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Résumé :

Le klystron optique est une version modifiée de l'onduleur utilisé pour améliorer le gain du Laser à Electrons Libres. L'émission spontanée et le gain sont calculés en fonction de l'énergie des électrons et de la longueur d'onde. Quelques effets limitant l'augmentation du gain sont étudiés : dispersion en énergie, dispersion angulaire, tailles transverses du faisceau d'électrons. Je discute brièvement l'utilisation de la modulation en densité introduite par le klystron optique sur le faisceau d'électrons pour générer du rayonnement synchrotron cohérent.

Abstract :

The Optical Klystron is a modification of the undulator which can be used to improve the gain in a Free Electron Laser. Spontaneous emission and gain are theoretically studied as function of electron energy and wavelength. Several effects limiting the gain enhancement are calculated : energy spread, angular spread, beam dimensions. I briefly discuss how one can use the electron beam bunching generated by the Optical Klystron to emit coherent synchrotron radiation.

1. INTRODUCTION

The Optical Klystron (O.K.) is a magnetic device consisting of two undulators separated by a dispersive section in which the transit time of electrons depend on their energy. As an example, figure 1 shows the vertical magnetic field and the electron horizontal trajectory of the Orsay O.K. It is also called Transverse Optical Klystron because the energy exchange process between electrons and light is due to the transverse electric field of the light.

There are two main applications of the O.K. :

- One is as a light amplifier to be used in a Free Electron Laser [1] (FEL). It has first been proposed by Vinokurov and Skrinsky [2] and has been theoretically studied by R. Coisson [3], C. Shih and A. Yariv [4] and P. Elleaume [5]. So far, two experimental investigations have been undertaken in Novosibirsk (USSR) and in Orsay (France). I shall deal with O.K. as a light amplifier in section 2. The experimental results will be considered in section 3.

- Another purpose is to use it as a source of coherent synchrotron radiation in which the emission is enhanced by the bunching of the electron beam. An energy modulation is created by sending a high power laser. This energy modulation develops a density modulation in the dispersive section. The last undulator is there as a source of synchrotron radiation. This scheme has been extensively studied theoretically [3, 6 to 19] . However, it has not yet received any experimental investigation. The discussion of the O.K. as a source of coherent synchrotron radiation is presented in section 4.

Let's detail the principle of operation of the O.K. We consider an electron beam and

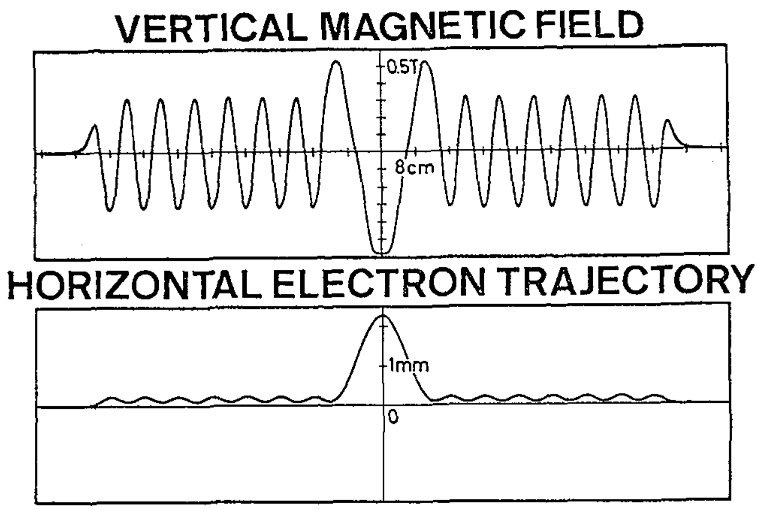

Figure 1 Vertical magnetic field calculated for the Orsay Optical Klystron (gap = 33.00 mm) and the corresponding horizontal electron trajectory at an energy of 240 MeV.

a plane wave entering an undulator. At first order in the field of the plane wave, there is no gain but, according to their initial phase with respect to the plane wave, some electrons are accelerated, some are decelerated. We have at the exit of the undulator what we call an energy bunching. Then electrons with higher energy tends to pass over the slower electrons, and all along the undulator the energy bunching creates a longitudinal density modulation or a density bunching which is also linear in the plane wave field. This density bunching is then responsible for the gain process which is proportional to the density bunching times the plane wave field. This process is the basic process of the classical gain of the undulator, originally developed by W.B. Colson [20]. Since the gain is due to the density bunching, is there any way of increasing the density bunching ? In other words, is it possible to make the bunching process faster for a given undulator length. The answer is yes, just by inserting a dispersive section inside the undulator in which the energy modulation will generate a density modulation. The simplest dispersive section is a drift space, it has the disadvantage of being too long (as we shall see later) and one prefers the magnetic dispersive section consisting of three poles forcing the electron trajectory to be a single large wiggle (fig. 1). This is the Optical Klystron. All has been done to increase the density bunching which is responsible for the coherent synchrotron radiation and for the gain process when the system is used as an amplifier. In fact, the main advantage of the dispersive section is to achieve a larger bunching (and therefore larger gain and coherent emission) than an undulator of the same total length. This is crucial in devices where space is limited like storage rings.

2. OPTICAL KLYSTRON AS AN AMPLIFIER

Several methods have been used to calculate the bunching and the gain. One of them consists in calculating the exact trajectory of an electron and then to average over the initial phase [4,5] as in the original classical theory of the FEL [20]. Another is to use the Vlasov equation to describe the density evolution [11, 12, 3] Both these methods give heavy calculations that can be simplified by assuming the dispersive section to be much more dispersive than the undulators. One then neglects the density bunching inside the first undulator and considers it as constant in the second undulator. With this approximation, sometimes called "impulse" approximation, the O.K. gain calculation is simpler than the undulator's case.

I will use a different approach that has the advantages of not necessitating the "impulse" approximation and giving limited calculations. I shall first derive the spontaneous emission. The gain will then be deduced from the well known " Madey's theorems" [21 to 25] that reads in MKS units :

$$\frac{dI(\omega)}{d\omega d\Omega} = \frac{m^2 c\omega^2}{2\pi^2 E^2} \quad <(\delta\gamma^{(1)})^2>_\phi \qquad (1)$$

and

$$\text{low field gain} \quad \propto \quad <\delta\gamma^{(2)}>_\phi \quad = \frac{1}{2}\frac{\partial}{\partial\gamma}<(\delta\gamma^{(1)})^2>_\phi \qquad (2)$$

where $\frac{dI(\omega)}{d\omega d\Omega}$ is the energy radiated per electron, per pass in the undulator, per unit solid angle and per frequency in the forward direction (electron beam propagation direction). m is the electron mass, c the speed of light, ω the light frequency, E the electric field amplitude of an incident planewave and γmc^2 the electron energy. $\delta\gamma^{(i)}mc^2$ is the electron energy variation in the O.K. induced by the plane wave interaction at i^{th} order of its amplitude E. $<.>_\phi$ stands for averaging over the initial random phase of the electron with respect to the plane wave.

This method reduces the low field gain (gain at low power of the FEL) calculation to the spontaneous emission calculation. However it is almost impossible to derive the high field gain by this method.

