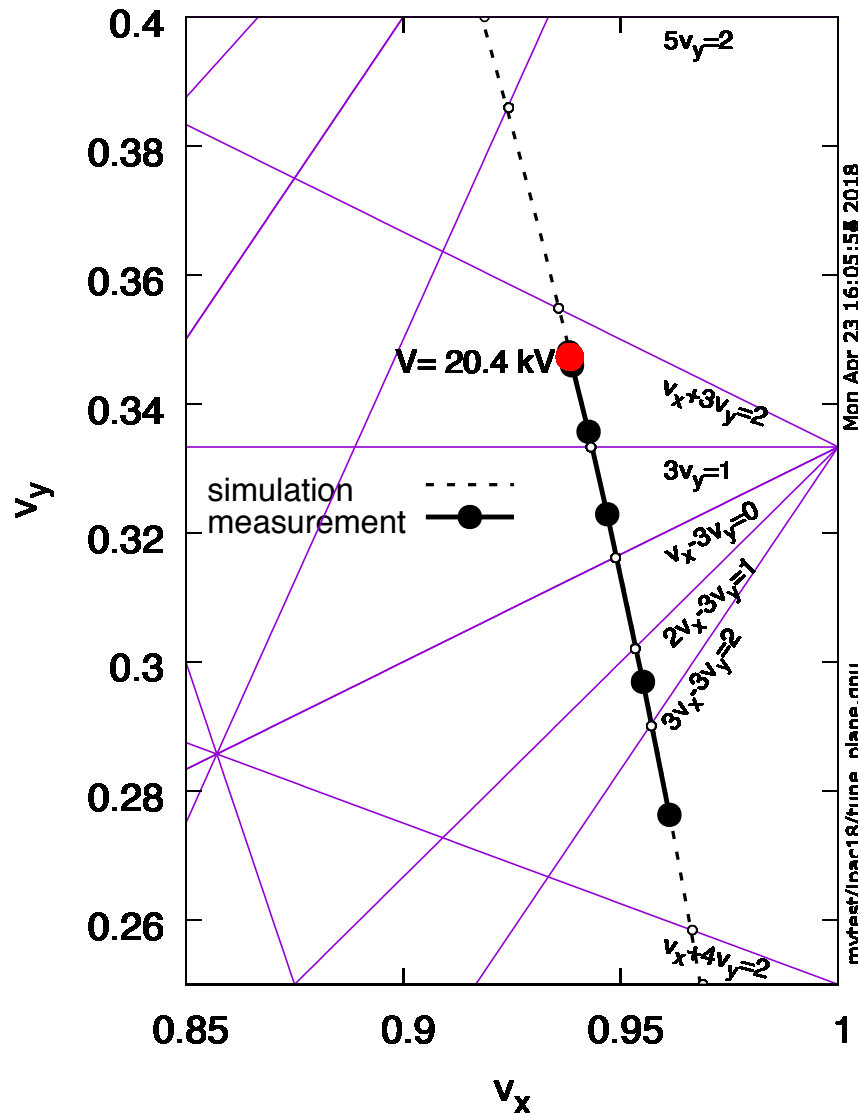




# Tune scan

