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OPTICAL KLYSTRONS

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"Optical klystrons" are free-electron lasers with separated functions: energy modulation, dispersive drift and emission. Different proposals are reviewed, and the basic physics is discussed, showing in particular the difference between devices based on "coherent" emission and on "stimulated" emission, and pointing out some possible limitations.

I. INTRODUCTION

The "free electron laser" (FEL) is an amplifier or oscillator device based on stimulated synchrotron radiation from relativistic ($\gamma^2 \gg 1$) electrons in an undulator (periodic transverse magnetic field, or transverse e.m. wave). In this device, the electrons in the beam experience a longitudinal force that is a periodic function of position, and therefore a velocity modulation; they then tend to bunch at distances $\lambda/2$ (where λ is the output wavelength), giving rise to coherent synchrotron emission (which interferes with the input wave). The effect decreases for large electron energies γmc^2 because the dispersion $d\beta/d\gamma$ (βc = velocity) for a free particle is proportional to γ^{-3} (see Refs. 1–4).

The bunching can be made more rapid by introducing a magnetic dispersive element (where faster (slower) particles move on a shorter (longer) path) or by producing the bunching with a high-power pulsed laser. In these proposals the functions of bunching and "coherent" or "stimulated" scattering are usually separated, so that in analogy with the microwave klystron, such a device is called an "optical klystron" or "transverse optical klystron".[5]

The idea of having a bunched beam emitting "coherently" on harmonics was used in microwave tubes (TWT and klystrons) to produce mm-waves (see for example, Ref. 6). Csonka proposed[7] the use of a FEL as an "energy modulator" (with high-power laser input) to produce electron bunches much smaller than the modulating wavelength, with the aim of producing "coherently" X-rays from, for example, a bending magnet.

Alferov, Bashmakov and Bessonov,[8] from an analysis of the bunching of the electron beam in a FEL and of "coherent" emission, arrived at a proposal of an "optical klystron" composed of a FEL amplifier followed by an undulator as a "radiator" on a harmonic.

The "optical klystron" (OK) proposed by Vinokurov and Skrinskii,[9,10] made of two undulators with a dispersive magnetic drift space in between, is an amplifier-oscillator aimed at reducing the length of a FEL in high-energy electron beams. In this case the input wave interacts with the electron beam also in the "radiator".

The possibility of getting "coherent" radiation from an electron beam modulated by a FEL is also suggested by Brautti et al., [11,12] and the use of dispersion (positive or negative) to enhance modulation is described by Boscolo et al.[13] In a proposal of OK by de Martini and Madey[14,15] the second undulator is a traveling wave and the emission is on a very high harmonic.

A numerical treatment of the evolution of harmonics by the Vlasov equation has been made by Stagno et al.,[16,17] and an equivalent one using a Monte Carlo method by de Martini and Edighoffer,[15] An analytical solution for the electron density in a FEL or OK has been found by Leo et al.[18]

A detailed energy-loss analysis of a device similar to that of Refs. 9,10 is given by Shih and Yariv.[19] An "energy separator" has been proposed by Csonka[20] to reduce the limitations due to the electron energy spread. Recently the OK of Vinokurov and Skrinskii has been realized, its spontaneous spectrum measured,[21] and gain has been observed.[22]