In the following I shall restrict myself to an O.K. made of two identical planar undulators (which maximizes the gain [5]) separated by a planar dispersive section. Planar just means that the field is always parallel to one given direction that I shall call vertical and which is the same for undulators and dispersive section. I shall assume the dispersive section to be "fully" compensated, namely :

$$\int_{-\infty}^{+\infty} B(z)\, dz = 0 \qquad (3)$$

and

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{u} B(z)\, dz\, du = 0 \qquad (4)$$

where B(z) is the dispersive section magnetic field at the longitudinal coordinate z (taken along the undulator axis). These conditions mean that the electron trajectory is exactly on the same axis in both undulators (see figure 1). This insures maximum interaction between the electron beam and the light beam which is in fact a cavity mode confined to the cavity axis. The two O.K. designed in Novosibirsk and Orsay satisfy these restrictions (*).

In the following, I shall assume the reader is familiar with the physics of the spontaneous emission and gain in a regular planar undulator [26].

2.1. Spontaneous emission

As for the calculation of the spontaneous emission from an undulator, one starts from the classical formula (in MKS units) [27] :

$$\frac{dI}{d\omega d\Omega} = \frac{e^2\omega^2}{16\pi^3\varepsilon_0 c}\left|\int_{-\infty}^{+\infty} \vec{n} \times (\vec{n} \times \vec{\beta}) e^{i\omega(t-\frac{\vec{n}\vec{r}}{c})}\, dt\right|^2 \qquad (5)$$

(*) *The case of non identical undulators and violation of (3) and (4) are studied in reference [5]. A straightforward modification of this calculation would apply to helical undulators.*

where e is the electron charge, ε_o the vacuum permeability, $\vec{n}$ the unit vector in which direction $\frac{dI}{d\omega d\Omega}$ is calculated, $\vec{\beta}$ the electron speed divided by the light speed and $\vec{r}$ the electron position at time t. The integral in (5) can be split into three parts representing the integral in the first undulator, the dispersive section and the second undulator. I shall neglect the contribution of the dispersive section to the spontaneous emission for the following two reasons :

- the dispersive section is much shorter than the undulators
- the electron motion in the dispersive section is not resonant with that in the undulators. Therefore the dispersive section contribution is expected to be smaller and broader than the one from the undulators.

Since the two undulators are identical Eq. 5 can be rewritten as :

$$\frac{dI}{d\omega d\Omega}\,(\text{O.K.}) = 2\,\frac{dI}{d\omega d\Omega}\,(\text{1 undulator})\,(1+\cos\alpha) \tag{6}$$

with :

$$\alpha = \alpha_u + \alpha_d \tag{7}$$

$$\alpha_u = \omega\,\Delta_u\left(t - \frac{\vec{n}\vec{r}}{c}\right) \tag{8}$$

$$\alpha_d = \omega\,\Delta_d\left(t - \frac{\vec{n}\vec{r}}{c}\right) \tag{9}$$

where $\Delta_u(t - \frac{\vec{n}\vec{r}}{c})$ and $\Delta_d(t - \frac{\vec{n}\vec{r}}{c})$ stand for the total variation of $t - \frac{\vec{n}\vec{r}}{c}$ in one undulator and in the dispersive section. Let's define K the usual deflection parameter in the undulators :

$$K = \frac{e\,B_o\lambda_o}{2\pi mc} \quad \text{(in MKS units)} \tag{10}$$

where B_o and λ_o are the maximum field and period of the undulator. Then for an electron injected at a small angle Θ with respect to the observation direction one has :

$$\alpha_u = \frac{\pi L}{\lambda\gamma^2}\,(1 + K^2/2 + \gamma^2\Theta^2) \tag{11}$$

$$\alpha_d = \frac{\pi d}{\lambda\gamma^2}\,\left(1 + \frac{e^2}{dm^2c^2}\int_0^d \left[\int_0^u B(z)\,dz\right]^2 du + \gamma^2\Theta^2\right) \tag{12}$$

where L is the length of one undulator, d the length of the dispersive section, λ the light wavelength and B(z) the dispersive section field at longitudinal coordinate z.

$\int_0^d$ denotes the integral along the dispersive section, I assume the field to be zero outside the interval $[0,d]$. In any practical case the fringe fields of the undulators and of the dispersive section add together making impossible the decomposition $\alpha = \alpha_u + \alpha_d$. Since the dispersive section field is larger than the undulator field one can integrate the dispersive section field between $-\infty$ and $+\infty$; the error resulting in the determination of α will usually be negligible (it is estimated to be of the order of 5 % for the Orsay O.K. and much lower for the Novosibirsk O.K.).

Let's define the dimensionless parameter $N_d(\lambda,\gamma)$:

$$\alpha_d \equiv 2\pi N_d(\lambda,\gamma) \tag{13}$$

From eq. (9) and (13) one understand the physical meaning of N_d. N_d is exactly the number of wavelength of light passing over an electron energy γmc^2 in the dispersive section. In fact N_d is the unique new parameter taking into account all the dispersive section effects. It can be compared to N the number of periods of an undulator which is also the number of wavelength of light passing over a resonant electron in the undulator.

Note that in a preceding paper /‾5_7 I defined N_d independent of λ and γ but always computed at resonance. The two points of view are evidently equivalent although it turns out to be more convenient to consider N_d as a function of λ and γ in the interpretation of experimental emission and gain curves.

Instead of N_d, some people prefer the use of $\frac{\delta s}{\delta\gamma}$ to characterize the dispersive section. δs is the longitudinal distance delay between two electrons crossing the dispersive section with a difference in energy equal to $\delta\gamma mc^2$. One can relate N_d to $\frac{\delta s}{\delta\gamma}$ by the relation :

$$N_d = \frac{\gamma}{2\lambda}\,\frac{\delta s}{\delta\gamma} \qquad (14)$$

This point of view has the advantage of showing how the dispersive section can efficiently bunch an energy modulated electron beam. I find easier to use N_d when discussing emission and gain because it is dimensionless.

The Θ dependence of N_d is usually weak (see eq. 12 and 13). I shall neglect it in the following except when dealing with off-axis spectrum or angular spread effects.