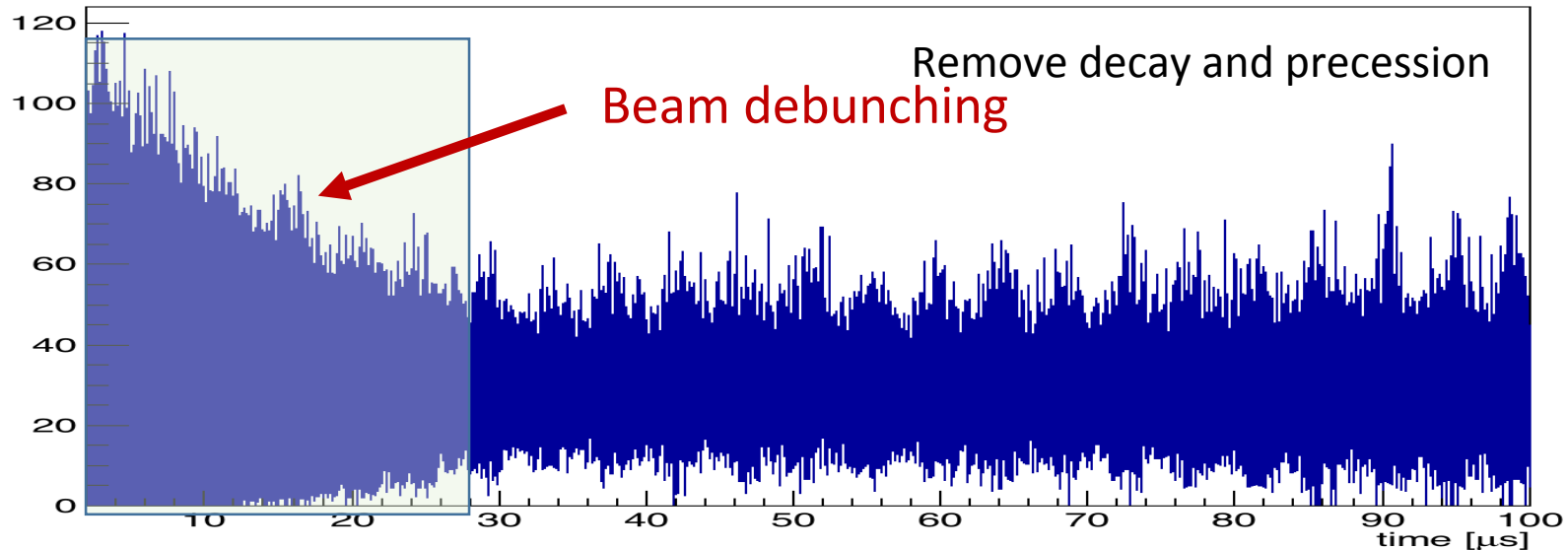
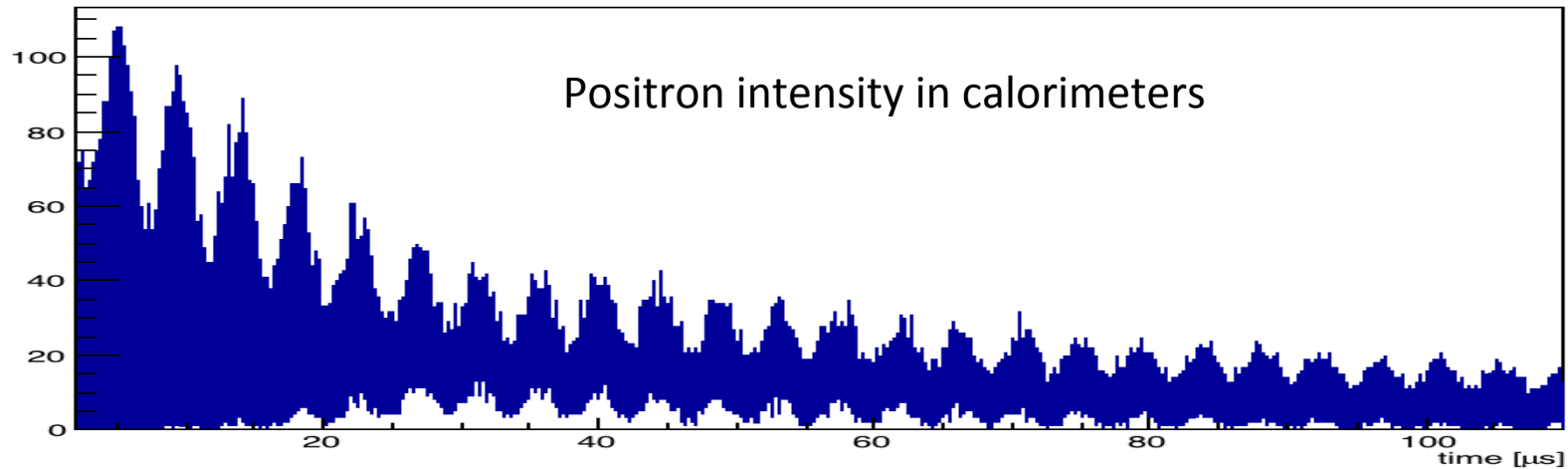