We want now to give a rough description of an

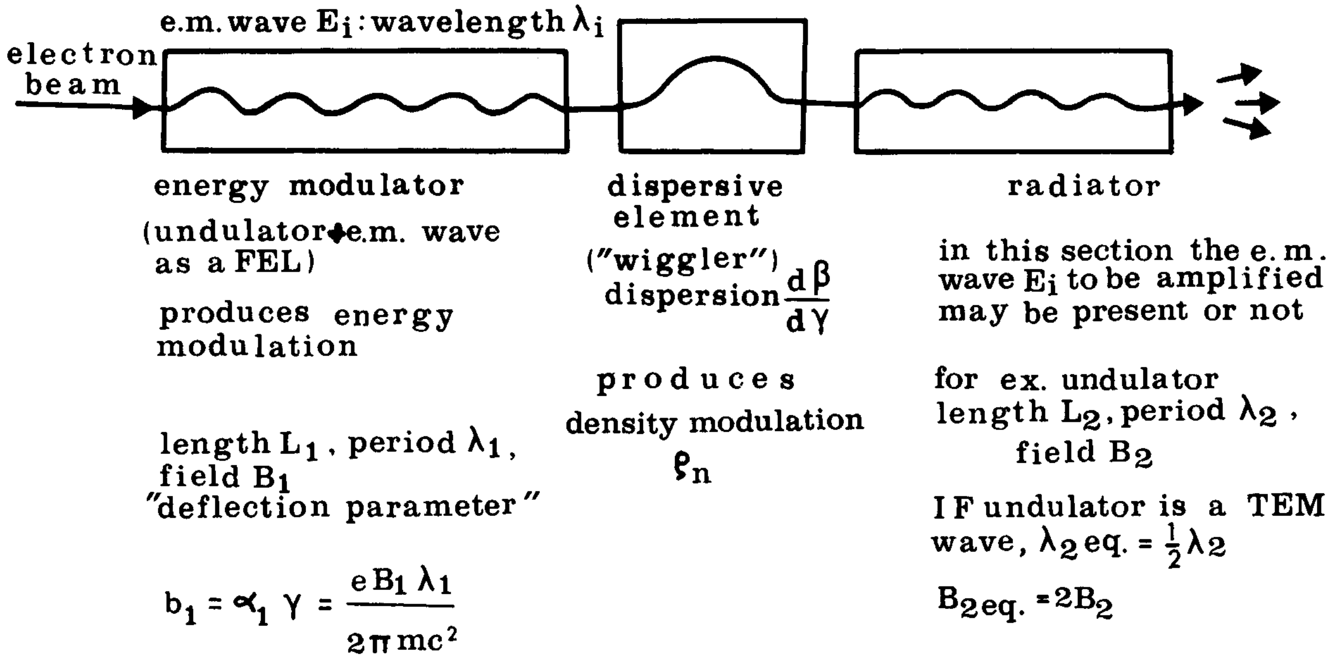

FIGURE 1 General scheme of an OK and definition of symbols (cgs units).

OK and clarify the physical mechanisms involved and to give simple practical formulas for order-of-magnitude calculations and for the discussion of possible limitations. The description will be in two parts:

(i) Dynamics of electrons in the buncher (energy modulator + drift): with a given input, what will be the modulation index (fundamental and harmonics) and how many harmonics?

(ii) Emission: in the radiator with a given modulation, calculate power (and $\Delta\nu$ and $\Delta\Omega$) of the emitted radiation.

There are essentially two different kinds of devices, with different aims: small-signal amplifier-oscillators with the input wave present also in the radiator, and strong-signal (pulsed-laser input) frequency multipliers, and they will be treated separately in Sec. 5 and 6.

We start with Fig. 1. We use the approximations $\gamma^2 \gg 1$ and $\alpha^2 \ll 1$. The longitudinal velocity β_x depends on energy γ and angle θ with respect to x; then

$$\beta_x - \beta_0 = \frac{\gamma - \gamma_0}{\gamma^3} - \tfrac{1}{2}\theta^2, \qquad (1)$$

where $\beta_0 = \beta_x(\gamma = \gamma_0, \theta = 0)$.

II. BUNCHING: ENERGY MODULATION AND DISPERSION.

A. ENERGY MODULATION

The function of energy modulation in an OK is performed by a FEL (= undulator + input wave) (see Fig. 1). The dynamics of the electron beam in a FEL is well known,[4] and we merely summarize it with Fig. 2. The incoming particles, with a random longitudinal distribution (Fig. 2a),

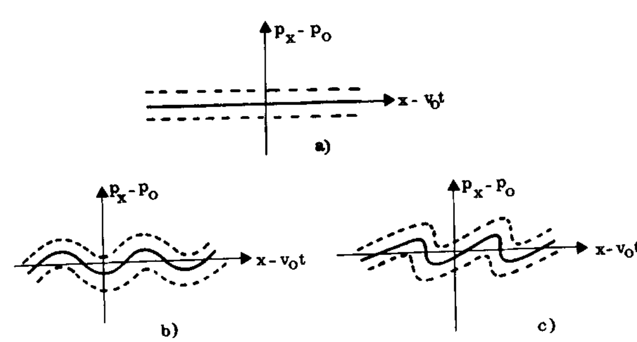

FIGURE 2 Longitudinal phase space (momentum $p_x - p_{x0}$ versus position $x - \sigma_0 t$) electron density distribution: a) initially, b) energy-modulated but not yet bunched, c) bunched. The continuous line indicates the distribution for $\Delta p_x = 0$ (or $\Delta\gamma = 0$, $\Delta\theta = 0$) (ideal), or the maximum of the distribution whose width is indicated by the dashed lines. The density is the integral of this distribution over p_x.