Defining λ_R the fundamental resonant wavelength for a given energy as

$$\lambda_R = \frac{\lambda_o}{2\gamma^2}\,(1 + K^2/2) \qquad (15)$$

and γ_R the fundamental resonant energy for a given wavelength as

$$\lambda = \frac{\lambda_o}{2\gamma_R^2}\,(1 + K^2/2) \qquad (16)$$

and N as the number of periods in one undulator.
One can rewrite α_u, α_d and α :

$$\alpha_u = 2\pi N\,\frac{\lambda_R}{\lambda} = 2\pi N\,\left(\frac{\gamma_R}{\gamma}\right)^2 \qquad (17)$$

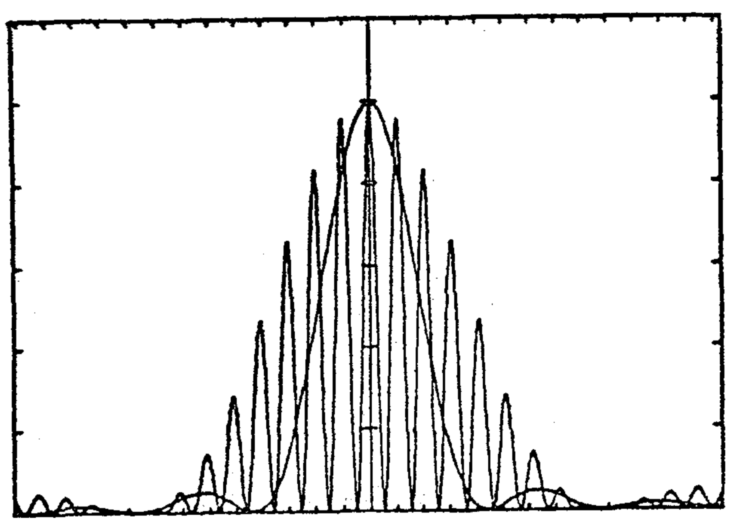

Figure 2 Curve of $\lambda^4 \frac{dI}{d\lambda d\Omega}$ calculated for a single electron as a function of $\frac{1}{\lambda}$ around the fundamental resonance wavelength in an optical klystron made of two 7 periods undulators. The dispersion corresponds to $N_d = 53$ for the fast oscillating curve and $N_d = 0$ for the smooth curve.

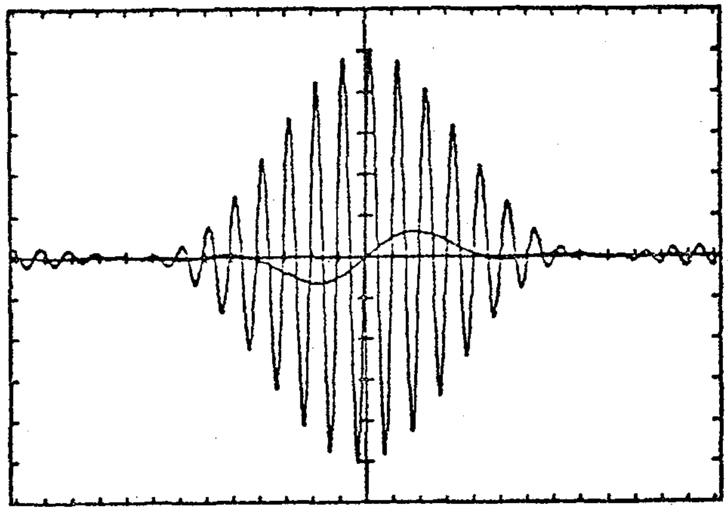

Figure 3 Gain = $\frac{\partial}{\partial\gamma}(\lambda^4 \frac{dI}{d\lambda d\Omega})$ for the conditions of figure 2.

$$\alpha_d = 2\pi N_d (\lambda_R,\gamma) \frac{\lambda_R}{\lambda} = 2\pi N_d(\lambda,\gamma_R) (\frac{\gamma_R}{\gamma})^2 \qquad (18)$$

$$\alpha = 2\pi(N + N_d(\lambda_R,\gamma)) \frac{\lambda_R}{\lambda} = 2\pi (N + N_d (\lambda,\gamma_R)) (\frac{\gamma_R}{\gamma})^2 \qquad (19)$$

The spontaneous emission spectrum of a perfect N periods undulator can be rewritten :

$$\frac{dI}{d\lambda d\Omega} = \frac{2\pi c}{\lambda^2} \frac{dI}{d\omega d\Omega} \propto \frac{1}{\lambda^4} (\frac{\sin \delta}{\delta})^2 \qquad (20)$$

with

$$\delta = \pi N (1 - \frac{\lambda_R}{\lambda}) = \pi N (1 - \frac{\gamma_R^2}{\gamma^2}) \qquad (21)$$

From Eq. 6, 19, 20 and 21 one is able to draw any emission curve as function of wavelength or energy.

Figure 2 shows $\lambda^4 \frac{dI}{d\lambda d\Omega}$ as a function of $\frac{1}{\lambda}$ around the fundamental resonance wavelength for an O.K. made of two 7 periods undulators (the Orsay case) and a dispersive section having $N_d(\lambda_R,\gamma) = 53$ (*fast oscillating* curve) and $N_d = 0$ (smooth curve). $\frac{dI}{d\lambda d\Omega}$, the experimentally accessible quantity, would be very similar with the lower wavelength secondary maximum higher than the higher wavelength one. Similar curves would be obtained as a function of $\frac{1}{\gamma^2}$ around the resonant energy. The fringes can be understood as the interference between emissions from both undulators, positive (negative) interference occurs when an exact integer (half integer) number of wavelength passes over the electron between the two undulators.

Figure 2 demonstrates the effect of turning on the dispersive section. When $N_d=0$ that is if B(z) = 0 and d = 0 we have the expected result of the emission spectrum of a 2N periods undulator which is narrower by a factor of two than the one of an O.K. made of two N periods undulators.

If B(z) = 0 and Θ = 0 one has from Eq. 12 and 13 $N_d = \frac{d}{2\lambda\gamma^2}$. This corresponds to the dispersivity of the free space. B(z) can therefore be understood as a way of decreasing the dispersive section length keeping the same dispersivity N_d.

Typical length reduction is of the order of 100 for the Orsay O.K. ($N_d \sim 75$ to 100) and much more for the Novosibirsk O.K. ($N_d \sim 450$).

Finally, let's note that so far we looked at the emission from one electron. If a bunch of electrons is injected each with different energies, initial angle and transverse position, the contrast of the measured interference pattern is no longer maximum. I shall discuss this problem in connection with its effect on the gain in section 2.3.

2.2. Low field gain

As discussed earlier, I shall now make use of the "Madey's theorems" to derive the gain as the derivative of the spontaneous emission with respect to energy. The first Madey's theorem (Eq. (1)) only deals with the on axis gain. However the gain and emission only depend on the electron trajectory. The trajectory of an electron injected with a small angle Θ with respect to the undulator axis is equivalent to the trajectory of the same electron injected without angle but rotated by Θ as soon as one neglects the off-axis field variation in the undulators and dispersive section (*) . Therefore, gain in the direction $\vec{n}$, sufficiently close to the undulator axis, is proportional to $\frac{1}{\omega^2} \frac{\partial}{\partial\gamma} \left(\frac{dI}{d\omega d\Omega} (\vec{n}) \right) \equiv \frac{\lambda^4}{(2\pi c)^3} \frac{\partial}{\partial\gamma} \left(\frac{dI}{d\lambda d\Omega}\right)$. I shall not deal with the constant of proportionality, which is essentially the same as for an undulator. Analytical formulas for the gain are given in ref. [5].