Tune along quad  
voltage contour





# Momentum distribution



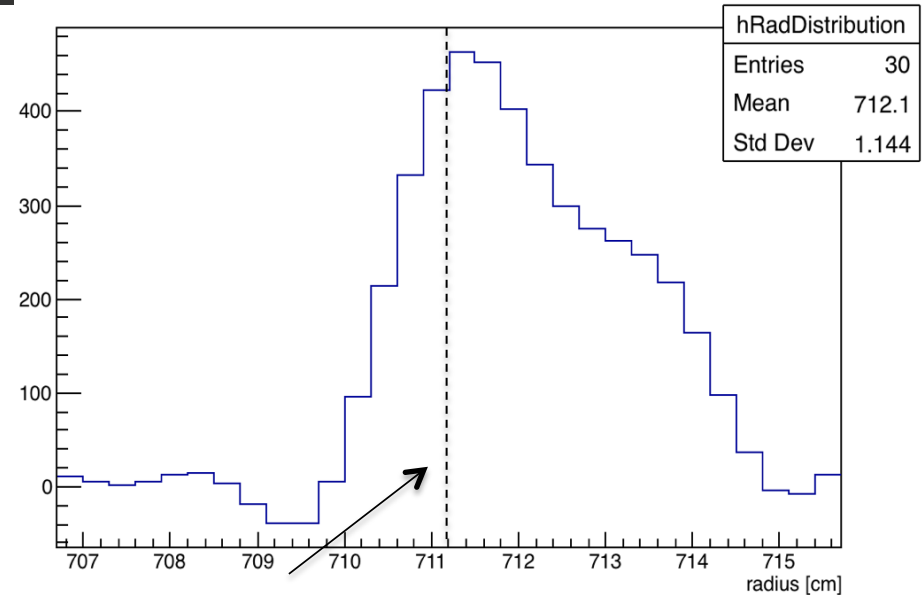


# Momentum Distribution

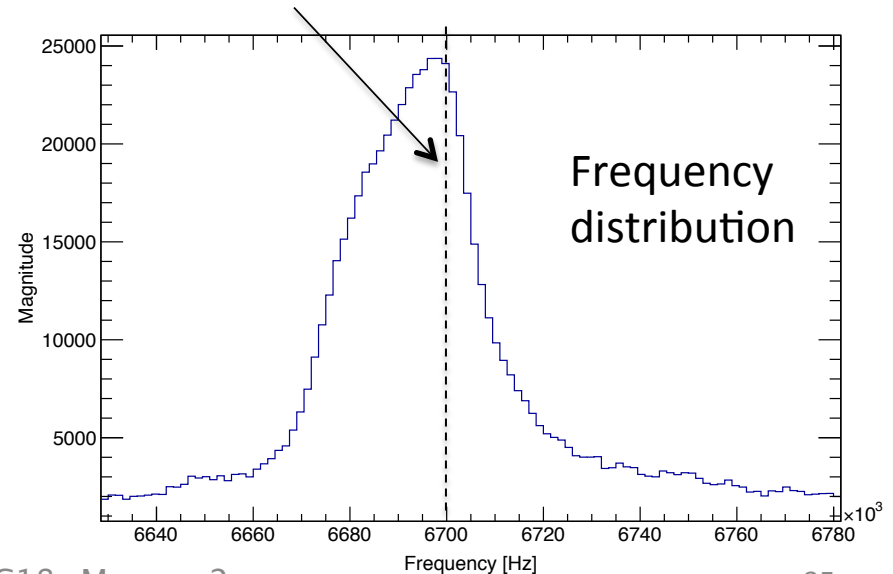
*Fit momentum distribution to  
debunching signal*

*Fourier analysis of fast rotation signal*

Distribution of revolution frequencies **is** the  
distribution of momenta. Fourier transform  
of the fast rotation signal => frequency  
distribution



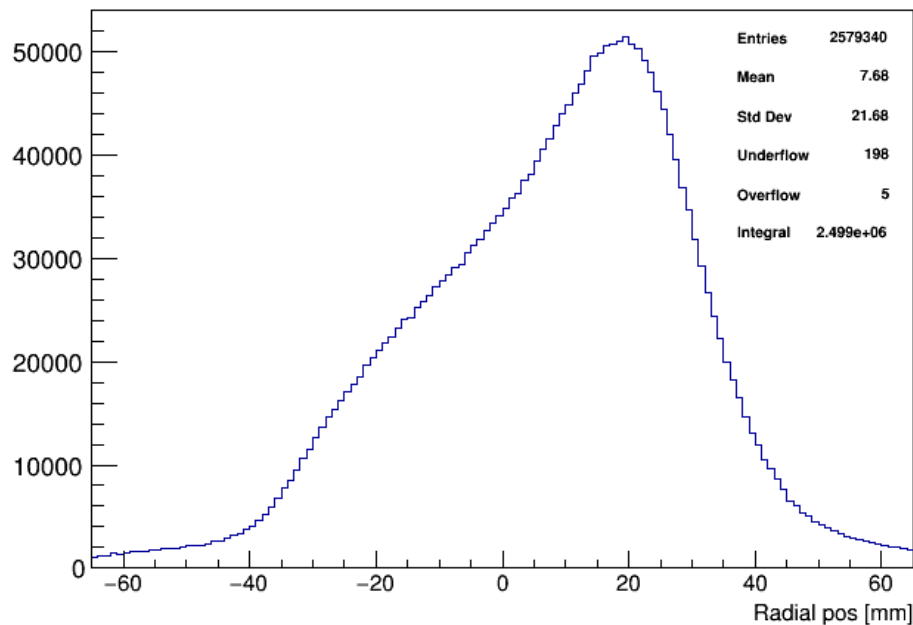
Magic momentum





# Radial distribution

Average radial distribution measured with traceback tracker  
(Radial distribution convoluted with detector acceptance)