experience an increase or decrease of momentum depending on the relative phase of the input wave and the undulator (then on the position of the electrons in the beam). Then the phase-space distribution becomes a wavy line (or strip) (Fig. 2b). The different velocities of the particles with different p_x distort Fig. 2b to something similar to Fig. 2c. Within the FEL, this distortion is the one corresponding to a pendulum potential (with closed orbits for "trapped" particles) while in a drift space they drift horizontally at a rate proportional to $p_x - p_{x0}$. This can be described quantitatively by the Vlasov equation for the phase-space density distribution $f(x, p)$ ($\int f(x, p)dp = \rho(x)$). In the electron average rest frame,

$$\frac{\partial f}{\partial t} + \frac{p}{m}\frac{\partial f}{\partial x} + \frac{dp}{dt}\frac{\partial f}{\partial p} = 0.$$

If the energy modulator is sufficiently short (length $\ll c/4$ times the pendulum period so that there is negligible bunching within the energy modulator), the amplitue $\delta\gamma$ of the energy modulation can be easily calculated[4] in the "impulsive" approximation by the work done by the incoming field E_i. Thus

$$\delta\gamma = \frac{e}{mc^2}\int_0^L E_i v_\perp \frac{dx}{c} = \frac{e}{2mc^2}\alpha L E_i = \frac{20}{\gamma} bLP_2{}^{1/2}, \tag{2}$$

where in the last expression P_L is the input power in MW/mm^2 and L is in meters.

B. DISPERSION

After the beam is energy-modulated, it must travel a distance such that the faster particles reach the slower ones to get bunching (Fig. 2b to 2c). If $ds/d\gamma$ is the change in effective length travelled by two particles with energy difference $d\gamma$, the maximum bunching will happen (if $\Delta\gamma = 0$) when the line describing the longitudinal phase-space distribution will have a vertical tangent, i.e., let $\Delta\gamma \neq 0$ when electrons with energy $\gamma + \delta\gamma + \Delta\gamma$ will gain a distance $\lambda/2\pi$

$$\frac{ds}{d\gamma}(\delta\gamma + \Delta\gamma) \simeq \frac{\lambda}{2\pi}. \tag{3}$$

In general this path difference will result in part within the energy modulator, and in part in the drift, the contribution of each depending on $\delta\gamma$ and on input power. For $\delta\gamma \ll \gamma/4\pi N$ all the dispersion will have to be provided by the drift, while for $\delta\gamma \simeq \gamma/4\pi N$ the radiator will be just at the exit of the energy modulator.

To calculate the length of the drift or the magnetic field of the "wiggler" to get the desired path difference $ds/d\gamma$ we remark that in a free space of length L_d

$$\frac{ds}{d\gamma} = \frac{1}{\gamma^3} L_d, \tag{4}$$

and for a (sinusoidal) 3-pole "wiggler" with magnetic field $B_w = 2\pi mc^2 b_w/eL_w$ $(b_w \gg 1)$

$$\frac{ds}{d\gamma} = \frac{b_w}{2\gamma^3} L_w. \tag{5}$$

To estimate the bunching within the FEL, we remark that

$$\frac{ds}{d\gamma} = \frac{1 + \frac{1}{2}b_1{}^2}{\gamma^3} L. \tag{6}$$

C. LIMITATIONS

Ideally (electron energy spread $\Delta\gamma = 0$ and electron angular spread $\Delta\theta = 0$) at the bunching point the density of electrons in the equilibrium positions would tend to ∞, and the modulation index $\mu_n = \rho_n/\rho_0$ would be of the order of 50%, decreasing slowly up to very high harmonics, but it is limited in practice by the spreads $\Delta\gamma$ and $\Delta\theta$ which give a spread Δs in path lengths s. Then the ideal density distribution at the bunching point must be convoluted by a point spread function of width Δs depending on $\Delta\gamma$ and $\Delta\theta$; the modulation spectrum is multiplied by the Fourier transform of this function (for the convolution theorem).