Figure 3 shows the derivatives of curves from figure 2. Evidence of the higher O.K. gain appears connected to the presence of the fine structure in the emission curves. When $N_d >> N$, the O.K. peak gain is $.926 \left(1 + \frac{N_d}{N}\right)$ times greater than the one of the 2N periods undulator (or same O.K. with $N_d = 0$). When $N_d/N >> 1$, we have a "strong" Optical Klystron that has much more gain than the undulator. Emission and gain as function of $\frac{1}{\lambda}$ and $\frac{1}{\gamma^2}$ are very close to sinewaves phase shifted by 90° with respect to each other. This simple result is the same as the one obtained from the "impulse" approximation [3, 16] that I discussed in the introduction of section 2. A condition of validity of this approximation is therefore $N_d/N >> 1$. With this approximation, one usually underestimates the O.K. maximum gain by a factor $\frac{N}{N_d}$ ($\sim$ 10 % for the Orsay O.K. and less than 1 % for the Novosibirsk O.K.).

So far I have studied the gain on an incident plane wave, but for FEL operation, one needs to know the gain on a cavity mode. For weakly diverging modes, gain curves as function of energy or wavelength are just found to be wavelength shifted from the corresponding gain curves of a planewave [29].

Inhomogeneous effects on the gain are discussed in the next section.

2.3. Inhomogeneous effects

We know that the emission $\frac{dI}{d\omega d\Omega}$ depends on the electron energy, initial angle and initial transverse position (via field inhomogeneities) .For a bunch of electrons, the measured emission is proportional to $\left\langle\frac{dI}{d\omega d\Omega}\right\rangle$ the average of emission over all the electrons. The resulting curve is usually broader than the emission of only one electron, this effect is called inhomogeneous broadening [30]. For an O.K. one has to average on α (see Eq. 6). In the following I shall assume $N_d >> N$, where the sensitivity of α to the spread in initial parameters is the dominant effect and the small broadening of the envelope can be neglected. With this approximation, one can rewrite Eq. 6 after averaging over all the electrons :

(*) *Such effects are studied in reference [28] .*

$$<\frac{dI}{d\omega d\Omega}\ (\mathrm{O.K.})> \ = \frac{dI}{d\omega d\Omega}\ (1\ \mathrm{undulator})\ \ <1 + \cos\alpha> \tag{22}$$

where

$$\alpha \ = \omega\Delta(t - \frac{\vec{n}\vec{r}}{c}) \tag{23}$$

In the most general case α is distributed around α_m according to some distribution and some spread and one can rewrite (22) :

$$< \frac{dI}{d\omega d\Omega}\ (\mathrm{O.K.}) > \ = \frac{dI}{d\omega d\Omega}\ (1\ \mathrm{undulator})\ (1 + f\cos(\alpha_m + \mu)\ ) \tag{24}$$

where $f < 1$ and μ are function of the shape and spread of the distribution. From Eq. (1) and (2) one has since $N_d/N >> 1$:

$$< \mathrm{Gain}\ (\mathrm{O.K.})> \simeq f \times G_m\ (\mathrm{O.K.}) \tag{25}$$

where G_m is the gain of the electron having $\alpha = \alpha_m$ which fine structure is shifted by μ.

Inhomogeneous effects decrease the amplitude of modulation of the spontaneous emission and merely decrease the gain by the same factor f. In the appendix I show that the bunching created by the dispersive section is also decreased by f. We shall see in section 2.3.4 that an optical klystron optimized for the gain operates in a regime where $f < 1$. Control of the inhomogeneous broadening can be accomplished with the aid of a simple diagnostic : the modulation rate of the spontaneous emission.

In the following, I shall calculate f and μ due to energy spread, angular spread and beam transverse dimensions. I shall assume all these contributions (labelled "i") to be independent of each other, which implies :

$$f = \prod_i f_i \tag{26}$$

$$\mu = \sum_i \mu_i \tag{27}$$

I shall also assume the electron to be Gaussian distributed in energy, transverse position and angle as theoretically predicted for low current electron beams in storage rings /̲ 31 7̲. Finally I will calculate f around the fundamental resonances. The calculations will be presented in detail since the emission spectrum of the klystron is also a valuable diagnostic tool on beam quality in storage rings.

2.3.1. Energy spread

From Eq. 12 one has :

$$\alpha = 2\ \pi(N + N_d(\lambda,\gamma_R))\ \frac{\gamma_R{}^2}{\gamma^2} \tag{28}$$

Let's assume a gaussian energy spread around γ_m with relative RMS spread $\sigma_\gamma/\gamma <<1$ we find

$$f = e^{-\ \sigma^2/2} \tag{29}$$

$$\mu = 0 \tag{30}$$

with

$$\sigma = 4\ \pi(N + N_d\ (\lambda,\gamma_R)\ )\ \sigma_\gamma/\gamma\ \ \frac{\gamma_R{}^2}{\gamma_m{}^2} \tag{31}$$

as a consequence the modulation rate varies inside the fundamental resonance according to :

$$\exp\left[- \frac{\mathrm{constant}}{\lambda\gamma_m^2}\right] \tag{32}$$

In storage rings, one can have a non gaussian energy spread due to various oscillations of the electron bunch. Let's consider the simplest case of coherent motion (phase oscillation) :

$$\gamma(t) = \gamma_m + \delta\gamma \cos \omega t \tag{33}$$

where t is the time and ω the frequency of the motion.

Then :

$$f = J_o\left(4\pi(N + N_d) \frac{\delta\gamma}{\gamma_m} \right) \tag{34}$$

where J_o stands for the zeroth order Bessel function.

If $4\pi(N + N_d) \frac{\delta\gamma}{\gamma_m} << 1$

$$f \simeq 1 - \frac{\sigma^2}{2} \simeq e^{-\sigma^2/2} \tag{35}$$

with :

$$\sigma = 4\pi(N + N_d) \left(\frac{\sigma_\gamma}{\gamma} \right)_{coh} \tag{36}$$

where $\left(\frac{\sigma_\gamma}{\gamma}\right)_{coh} = \frac{1}{\sqrt{2}} \frac{\delta\gamma}{\gamma_m}$ is the coherent energy spread.

The modulation rate is a direct measurement of the beam energy spread.

2.3.2. Angular spread

From Eq. (11) and (12) one has :

$$\alpha = cste + \pi \left(\frac{L + d}{\lambda}\right) \Theta^2 \tag{37}$$

Let's assume the two dimensional angular distribution to be gaussian with RMS spread $\sigma_{\Theta 1}$ and $\sigma_{\Theta 2}$. The observation is made in the direction (Θ_1, Θ_2) with respect to electron trajectory axis. Axis 1 and 2 are orthogonal and chosen to factorize the density. Let's also assume these distributions to be independent of the longitudinal coordinate z which is equivalent to the requirement that the betatron functions are much larger than L + d [31]. Then, one has :

$$f = f_1 \times f_2 \tag{38}$$

$$\mu = \mu_1 + \mu_2 \tag{39}$$

with :

$$f_i = (1 + 2\sigma_i^2)^{-\frac{1}{4}} \exp\left[-\frac{\Theta_i^2}{2\sigma_{\Theta i}^2} \times \frac{2\sigma_i^2}{1 + 2\sigma_i^2} \right] \tag{40}$$

$$\mu_i = \frac{1}{2} \operatorname{tg}^{-1}(\sqrt{2}\,\sigma_i) + \frac{\Theta_i^2}{\sigma_{\Theta i}^2} \times \frac{2\sqrt{2}\,\sigma_i}{1 + 2\sigma_i^2} \tag{41}$$

$$\sigma_i = \frac{\pi(d+L)}{\lambda} \sqrt{2}\; \sigma_{\Theta i}^2 \tag{42}$$

From Eq. 40 one deduces that the modulation decreases with σ_Θ and can be further seriously reduced if one looks in a direction $\Theta_1 \neq 0$.