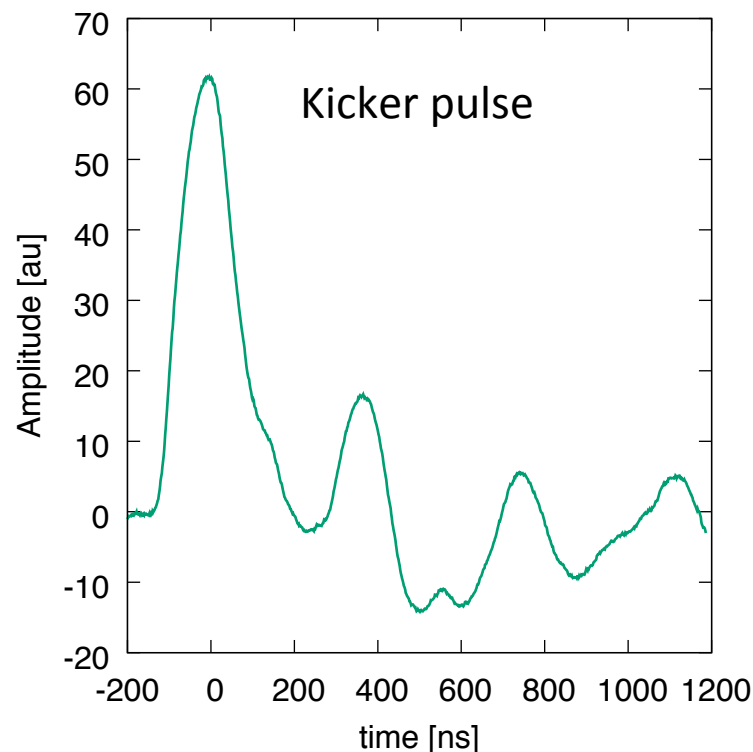
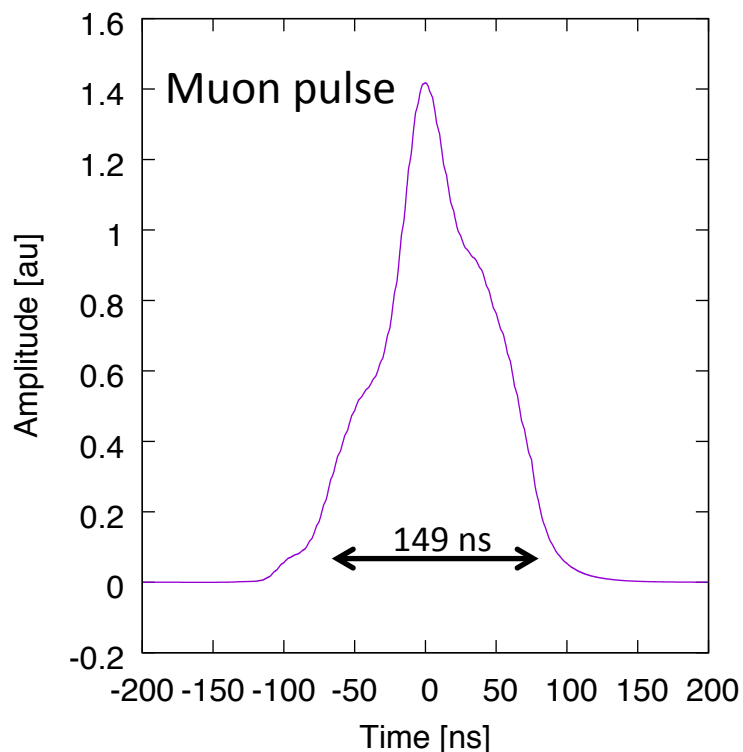




# Momentum distribution

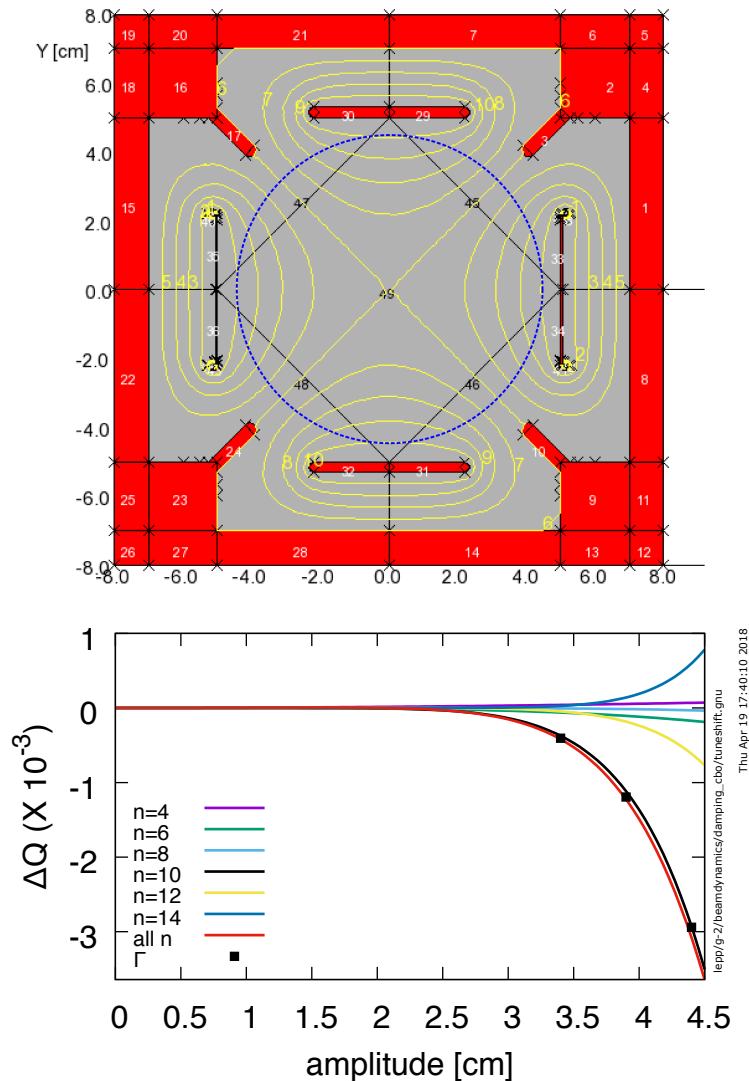
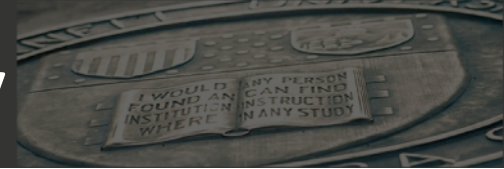
Momentum distribution is 'high'