The distributions of γ and θ are Gaussians. As $v_x = v\cos\theta = = v(1 - \frac{1}{2}\theta^2)$, the distribution in v_x is exponential up to $v_x = v$, and $= 0$ for $v_x > v$, then the two "transfer functions" for the modulation spectrum are a Gaussian (for energy spread) and a Lorentzian (for angular spread). Then, generalizing the result of Vinokurov and

Skrinskii, we can say that

$$\mu_n = \frac{\rho_n}{\rho_0} \simeq \frac{1}{2}\frac{\delta\gamma}{(\delta\gamma + \Delta\gamma)} \times \frac{\exp - \frac{1}{2} n^2 \left(\frac{\Delta\gamma}{\delta\gamma + \Delta\gamma}\right)^2}{\left[1 + n^2 \left(\pi N_1 \frac{\gamma^2 \Delta\theta^2}{1 + \frac{1}{2} b_1^2}\right)^2\right]^{1/2}}. \quad (7)$$

The Lorentzian factor expresses the qualitative remark of Vinokurov and Skrinskii that the angular spread should not produce a delay of more than $\lambda/2\pi$ at the end of the undulator. The harmonic number that can be reached, is then limited by $\Delta\gamma$ and $\Delta\theta$, but there are other problems limiting power and harmonic number.

As we have already seen [Eq. (3)] there is a maximum input power

$$P_L^{\max} = (\gamma^2/500\, b_1 L_1)^2 \text{ MW/mm}^2 \; (L \text{ in meters}) \quad (8)$$

which, for Eq. (7) means

$$n \leq \frac{\gamma}{4\pi N_1 \Delta\gamma}. \quad (9)$$

But the same problem (energy modulation $\delta\gamma$, then velocity modulation $\delta\beta$, now producing a debunching) arises in the radiator: if after a length L_2 in the radiator, the n-th harmonic disappears ($\delta\beta\, L_2 = \lambda/2\pi$), it is useless to have a radiator longer than

$$L_2 \leq \gamma\lambda_2/4\pi\delta\gamma. \quad (10)$$

In particular, if the bunching is all in the drift ($P_L \ll P_L^{\max}$)

$$L_2 \leq \frac{1}{n}\frac{s}{(1 + \frac{1}{2} b_2^2)}, \quad (11)$$

where $n = \lambda_2/\lambda_1 = \lambda/\lambda_i$ and s is the effective length of the drift, while if $P_L = P_L^{\max}$

$$N_2 \leq N_1 \quad (12)$$

or

$$L_2 \leq L_1/n, \quad (12')$$

where $N_2 = L_2/n\lambda_i$ (see also Sec. 3A). This reduces further the possible power output on higher harmonics.

III. INCOHERENT, COHERENT, AND STIMULATED EMISSION.

If a modulated electron beam enters an electromagnetic structure (for example, a bending magnet or an undulator or an e.m. wave), the emission from it can be considered as the sum of two parts: an "incoherent" one which is the sum of the intensities of the radiation emitted by each electron (with the same spectral and angular distribution) as if the beam was not modulated; and a "coherent" part which would be emitted by a smooth (modulated, but continuous) current distribution, with spectral and angular properties depending on the modulation, and intensity proportional to μ^2 (where μ is the modulation index).

To compare the different kinds of emission, let us take for simplicity an undulator with $b_2 \ll 1$ (see Fig. 1). The results can be easily generalized.