Let's consider the case $\Theta_i = 0$, then the gain reduction f_i reads :

$$f_i = (1 + 2\sigma_i^2)^{-1/4} \tag{43}$$

Note that this reduction is of the same order of magnitude as for an undulator of same length [30]. Therefore the assumption that the envelope of the fine structure was not broadened is no longer valid if $\sigma_i \gtrsim 1$. Moreover, the Θ^2 distribution responsible of the modulation extends non symmetrically to several periods of the fine structure, the modulation rate will depend on whether the envelope is increasing or decreasing with λ. The overall conclusion being that the modulation rate will be higher on the lower wavelength tail than on the higher wavelength tail of an emission spectrum. This effect is reversed of the one predicted for energy spread (Eq. 32) and should be very obvious when angular spread plays a dominant role, it could be a way to distinguish those two effects. This non symmetry in the modulation is a pure consequence of the non symmetry of the Θ^2 distribution and breaks down if one looks in a direction $\Theta_I \gtrsim \sigma_{\Theta I}$. However, for $\sigma_i >> 1$, Eq. 40 to 42 are still valid around the wavelength of the maximum of the envelope, and for $\sigma_i << 1$ they apply to any wavelength.

In practical situations on storage rings, one has $\sigma_i << 1$, Eq. 40 can be rewritten in a similar form as Eq. (29) :

$$f_i \simeq (1 - \frac{\sigma_i^2}{2}) \exp (- \frac{\Theta_i^2}{\sigma_{\Theta i}^2} \sigma_i^2) \simeq \exp \left[- \frac{\sigma_i^2}{2} (1 + 2 \frac{\Theta_i^2}{\sigma_{\Theta i}^2}) \right] \tag{44}$$

This last result can be used to measure the angular spread of the beam by simply measuring the modulation rate on the spontaneous emission off-axis from the electron beam. Although Eq. 44 was established for a gaussian Θ distribution and large betatron functions, a straightforward generalisation can be made to measure the lowest order moments of the angular distribution dropping these assumptions.

Finally let's note that σ_i is proportional to L + d, a non magnetic dispersive section would have a much longer length d and would be much more sensitive to angular spread.

2.3.3. Beam transverse dimensions

Because of the Poisson's equation, the magnetic field in the dispersive section is not uniform and electrons with different initial positions see different fields. It has been studied in ref. /5/. One make a quadratic expansion of the vertical (also referred as the y-component) field of the dispersive section around the axis :

$$B_y(z) = B_o(z) + x^2 B_{xx}(z) + y^2 B_{yy}(z) \tag{45}$$

where x denotes the horizontal direction.

Let's assume the betatron functions much larger than the dispersive section length in such a way that an off-axis injected electron will always stay off-axis in the dispersive section.

From Eq. 45 and 12, one has /5/ :

$$\alpha = \alpha_o (1 + 2 Q_x x^2 + 2 Q_y y^2) \tag{46}$$

where $\alpha_o = \alpha(x = 0, y = 0)$

and

$$Q_x = \frac{\int_{-\infty}^{+\infty} \left[\int_{-\infty}^{u} B_o(z)\, dz\right] \left[\int_{-\infty}^{u} B_{xx}(z)\, dz\right] du}{\int_{-\infty}^{+\infty} \left[\int_{-\infty}^{u} B_o(z)\, dz\right]^2 du} \tag{47}$$

Q_y is obtained from Eq. 47 if one change B_{xx} into B_{yy}.

Eq. 46 is very similar to Eq. 37. Making the assumption that x and y are independently gaussian distributed around x_o and y_o with RMS spreads σ_x and σ_y , one can calculate a similar gain reduction factor as the one given in Eq. 40 which, in the case of low gain reduction reduces to :

$$f \simeq 1 - \sigma^2/2 \simeq e^{-\sigma^2/2} \qquad (48)$$

with

$$\sigma^2 = 16\,\pi^2\,(N + N_d(\lambda,\gamma))^2 \left[2\,Q_x^2\,\sigma_x^4\,(1 + 2\,\frac{x_o^2}{\sigma_x^2}) + 2\,Q_y^2\,\sigma_y^4\,(1 + 2\,\frac{y_o^2}{\sigma_y^2}) \right] \qquad (49)$$

Note that in a free space dispersive section, $Q_x = Q_y = 0$ and inhomogeneous broadening is only due to the undulator gradients. Using magnets sufficiently large (in the x dimension) one usually makes $B_{xx} << B_{yy}$ but B_{yy} cannot be decreased to zero because B(z) has to satisfy Poisson's Equation. Therefore for such a dispersive section, the most important broadening comes from the vertical dimensions of the beam.

2.3.4. O.K. maximum gain

From sections 2.2 and 2.3 one can write the maximum O.K. gain averaged over the electrons :

$$\langle G\ max\ (O.K.)\rangle \simeq .926\ f\ (1 + \frac{N_d}{N})\ x\ G_{max}\ \text{(2 N periods undulator)} \qquad (50)$$

Eq. 50 being valid if $N_d/N >> 1$, N_d being calculated at resonance.

However we have seen in section 2.3 that angular spread and transverse beam dimensions contributions to f depends on $N + N_d$ which is not true for angular spread contributions. From Eq. 29, 31, 35, 36, 48 and 49, one can write :

$$f = \exp\left[- Cste\ (4\,\pi (N + N_d)\,\frac{\sigma_\gamma}{\gamma_{eq.}})^2 \right] \qquad (51)$$

where $\frac{\sigma_\gamma}{\gamma_{eq.}}$ is an "equivalent" energy spread usually very close to real energy spread since beam transverse dimensions don't contribute very much. I shall not distinguish them in the remaining of this section.

From Eq. 50 and 51, one concludes that there exists an optimal N_d that maximizes the O.K. gain, namely :

$$N_d = \frac{1}{4\,\pi\,\frac{\sigma_\gamma}{\gamma}} - N \simeq \frac{1}{4\,\pi\,\frac{\sigma_\gamma}{\gamma}} \qquad (\text{when } N_d >> N) \qquad (52)$$

with

$$< G_{max}\ (O.K.) >_{optimum} \simeq \frac{.045}{N\,\frac{\sigma_\gamma}{\gamma}}\ G_{max}\ \text{(2N period undulator)} \qquad (53)$$

An optimum O.K. operates with $f \simeq .6 \times f'$ where $f' < 1$ accounts for angular spread and other small effects I shall discuss in the next section. It is therefore important to experimentally control f and to know how much of it comes from energy spread.

Let's note that all three contributions were seen to be approximated by :

$$f = \exp\,(-\frac{1}{2}\ \sum_i\ \sigma_i^2) \qquad (54)$$

It is seen from Eq. 31, 42 and 49 that all σ_i are proportional to $1/\lambda$. Therefore, $f = \exp\,(-\frac{cste}{\lambda^2})$ decreases with decreasing wavelength.