⇒ Injection kicker is under-kicking and/or tails of the muon pulse  
are in the rising and falling tails of the kick

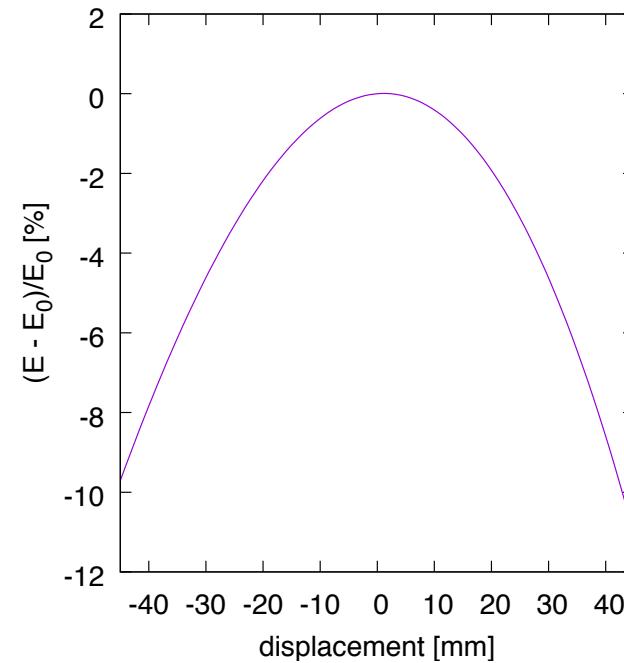




# Nonlinearity



Quad field error



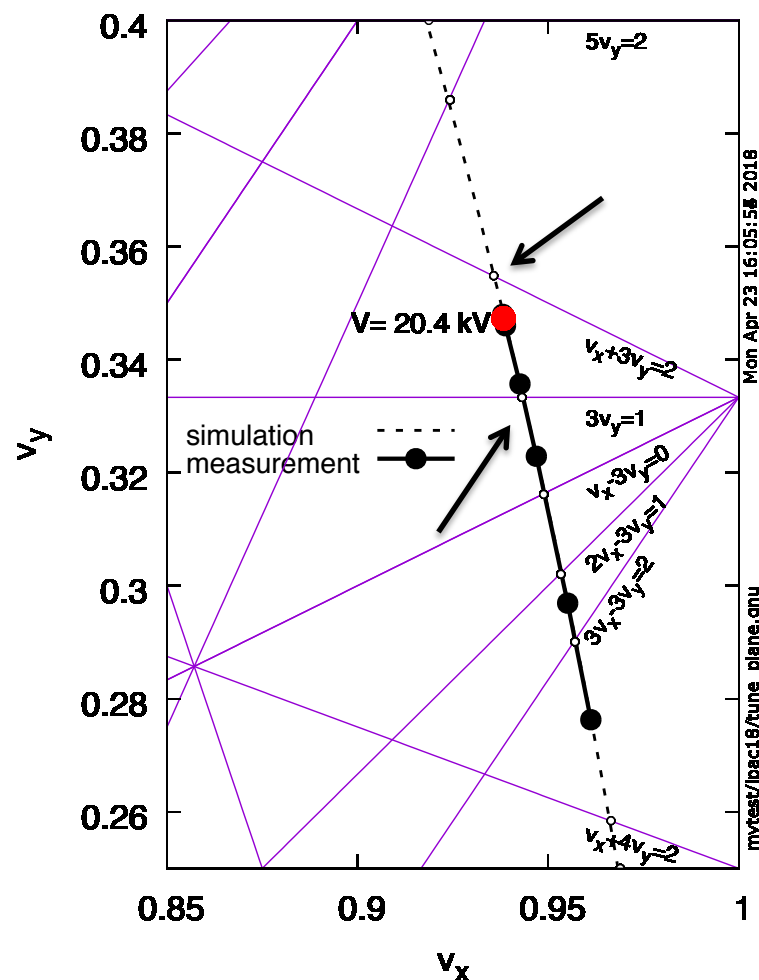
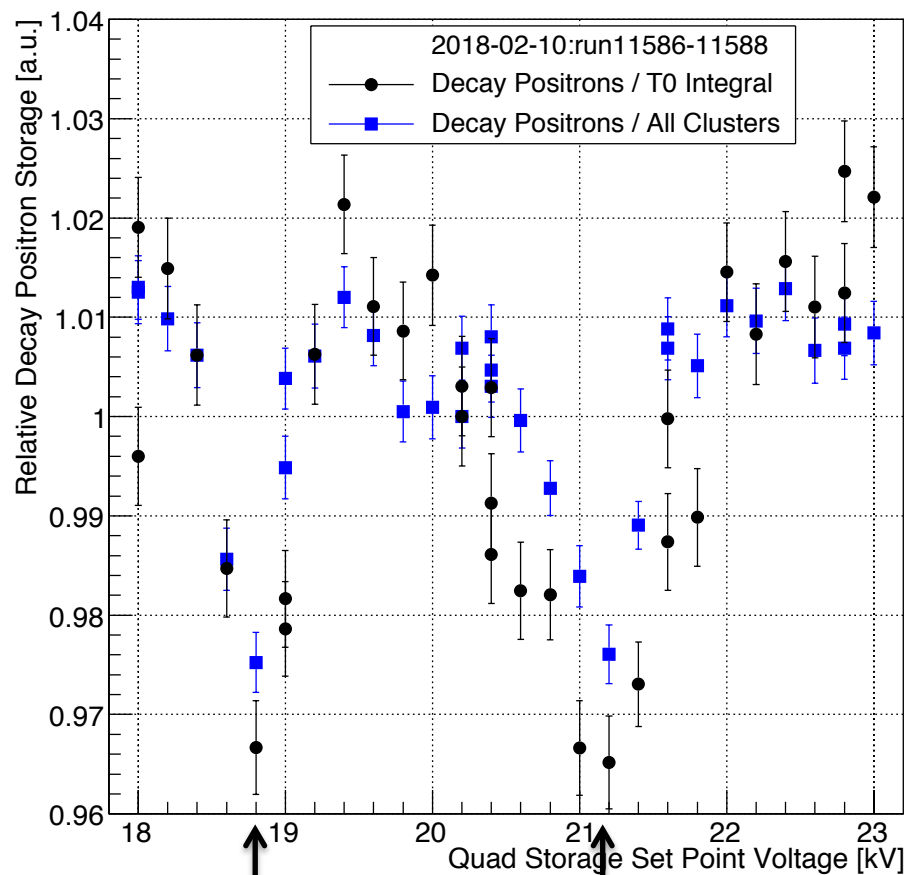


# Resonances



## Tune scan

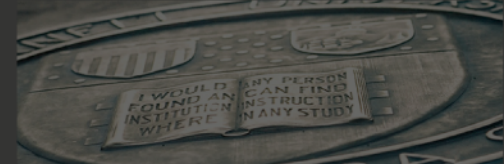
Stored muons



Quad voltage



# Summary



- A quantitative understanding of the evolution of the muon distribution over the course of the fill is essential to limiting the systematic uncertainty in the measurement of the anomalous magnetic moment to the 70 ppb (systematic) target.
