

## Optical stochastic cooling for RHIC using optical parametric amplification

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(Received 17 June 2003; published 5 January 2004)

We propose using an optical parametric amplifier, with a  $\sim 12 \mu\text{m}$  wavelength, for optical-stochastic cooling of  $^{79}\text{Au}$  ions in the Relativistic Heavy Ion Collider. While the bandwidth of this amplifier is comparable to that of a Ti:sapphire laser, it has a higher average output power. Its wavelength is longer than that of the laser amplifiers previously considered for such an application. This longer wavelength permits a longer undulator period and higher magnetic field, thereby generating a larger signal from the pickup undulator and ensuring a more efficient interaction in the kicker undulator, both being essential elements in cooling moderately relativistic ions. The transition to a longer wavelength also relaxes the requirements for stability of the path length during ion-beam transport between pickup and kicker undulators.

DOI: 10.1103/PhysRevSTAB.7.012801

PACS numbers: 41.75.Ak, 42.65.Lm

### I. INTRODUCTION

Electron cooling was proposed as a mechanism to counteract the growth of emittance and beam loss due to intrabeam scattering (IBS) in the Relativistic Heavy Ion Collider (RHIC) [1]. In this paper, we present a feasibility study of another potential cooling method, optical-stochastic cooling (OSC). It may be used as a stand-alone technique, or to complement electron cooling, acting mainly on halo particles for which electron cooling approach is less efficient.

OSC [2] and its transit-time method [3] were suggested to extend the stochastic cooling technique [4] into the optical domain, with a broad-band optical amplifier and undulators (wigglers) for coupling the optical radiation to the charged-particle beam. Cooling results from a particle's interaction in the kicker undulator with its own amplified radiation, emitted in the pickup undulator. The path of the particles between the pickup and kicker (called a bypass) can be designed such that each particle receives a correction kick from its own amplified radiation toward equilibrium orbit and energy. The interaction of a particle with amplified radiation from other particles results in heating. It was shown in [5] that the balance between cooling and heating define the optimal power of the amplifier needed to achieve the ultimate cooling rate that is limited only by the bandwidth of the cooling loop, pickup-amplifier-kicker. However, in all possible applications of OSC to heavy particles, including  $^{79}\text{Au}$  ions in the RHIC, the power required in such a system appears to be several orders of magnitude larger than that feasible with modern optical amplifiers [6]. In this case, the amplifier's power limits the cooling rate.

### II. DAMPING TIME

The theory of OSC suggests that nonzero dispersion in the vicinity of the pickup and kicker undulators couples

the energy kick to the particles' transverse motion, and thus provides both transverse cooling and longitudinal cooling [3]. Many thousand passes through the cooling system, pickup-amplifier-kicker, are required to significantly change the particles' energy and/or betatron amplitude. The damping time, expressed in the number of active passes through the cooling system, can be estimated by the equation (see the Appendix)

$$\frac{1}{n_{x,\varepsilon}^2} = \frac{16}{e} \frac{P}{I} \frac{\delta E}{\Delta E} \frac{\Delta E}{q} N_u \Gamma f_{x,\varepsilon}^2, \quad (1)$$

where  $P$  is the average amplifier power,  $I$  is the average beam current,  $e = 2.718$ ,  $q$  is the particle charge,  $\Delta E$  is the beam's energy spread (rms),  $N_u$  is the number of undulator periods,  $\Gamma = \Delta\omega/\omega$  is the relative bandwidth of the amplifier ( $N_u\Gamma < 1$  is assumed), and,  $f_x$  and  $f_\varepsilon$  are, respectively, the damping partition numbers for transverse and longitudinal oscillations ( $f_{x,\varepsilon} = 1/2$  for a case of equal damping).  $\delta E$  is the energy radiated by the particle in the pickup undulator into the fundamental mode defined by

$$\delta E = 4.12 q^2 k \frac{K^2}{2 + K^2} \left[ J_0 \left( \frac{1}{2} \frac{K^2}{2 + K^2} \right) - J_1 \left( \frac{1}{2} \frac{K^2}{2 + K^2} \right) \right]^2, \quad (2)$$

where  $J_0$  and  $J_1$  are zero- and first-order Bessel functions, respectively,  $K = \frac{qB\lambda_u}{2\pi Mc^2}$  is the undulator parameter,  $B$  is the undulator's peak magnetic field,  $\lambda_u$  is its period,  $M$  is the particles' mass,  $k = 2\pi/\lambda$  is the wave number,  $\lambda = \frac{\lambda_u}{2\gamma^2} (1 + \frac{K^2}{2})$  is the wavelength of the undulator radiation, and  $\gamma$  is the particles' relativistic factor.