2.3.5. Other effects decreasing the measured modulation rate

So far I discussed the electron beam effects on the modulation rate, let's call f_e their contribution to f. Their also exists what I call field effects, refered as f_f and monochromator effects refered as f_m.

The final measured f being :

$$f = f_e \, f_f \, f_m \tag{55}$$

Field effects give a non total modulation on a one electron emission spectrum analyzed by a perfect monochromator. It is due to :

- non identical undulators because of field imperfections
- dispersive section emission (which was neglected from the beginning)
- imperfect dispersive section compensation /¯5_7 .

Monochromator effects are due to :

- the wavelength bandwith of the monochromator used to measure the emission

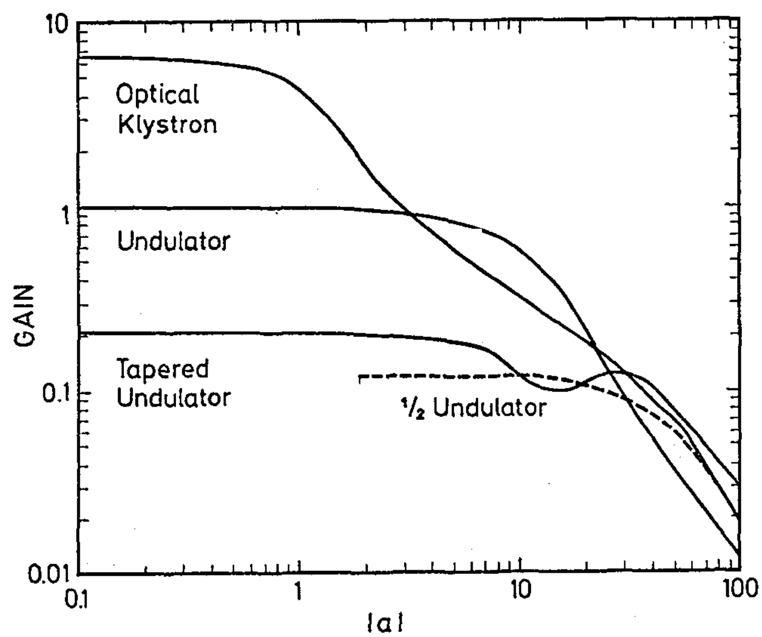

Figure 4 Maximum gain in relative units, as a function of $|a|$ the generalized dimensionless laser amplitude for an optical klystron with D = 3, a regular undulator and a tapered undulator with $\delta = 10\pi$ (respectively), all three having the same total length

- the angular aperture of the monochromator which decreases f just like an angular spread
- error of the monochromator positioning. Eq. 5 is only valid when looking at infinity. Looking at a finite distance would make the two undulators contributions in the interference pattern non equivalent.

f_m can be easily measured experimentally. f_f is fixed by construction of the O.K. and is usually close to 1 (> .98 for the Orsay O.K.).

2.4. High field gain

In this section, I shall make use of results established by W.B. Colson [32] on the Pendulum Equations. A reader not familiar with these results and notations should refer first to the reference.

For an undulator, these equations read :

$$\frac{d\zeta}{d\tau} = \nu \qquad (56)$$

$$\frac{d\nu}{d\tau} = |a| \cos \zeta \qquad (57)$$

where ζ is the generalized electron phase initially uniformly distributed between 0 and 2π. ν is called the resonance parameter. $|a|$ is the dimensionless square root of the laser intensity. τ is the dimensionless time varying between 0 and 1.

In the low gain case, one can neglect the change of $|a|$ along the undulator and the gain reads :

$$G \propto \frac{\langle\delta\nu\rangle_{\zeta_o}}{|a|^2} \qquad (58)$$

where $\langle\delta\nu\rangle_{\zeta_o}$ denotes the variation of the resonance parameter in the undulator averaged over the initial phase. Eq. 56, 57 and 58 allow the calculation of the gain at any power. Transition from low to high power occurs when $|a| \sim 1$.

Similar equations are true for the Optical Klystron, one just has to add [29] :

$$\Delta\zeta = D\nu \qquad \text{at } \tau = \frac{1}{2} \qquad (59)$$

where $D = \lambda_o N_d$ / O.K. length.

Eq. 59 simply means there is an instantaneous phase shift due to the dispersive section, this shift being proportional to the resonance parameter.

Figure 4 shows the maximum gain as function of the field $|a|$ for an O.K. with D = 3, an undulator and a tapered undulator. All devices have a same total length, period and K factor and gain is maximized over the initial resonance parameter. These curves are obtained by integration of Eq. 56, 57, 58 and 59. For the tapered undulator one adds $\delta \approx 2\pi N \,\Delta\lambda_o/\lambda_o$ at the right hand side of Eq. 57 [33] . $\Delta\lambda_o$ is the period variation in the tapered undulator. $\delta = 10\pi$ has been used in figure 4.

At low field, figure 4 clearly shows the higher gain of the O.K. compared to the undulator and tapered undulator. However saturation occurs earlier, and for $|a| \gtrsim 3$ the O.K. gain is lower than the undulator gain. At very high power, the O.K. gain tends to the one of a half undulator, everything happens as if only the first undulator contributes to the gain, and the remaining undulator and dispersive section does not add anything. This clearly appears in the shape of the gain curve as a function of initial resonance parameter where the fine structure vanishes [34].

At very high power, $|a| \gtrsim 22$, the O.K. gain surprisingly becomes higher than the undulator gain. However, in this range, one can design a tapered undulator that has a higher gain. One conclusion is that the O.K. is not useful for FEL operation on Linear Accelerators (LINAC) since for a given high power one can always achieve a higher gain using a tapered undulator. However, it is known that if the tapering parameter δ is big, one can have tapered undulators with a higher high field gain than low field gain [35]. Starting such an FEL could be a problem if the threshold gain is above the low field gain. In that case, improvement could be achieved by inserting a dispersive section somewhere in the tapered undulator that would keep the high field gain and efficiency and enhance the low field gain [35]. Such an hybrid undulator is sometime called a "multicomponent wiggler".

On storage rings, space is usually limited and FEL saturation is theoretically expected at less power than for a LINAC in a region where $|a| \ll 1$ [36 to 40]. Therefore,

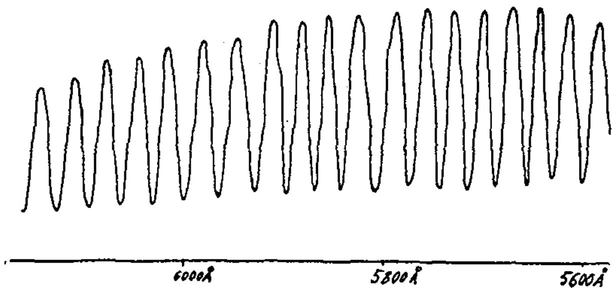

Figure 5 Novosibirsk OK-1 Spontaneous emission spectrum

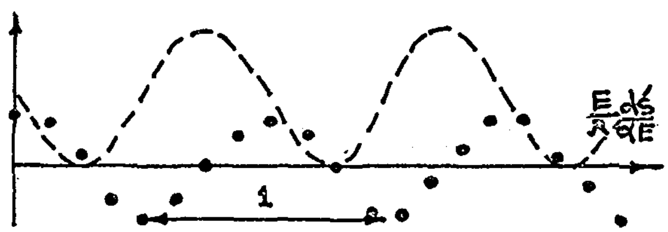

Figure 6 Novosibirsk OK-1 gain as function of energy (dotted curve) compared to the spontaneous emission (dash curve)

efficient use of an O.K. can be made to improve the usual low gain. An O.K. is useful to reach the oscillation threshold.