- The Muon g-2 experiment is equipped with detectors that can measure phase space and momentum distribution in some detail.
- Beam dynamics simulations informed by the measurements complete the description.
- The preliminary data presented was collected during the commissioning phase of the experiment.
- Based on that data it is clear that the fiber harp and tracker systems are an effective window on the behavior of the circulating distribution.

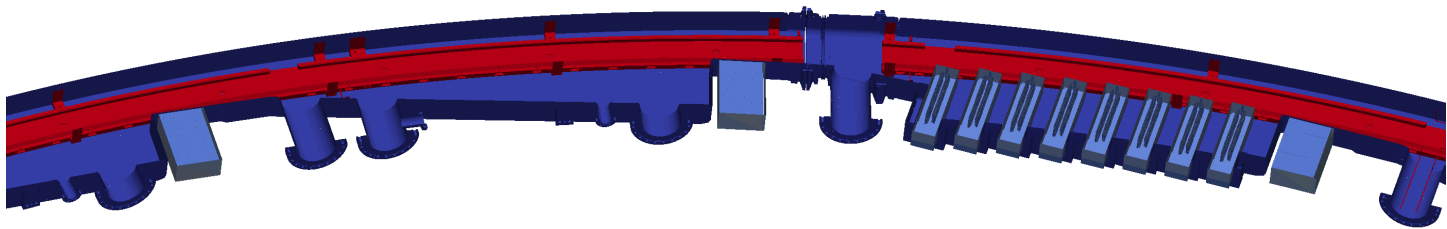
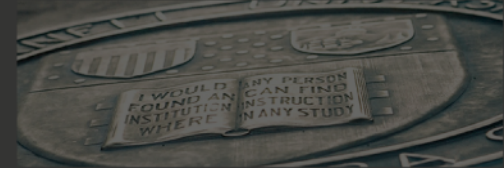