A. INCOHERENT EMISSION

The power emitted is (from Liénard's formula)

$$\frac{dW_{\rm inc}(\theta = 0)}{d\nu d\gamma} = \frac{2\pi e}{c} I_0 b_2^2 N_2^2 \gamma^4, \quad (13)$$

where I_0 is the average current, with relative bandwidth

$$\frac{\Delta\nu}{\nu} \simeq \frac{1}{N_2} \quad (14)$$

around a frequency

$$\nu = 2\gamma^2 c/\lambda_2 \quad (\text{at } \theta = 0) \quad (15)$$

and within a solid angle

$$\Delta\Omega \simeq 1/\gamma^2. \quad (16)$$

In the case $b_2 \gtrsim 1$, there is also emission on harmonics n_2 of the radiator, where

$$\lambda = \frac{1}{n_2}\frac{\lambda_2}{2\gamma^2}(1 + \tfrac{1}{2} b_2^2), \quad (17)$$

and b_2^2 becomes a more complex function $F_n(b)$.[1,23] In case the emission is on a harmonic n_2 of the radiator, in Eqs. 10 and 12, we must understand $\lambda_2 \rightarrow \lambda_2/n_2$ and $N_2 \rightarrow n_2 N_2$.

B. COHERENT EMISSION

The power emitted can be obtained by integrating Liénard's formula over the modulated (smooth) current distribution: in general we can say[24,16] that for a monoenergetic beam

$$\frac{dW_{\mathrm{coh}}(\theta = 0)}{d\Omega d\nu} = \frac{dW_{\mathrm{inc}}(\theta = 0)}{d\Omega d\nu} \frac{M^2(\nu)}{\frac{1}{e} I_0}, \quad (17)$$

where W_{inc} is the incoherent power, and $M(\nu)$ is the power spectrum of the current $\frac{1}{e} I(t)$,

$$M^2(\nu) = \frac{I_0^2}{e^2} \sum_n \mu_n{}^2 \delta(\nu - n\nu_1) \quad (18)$$

for an infinite beam. Then

$$\frac{dW_{\mathrm{coh}}/d\Omega}{dW_{\mathrm{inc}}/d\Omega} \simeq \mu^2 \zeta, \quad (19)$$

where ζ is the number of electrons in the coherence length $N\lambda$ of the (incoherent) radiation.

From another, equivalent,[25] point of view, the coherent emission can be viewed as a reflection of the wave equivalent (in the electron rest frame) to the undulator on the modulated refractive index

$$\sqrt{e} = \left(1 - \frac{4\pi\rho e^2}{m\omega^2}\right)^{1/2} \simeq 1 - \frac{2\pi e^2}{mw^2} \times \left(\rho_0 + \rho_1 \cos \frac{4\pi}{\lambda^1} x\right) \quad (20)$$

then (primed variables in electron rest frame, neglecting $e^{i\omega t}$, and $r_0 = e^2/mc^2$ classical electron radius) the reflected amplitude R' is

$$\frac{dR'}{dx} + ik'R' = \frac{1}{2\sqrt{\epsilon}} \frac{d\sqrt{\epsilon}}{dx} E'.$$

Then $R' = \frac{1}{4}\, i r_0 \rho_1 \lambda' L' E_0' e^{-ik'x}$ or, in the laboratory frame

$$R = \frac{\pi}{2} e \frac{b_2}{\gamma} \rho_1 L_2. \quad (21)$$

If the cross-sectional area of the beam is o, the power emitted is approximately

$$W_{\mathrm{coh}} = \frac{c}{4\pi} \sigma R^2 \simeq \frac{\pi}{16} \left(e \frac{b_2}{\gamma} \rho_1 L_2\right)^2 \sigma c. \quad (22)$$

For a thick beam ($\sigma > L\lambda$) this power is emitted within a diffraction-limited angle

$$\Delta\Omega \sim \lambda^2/\sigma. \quad (23)$$

If we have a thin beam ($\sigma < L\lambda$), the power per unit solid angle

$$\frac{dW_{\mathrm{coh}}}{d\Omega} = c\sigma^2 \left(\frac{eb_2\rho_1 L_2 \gamma}{\lambda_2}\right)^2 \quad (24)$$

is the same, but is emitted within an angle

$$\Delta\Omega \sim 1/N\gamma^2. \quad (25)$$

Ideally (ρ_1 = constant) the bandwidth is

$$\Delta\nu \sim c/L_b, \quad (26)$$

where L_b is the length of the electron beam pulse.