An other question arises : would an optical klystron storage ring free electron laser (OKSRFEL) have a higher power than a regular SRFEL (made with an undulator) ? Existing theories [36 to 40] have not yet been experimentally confirmed. So far one identified two processes of saturation, they both start from the energy spread induced at first order of the laser field and stored spontaneous emission field [41], sometimes called beam heating. This energy spread directly reduces the gain by inhomogeneous broadening of the gain curve (this is the first process). On storage rings, energy spread increase is followed by bunch lengthening [42] (although the "anomalous bunch lengthening" complicates this scheme). This bunch lenghtening decreases the peak electron density and peak gain (it is the second process). From the first Madey's theorem (Eq. 1), the electron beam heating is proportional to the spontaneous emission and is therefore of the same order of magnitude for an O.K. as for an undulator.

So far, all the undulators designed to be used for a SRFEL have too low a gain and not enough periods number to saturate by direct inhomogeneous broadening (first process) and are more likely going to saturate by bunch lengthening (second process).

An optimized O.K. as discussed in section 2.3.4. would not admit any energy spread increase and would saturate by the first process. However, an O.K. standing somewhere between the undulator and the optimized O.K. (optimized for low field gain) saturating by bunch lengthening would have a higher gain than the corresponding undulator of the same length and should give a higher FEL power.

In other words, starting from a regular SRFEL saturating by bunch lengthening, one should be able to achieve a higher power OKSRFEL, the optimized Nd being lower than the one optimizing the very low field gain.

3. EXPERIMENTAL INVESTIGATIONS

Experiments on Optical Klystrons are under way at Novosibirsk (USSR) and Orsay (France). Both of them are using relativistic electrons from a storage ring. Their purpose is to build a visible FEL.

3.1. Novosibirsk

Two optical klystrons, OK-1 and OK-2 have been built so far. OK-2 is an improved version of OK-1. Their characteristics are the following [3, 43] :

	OK-1	OK-2
Undulator period (cm)	10	6.5
Number of periods	2 x 3	2 x 4.5
Dispersive section length (cm)	34	34
Maximum magnetic field in the dispersive section (kG)............................	5.7	11

The electron beam from VEPP-3 storage ring has the following typical characteristics [43]:

Energy	370 MeV
Relative energy spread	$1.5\ 10^{-4}$
Horizontal angular divergence	.05 mrad
Vertical angular divergence	.04 mrad

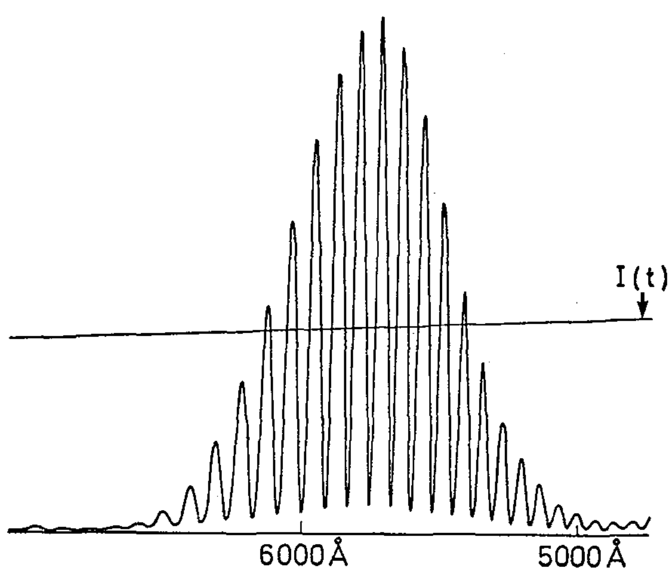

Figure 7 Spontaneous emission $dI/d\lambda d\Omega$ of the Orsay optical klystron measured for an electron energy of 240 MeV and a magnetic gap of 37.79 mm. The current decay I(t) is superimposed.

Figure 5 gives OK-1 spontaneous emission spectrum. The fine structure has been observèd to have a 34 Å period at 6000 Å giving Nd $\sim$ 180 at this wavelength. OK-2 is said to have a smaller fine structure period corresponding to Nd $\sim$ 450 at the s me wavelength [44]. Using results from section 2.3, one calculates the following inhomogeneous broadening parameter for OK-2.

f = 0.70 for energy spread
f = .999 for angular divergence.

Neglecting inhomogeneous effects, OK-2 has 40 times more gain than an undulator occupying the same total length.

OK-1 gain has been measured [43] as a function of energy at λ = 6328 Å and was found to be approximately sinusoïdal as expected since $N_d/N \sim 60 >> 1$ (see figure 6). The gain was also measured to be approximately 90° phase shifted from spontaneous emission as expected from Madey's theorem.

3.2. Orsay

An O.K. has been installed on the ACO storage ring. It has the following characteristics [45] :

undulator period	8 cm
number of periods	2 x 7
dispersive section length ..	32 cm
maximum dispersive section magnetic field	5.3 kG

The electron beam from the ACO storage ring has the following typical characteristics :

Energy	240 MeV	
Relative energy spread .... from	$2\ 10^{-4}$ (low current)	
to	$1.5\ 10^{-3}$ (high current)	
Horizontal angular divergence	$\sim$. 2mrad	at low current
Vertical angular divergence	$\sim$. 1mrad	at low current
Horizontal and vertical transverse sizes	$\sim$. 35mm	at low current

The first experimental results are discussed in [45]. Figure 7 shows the observed emission spectrum on the fundamental resonance at low current in the ring. The envelope of oscillations has the FWHM of a perfect 8.1 periods undulator instead of 7. This small discrepancy is probably due to the dispersive section field which could be partly resonant with the undulators. Dependence of Nd as $1/\lambda$ has been checked from spontaneous emission. A value of N_d = 64 was found at λ = 6328 Å. No precise gain measurement has yet been made on the optical klystron. However, from the emission curves, one can expect a maximum gain enhancement of 5.2 x f compared with the 17 periods undulator. For a perfectly injected electron beam, transverse beam dimensions contributions were found to be negligible. Experimentally confirmed contribution from relative energy spread and angular spread are :

$f = \exp\left(-\frac{\sigma^2}{2}\right)$ with $\sigma = 890\ \frac{\sigma_\gamma}{\gamma}$ (for energy spread) and f = .97 (angular spread).