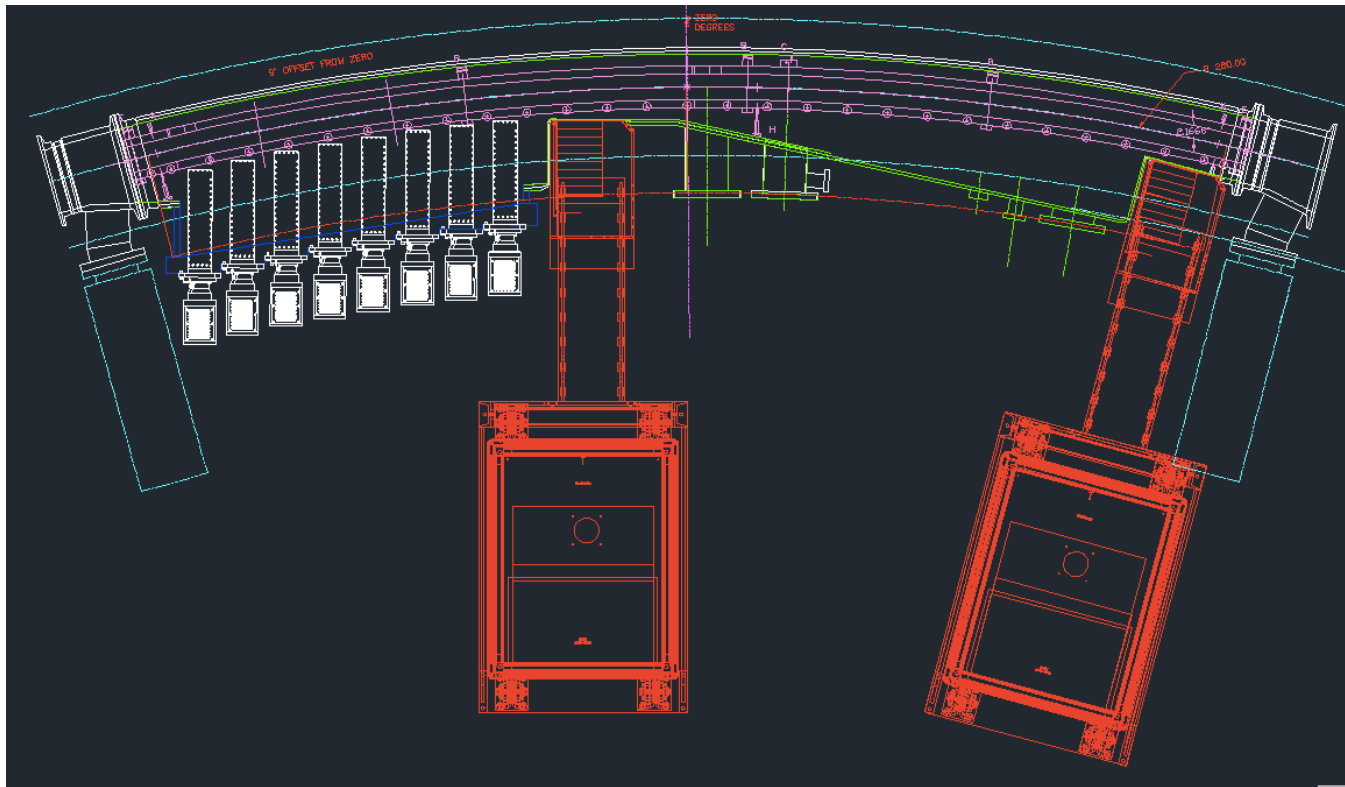
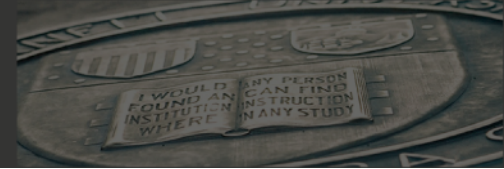


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W. Wu, University of Mississippi  
K. S. Khaw, A.T. Feinberg, University of Washington

All of the Muon g-2 Collaboration (E989)