In case the electron beam has a non-negligible energy and angular spread, incoherent emission bandwidth and angle of emission will be broadened (convoluted) by these spreads, but coherent emission spectrum and angular distribution depend on the macroscopic modulation and not on the properties of individual electrons. In practice, if the modulation is produced by a partially coherent laser pulse, then the relative bandwidth and solid angle will be the $\Delta\nu_L/\nu_L$ and $\Delta\Omega_L$ of the laser (if $\Delta\nu_L/\nu_L > \Delta\nu/\nu$ and the coherence distance $< n$ times the beam diameter).

C. STIMULATED EMISSION

If now, together with the reflected wave R, there is an incident wave E_i of the same wavelength in the same direction and phase Φ with respect to R, the intensity of the resulting wave is

$$I = (E_i + R)^2 = E_i^2 + R^2 + 2E_i R \cos \Phi$$

$$= E_i^2 + (e\alpha_2\rho_n L_2)^2 + 2E_i e\alpha_2\rho_n L_2 \cos \Phi. \quad (27)$$

The first term is the incoming wave, the second the coherent emission (undulator-equivalent wave reflected by density modulation ρ_n), present even when E_i is absent, and the third one is an interference term between the external field E_i and the coherent-emission field; we will call this term "stimulated emission."

D. REMARKS

As in free induction decay, we have here (Sec. 3B) coherent emission from a system prepared in a "coherent" way, i.e., with nonvanishing offdiagonal elements of the density matrix. In fact, for a free electron beam, the presence of off-diagonal elements of the density matrix is equivalent to modulation of the beam.

If the electron is described by the Schrödinger (or Klein-Gordon) equation and the e.m. field is quantized, the physical origin of incoherent (spontaneous) emission is the zeropoint fluctuations of the e.m. field. In a classical description (Maxwell eqs. + point-like electrons) incoherent (spontaneous) emission also exists (along with coherent and stimulated emission), and its origin is in the point-like nature of the electron. In a semiclassical description (Schrödinger-Maxwell) incoherent (spontaneous) radiation does not appear.

This power is emitted by the electron beam: correspondingly, the beam must lose energy: what is the physical origin of the work performed on the beam? For incoherent emission, the work is done by the radiation reaction $\mathbf{F} = (2e^2/3c^2)(d^2\boldsymbol{\beta}/dt^2)$($\boldsymbol{\beta}c$ = velocity), which is always negative (as the phase shift between f and $\beta_\perp$ is π). For coherent emission, the energy loss is the (always negative) work done by the field R on the modulated beam: $\int eRv_\perp\rho\sigma dx$ which is equal to W_{coh}. Conversely, the coherent amplitude R can be obtained by an energy-loss calculation, by equating this power to the power $c\sigma R^2/4\pi$ of the wave. For the stimulated emission, the work is done by the external field E_i, and can be positive (absorption) or negative (emission) depending on the phase shift Φ.

The discussion in Sec. 2 and 3 of energy modulation, dispersion and coherent emission is of course relevant also for a conventional FEL, i.e., when the three functions are not separated, and bunching (and then emission) increases gradually as the beam passes through the FEL. In that case ρ_1 is proportional to x^2 (x = coordinate in the FEL: $0 \le x \le L$), and so is the coherent amplitude per unit undulator length: then [see Eq. (27)] the outgoing intensity is

$$I = E_i^2 + E_i \cos\Phi \int_0^L \frac{da}{dx}\,dx + \left| \int_0^L \frac{da}{dx}\,dx \right|^2 = E_i^2 + E_i^2 r_0\rho_0 \frac{\alpha_0{}^2L^3}{\gamma\lambda_0}\cos\Phi + E_i^2 r_0{}^2\rho_0{}^2 \frac{\alpha^4L^6}{\gamma^2\lambda_0{}^2} \tag{28}$$

If the electron energy is equal to the synchronous energy, $\gamma = \gamma_0$, bunching exists, but $\Phi = \pi/2$. Then the second term is zero, and the only contribution is the coherent one (third term: the factor E_i^2 appears,[26] but only because ρ_1 is proportional to E_i). The finite gain for stimulated emission comes from the fact that if $\gamma \neq \gamma_0$, the beam tends to bunch in a position displaced from the equilibrium position and then $\Phi \neq \pi/2$. In the case of an OK, the condition of maximum output is $\gamma = \gamma_0$.