4. OPTICAL KLYSTRON AS A SOURCE OF COHERENT SYNCHROTRON RADIATION

4.1. Principle

Eq. 5 shows the dependence of the radialid energy by an accelerated electron as the square of its charge. For an electron bunch, one has to sum on individual electrons inside the integral of Eq. 5 [27]. Therefore $\frac{dI}{d\omega d\Omega}$ should have a contribution proportional to $\mathcal{N}^2$ the square number of electrons. When calculating the synchrotron radiation emitted by electrons in a storage ring, one usually considers the electrons to be randomly positionned with respect to each other, in that case cross terms in the square brackets of Eq. 5 vanish and the emitted power is proportional to $\mathcal{N}$. However, it is known by storage ring physicists that for a wavelength larger than the bunch dimensions (of the order of several centimeters), the emitted power can be split into one contribution proportional to $\mathcal{N}$ called the incoherent part and one proportional to $\mathcal{N}^2$ called the coherent part which can be much larger than the incoherent part. Applying this idea in the optical domain of wavelength, Csonka [6] showed that one can have a significant coherent part with an electron beam modulated at an optical wavelength, and proposed the use of a laser sent into an undulator to create this modulation [9]. Optimization of the beam modulation then naturally leads to the use of an optical klystron in place of the undulator.

This $\mathcal{N}^2$ term is usually refered as coherent synchrotron radiation, even though that coherence is a priori loosely connected to the spatial and temporal coherence of a laser.

Note that in this scheme, one absolutely need a laser or a FEL to interact with the electrons in the first undulator and produce the bunching.

4.2. Properties of the coherent synchrotron radiation

Let's start with a periodically modulated beam with λ_1 the fundamental wavelength. Then the coherent emission spectrum consists of sharp peaks at each harmonic n of the fundamental wavelength. If one use a laser with a temporal coherence longer than the electron bunch length, one can relate the coherent and incoherent emission per solid angle and per frequency in the forward direction at harmonic n by [11, 46] :

$$\frac{dI}{d\omega d\Omega} \text{ coh } (\Theta=0) = \frac{dI}{d\omega d\Omega} \text{ incoh } (\Theta=0) \quad \frac{\rho_n \lambda_1^2}{\mathcal{N}} \tag{60}$$

where Θ is the angle between the observation direction and the bunching axis, $\mathcal{N}$ is the total number of electrons in the bunch and ρ_n is given by the following expansion of the total longitudinal density ρ (expressed in number of electrons per unit length) :

$$\rho = \sum_{n=0}^{\infty} \rho_n \cos \left(2\pi n \frac{z}{\lambda_1} + \phi_m\right) \tag{61}$$

Typically $\frac{\rho_n}{\rho_o} << 1$, therefore one only has significant coherent radiation for a large number of electrons in the bunch. The relative width of these peaks is just the ratio of their wavelength over the total electron bunch length. If one uses a laser which coherence length is less than the electron bunch length by a factor α , then the relative width is broadened by the same factor α. Assuming a perfect time overlap between laser and electron beam, the coherent emission would then be reduced by a factor α. For $\Theta \neq 0$, coherent emission decreases down to zero for typical angles of $\Theta_x \sim \lambda/\sigma_x$ and $\Theta_y \sim \lambda/\sigma_y$ where σ_x and σ_y denote the transverse beam dimensions.

4.3. Coherent synchrotron radiation from an optical klystron

As we have seen above, the coherent emission depends on the square of the amplitude of the bunching. Therefore, the main advantage of the O.K. over the undulator is of requiring less power from the external laser, improving the efficiency. From the Appendix, one can show that $(\frac{Nd}{N} f)^2$ less power is needed to create a bunching at wavelength λ with a N periods undulator followed by a dispersive section than for the same undulator alone. For a given laser power, the coherent emission ratio also grows like $(\frac{Nd}{N} f)^2$.

An other advantage is the use of a second undulator resonant on an harmonic of the fundamental wavelength of the laser, and use the bunching rich in harmonics created by a powerful external laser to realize a frequency upconverter [19, 47] and reach the UV and VUV range.

The short wavelength efficiency of a frequency upconverter will be limited by inhomogeneous effects which decreases the bunching by an experimentally confirmed [45] factor $f \sim \exp(-\text{cste}/\lambda^2)$ (see section 2.3).

A proposal has been made to realize an O.K. out of the LELA undulator mounted on the ADONE storage ring in Frascati [19] . The external laser is a pulsed YAG laser at the second harmonic of .53 μ. For a laser power of 10 MW/mm^2, a peak power of ∿ 1kW is expected at λ = .17 μ (third harmonic).

5. CONCLUSION

The Optical Klystron can be viewed as a modified undulator which has a higher gain but is more sensitive to inhomogeneous effects. It makes full use of the natural low energy spread and limited straigthsection length of storage rings.

The emission spectrum is easy to measure with a very good signal to noise ratio [48, 45] and presents a particular interference pattern that makes it a diagnostic tool for storage rings. The high sensitivity of spectra to energy makes it a suitable tool to measure any quantity connected to energy (energy spread, momentum compaction, energy calibration). Off-axis spectra give a measure of angular spread.

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APPENDIX

Inhomogeneous effects on the bunching in an Optical Klystron

Let's consider an electron entering the first undulator with initial phase ϕ with respect to an incident plane wave. Let's call ψ its phase with respect to the same plane wave when leaving the dispersive section.

From equations (7) to (9) one has :

$$\psi = \phi + \alpha \tag{A1}$$

However α depends on the electron energy which is modified in the undulator after interaction with the planewave. Since α is proportional to $1/\gamma^2$ (see eq. 12), assuming $N_d/N \gg 1$, one can expand α in power of ε the plane wave electric field :

$$\alpha = \alpha_o\,(1 + \beta\varepsilon\cos\phi\ \ldots) \tag{A2}$$

where $\alpha_o = \alpha$ in the absence of the plane wave and β is a constant. The $\beta\varepsilon\cos\phi$ term just means that the electron is accelerated or decelerated according to the cosine of its initial phase and proportionaly to the plane wave field.

Inverting A_1 and A_2 for a constant α_o and $\beta\varepsilon \ll 1$ (low field) one has :

$$\phi \simeq \psi - \alpha_o - \beta\alpha_o\,\varepsilon\cos(\psi - \alpha_o) \tag{A3}$$

Assuming a uniformly distributed initial phase ϕ , one can calculate $g(\psi)$ the phase distribution of electrons at the exit of the dispersive section [49] :

$$g(\psi) = \frac{1}{2\pi}\,\frac{d\phi}{d\psi} = \frac{1}{2\pi}(1 + \beta\alpha_o\varepsilon\,\sin(\psi - \alpha_o)\,) \tag{A4}$$

In the general case, one has to convolve $g(\psi)$ with the distribution of α_o giving the final density :

$$G(\psi) \simeq \frac{1}{2\pi}\,(1 + f\,\beta\alpha_m\varepsilon\ \sin(\psi - \alpha_m - \mu)\,) \tag{A5}$$

where f, α_m and μ are defined in the introduction of section 2.3.

From A5 one concludes that the bunching at the entrance of the second undulator is proportional to ε(the square root of the planewave power) $\alpha_m \sim N_d$ (the strength of the dispersive section) and f (the inhomogeneous reduction factor).

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