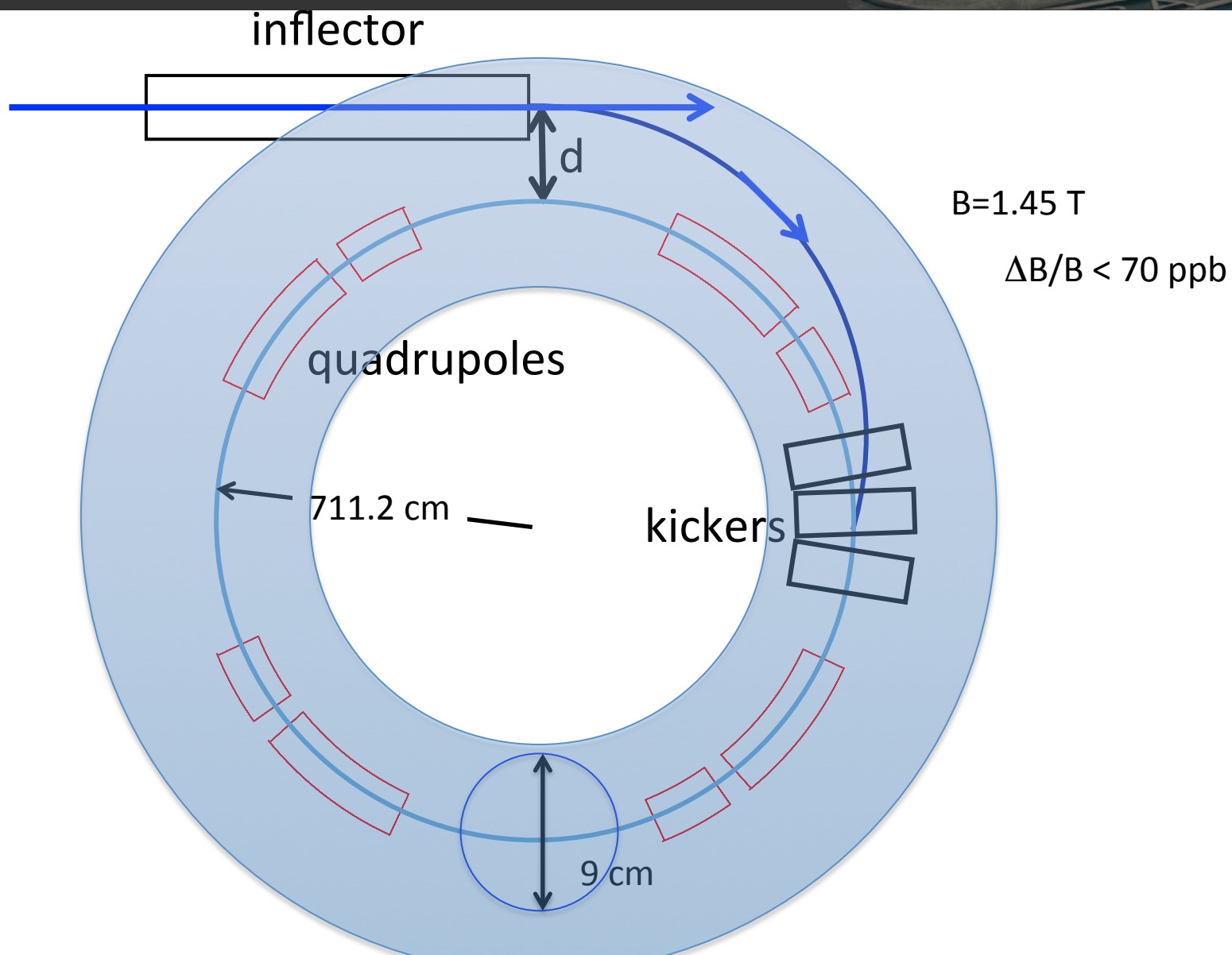
- Commissioning the Superconducting Magnetic Inflector System for the Muon g-2 Experiment – N.S.Froemming *et al.* WEPAF014
- Commissioning the Electrostatic Quadrupole System for Muon g-2 - J.D. Crnkovic *et al.* WEPAF015
- Application of Quad-Scan Measurement Techniques to Muon Beams in the Muon g-2 Experiment – J. Bradley *et al.* WEPAF016
- Lost Muon Function for the Muon g-2 Experiment – S.Ganguly & J.D.Crnkovic THPAK139
- New Fast Kicker Results from the Muon G-2 Experiment – A.P.Schreckenberger *et al.* THPML093
- Using Time Evolution of the Bunch Structure to Extract the Muon Momentum Distribution in the Muon g-2 Experiment – W. Wu, B. Quinn, J.D.Crnkovic
- Initial Studies into Longitudinal Inization Cooling for the Muon g-2 Experiment - J. Bradley *et al.*
- Correction of the Effect of the Coherent Betatron Oscillation with RF Electric Field for Muon g-2 experiment – O. Kim *et al.* THPAF010







# Muon storage ring





# Muon storage ring