IV. THE STIMULATED-EMISSION OPTICAL KLYSTRON (SOK)

In the work of Vinokurov and Skrinskii[9,10] (the only OK realized so far) and of Shih and Yariv,[19] the input wave is present also in the radiator, and the emission that is calculated is stimulated emission on the fundamental, with a small-signal ($\delta\gamma \ll \Delta\gamma$) input. The aim is to produce an oscillator.

Neglecting angular spread [$\Delta\theta \ll (1 + \tfrac{1}{2}b^2)/(N)^{1/2}\gamma$], from Eqs. (7) ($n = 1$, $\delta\gamma \ll \Delta\gamma$) and (22) one finds[9,21] a gain (with two equal undulators of length L)

$$G_{\text{sok}} \simeq \frac{\pi}{\sqrt{e}} \frac{I}{I_0\sigma} \frac{b^2L^2}{\gamma^2\Delta\gamma} \tag{29}$$

($I_A = mc^3/e \simeq 17$ kA, σ = effective cross-sectional area, including filling factor) then the ratio of G_{sok} to the gain of a FEL of length equal to the total length of the SOK (i.e. ~ 2.5 L) is ($N = L/\lambda_0$)

$$\frac{G_{\text{sok}}}{G_{\text{fel}}} \sim 2 \times 10^{-2} \frac{1}{N} \frac{\gamma}{\Delta\gamma}. \tag{30}$$

Then such a device would be very useful for high γ and the limited available length, if a good-quality electron beam (low $\Delta\gamma$ and $\Delta\theta$) is available. The incoherent spectrum[21] from such a device is a series of closely spaced bands due to interference of light emitted by an electron in both undulators. As the gain is proportional to the derivative with respect to γ of the spectral brilliance of the spontaneous radiation,[27] the gain can also calculated in that way.

Little can be said at present about saturation, but from the preceding remark (the closely spaced bands are easily washed out for a small increase in $\Delta\gamma$ and $\Delta\theta$), it can be foreseen that it will be at a lower power than in a FEL. An estimate has been made in Ref. 10.

The only OK that has been made is the one on VEPP-3 where spontaneous spectrum[21] and gain[22] have been measured, and roughly agree with theory. The two (equal) undulators have $N = 3$, $\lambda_0 = 10$ cm, $\beta_0 = 3$ kG, $\lambda = 0.6\ \mu m$, $\gamma mc^2 = 350$ MeV, $\Delta\theta \simeq 5 \times 10^{-5}$, $\Delta\gamma/\gamma = 1.5 \times 10^{-4}$, $\sigma \simeq \frac{1}{2}L\lambda$, $I = 20.A$ (peak). The gain under these conditions is expected to be approximately 30 times higher than a FEL of length $3L$ (total length of the OK).

On the LELA undulator on ADONE[28,29] (see below for data), using 5 poles for dispersion (NNSSSNN instead of NSNSNSN) and using two undulators with $N = 8$, one could increase the gain (if the dispersion is sufficient) approximately 10 times with respect to the full $N = 20$ FEL.

V. THE COHERENT-EMISSION OPTICAL KLYSTRON (COK)

In this kind of OK device, a pulsed laser gives a strong input ($\delta\gamma \gg \Delta\gamma$) producing a highly anharmonic modulation, to get coherent emission in the radiator on a harmonic of the laser frequency. Its aim is to produce short pulses of coherent vacuum ultraviolet radiation with very high spectral brilliance. This would be particularly useful because, for the lack of high reflectivity mirrors, it would be very difficult to make oscillators in this part of the spectrum. An estimate of the coherent power on the n-th harmonic with respect to the incoherent power, for $\sigma \geq L_2\lambda$, is

$$\frac{W_{\rm coh}}{W_{\rm inc}} \approx 2.10^4 I\lambda\mu_n^2 \frac{L_2\lambda}{\sigma} \qquad (31)$$

(I in amperes, σ in mm^2 and λ in μm) where μ_n^2 (modulation index of n-th harmonic) is given by Eq. 7. The highest harmonic number can be obtained with $P_L = P_L^{\max}$

$$\mu_n^2 \simeq \frac{1}{4}\frac{\exp - n^2\left(4\pi N_1 \frac{\Delta\gamma}{\gamma}\right)^2}{1 + n^2\left(\pi N_1 \frac{\gamma^2\theta^2}{1 + \frac{1}{2}b_2^2}\right)^2}, \qquad (32)$$

which is $\simeq \frac{1}{4}$ for small n, has a cutoff for $n \gtrsim \gamma/4\pi N_1\Delta\gamma$ and a rapid decrease for $n \gtrsim (1 + \frac{1}{2}b_2^2)/\pi N_1\gamma^2\Delta\theta^2$. For harmonics lower than these, it is convenient to use a $P_L < P_L^{\max}$, as the useful interaction length L_2 is longer [see Eq. (10)]. If one wants to see what is the minimum power to observe the 3rd or 5th harmonic, we can remark that for $\delta\gamma << \Delta\gamma$, we have from Eq. (7)

$$\mu_n^2 \simeq 10\left(\frac{b_1L_1}{\gamma\Delta\gamma}\right)^2 P_L e^{-n^2} \qquad (33)$$

(P_L in MW/mm^2, L_1 in meters).

In some earlier proposals[8,12] there are numerical estimates which are a bit optimistic for present accelerators. Let us consider a realistic example, the LELA undulator under construction on Adone ($I_{\rm peak} = 10\ A$ for 0.5 nsec, $\sigma \simeq 1$ mm^2, $\Delta\gamma/\gamma \simeq 2.3 \times 10^{-4}$; $\gamma\Delta\theta \simeq 0.15$ at $\gamma = 1194$) with $\lambda_0 = 11.6$ cm, $N_0 = 20$, $b = \alpha\gamma = 4.83$ ($B_0 = 4.46$ kG). Using 5 poles for dispersion and 8 periods for the energy modulator (as in the example above) maximum bunching could be obtained with $P_L = P_L^{\max} = 6.3$ GW/mm^2 ($\lambda = 0.5\ \mu$m, $\sigma \simeq 1$ mm^2, $\Delta\lambda_L/\lambda_L \sim 10^{-6}$, $\Delta\Omega_L \sim 10^{-6}$) producing a 43rd harmonic with $\mu_{43}^2 \simeq 0.02$ at $\lambda = 120$ Å with $W_{\rm coh}/W_{\rm inc} \sim 1/100$ but an increase in spectral brilliance (with respect to the incoherent one) of the order of 10^3, but the effective length L_2 of the radiator would be only approximately 2.5 cm. An increase of L_2 could be obtained with lower $\delta\gamma$, but with lower n. For ex. $L_2 \sim 35$ cm on the 11th harmonic ($\lambda = 450$ Å) at $P_L = \frac{1}{4}P_L^{\max}$; then $\mu_{11}^2 \sim 0.1$, $W_{\rm coh}/W_{\rm inc} \sim 10$ and coh/inc spectral brilliance $\sim 10^6$. Third-harmonic generation could already be observed with inputs of a few hundreds of kW.

VI. CONCLUSIONS

Many experiments still have to be made to make the OK a source of e.m. radiation (amplification, effects on electron-beam stability and momentum spread, oscillation, saturation, production of harmonics in the electron-beam modulation by pulsed lasers, ...) and it is probably too early to investigate details. Many problems also depend on developments in accelerator technology. Here we have made some remarks about orders of magnitude and pointed out some limitations which have to be kept in mind for further development.

Limitations are mainly of two types, those due to energy spread and angular spread of the electron beam, and those due to the variation of bunching both within the energy modulator and the radiator, (due to the velocity modulation corresponding to $\delta\gamma$). For the first limitations, improvements depend on the possibility of constructing electron machines with a lower emittance and lower energy spread, or of "modulating" the electron density in phase space. An interesting suggestion is due to Csonka,[20] who proposes an energy separator (electrons with different energies are laterally separated before entering an energy modulator with a transverse gradient) to reduce the effect of energy spread and thus reach higher harmonics.[30]

For the second type, a solution could be to make an energy demodulator (with a suitable phase-shifted undulator) to reduce $\delta\gamma$ before the beam bunching is complete.

If sufficient power can be obtained in a SOK it would be possible to extract the coherent emission on the 3rd or 5th harmonic (for ex. using a grating as a 2nd mirror). The FEL has also been proposed as an accelerator;[4a,11] the same can be said of an OK, and also the preceding remarks could be applied.

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