For moderately relativistic protons and heavy ions  $\delta E$  in Eq. (2) always scales proportionally to  $\lambda$  when  $K \ll 1$ . Then,  $n_{x,\varepsilon} \sim \lambda^{-1/2}$  by Eq. (1). Thus, for a given product  $PN_u\Gamma$ , cooling of  $^{79}\text{Au}$  ions in RHIC will be faster using a broad-band amplifier with the longest possible

wavelength. However, practical considerations (e.g., the undulators period and length, and the availability of pump sources) limit the long wavelength to the mid-infrared region.

### III. AMPLIFIER

OSC requires a high-gain wide-band amplifier. However, it is also important to minimize the time a signal takes to pass from a pickup to a kicker undulator. This requirement favors an amplifier with a single pass through the gain medium (compared to one where the light signal is bounced back and forth through the gain medium). Modern optical parametric amplifiers (OPAs) seem to have all these features [7] and possess another attractive one; namely, they do not need energy to be stored into the gain medium (typically a crystal) unlike other optical amplifiers that are based on population inversion in the gain medium.

Optical parametric amplification can be thought of as a nonlinear process in which a short wavelength photon from a narrow-band high-power pump laser is converted into two photons of longer wavelength known as a signal and an idler [8]. The input signal, properly polarized, may be introduced either at the idler or signal wavelength. It enhances conversion through stimulating emission, and thus, gets amplified. The amplification gain length (typically few mm) is limited by the damage thresholds of a given optical power density. In what follows, we give specific details for the OPA considered for use in OSC.

The approach we chose relies on the high average power available from gas lasers to provide a pump source in the mid-IR [9]. Working at the fundamental wavelength of a CO<sub>2</sub> laser near 10.6  $\mu\text{m}$ , several kW is attainable in continuous wave operation [9]. Active mode locking, using intracavity electro-optical CdTe modulators [10], generates MHz repetition rate, nanosecond pulse trains with high efficiency [11] whose frequency and phase can be synchronized to that of the signal optical beam. The output frequency of the laser then is doubled in a nonlinear crystal, e.g., AgGaSe [12] or GaAs [13], to reach a 5.3- $\mu\text{m}$  wavelength at conversion efficiency of at least 10%. An attractive alternative is to use the direct output of a CO laser operating at 5.3–5.6  $\mu\text{m}$  [14]. This beam then acts as the pump source for an OPA, ensuring the necessary gain for a radiator signal near 12  $\mu\text{m}$ . The generated idler beam, at approximately 9.6  $\mu\text{m}$ , is dumped. Another way is to seed the idler, and amplify the 9.6- $\mu\text{m}$  wavelength. Figure 1 is a scheme of this configuration.

There are several reasons for adopting this approach. First, the signal's wavelength at 12  $\mu\text{m}$  is favorable because some highly nonlinear materials are transparent both at this wavelength and the pump's wavelength. Operating near degeneracy, the condition under which the signal and idler wavelengths are equal, minimizes the requirements for transparency at three widely

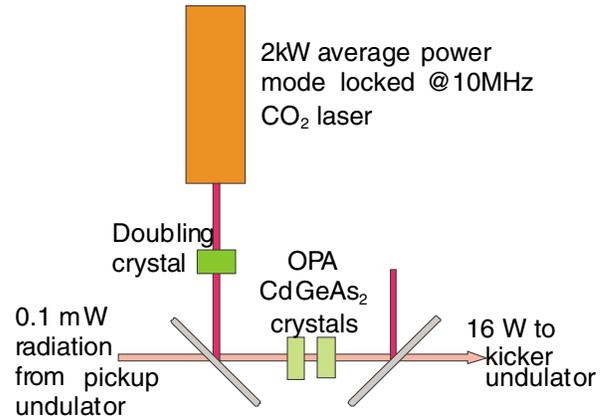


FIG. 1. (Color) Schematic diagram of the proposed optical amplifier.

separated wavelengths; also, the bandwidth of the parametric process can be enhanced near degeneracy.

Evaluating appropriate nonlinear materials involves varying several important parameters to achieve the largest gain and bandwidth. Specifically, the crystal orientation and geometry, the pump power and polarization, the optical mode sizes and divergences, and beam propagation angles will all affect the total gain achievable for a given crystal material. Different materials must be evaluated independently, as the set of parameters that produces the highest gain and bandwidth is different for each material. Practical considerations further limit the selection of materials since the average power is very high, and therefore only well-developed inorganic crystals can be considered. This is because when the optical power reaches a level at which the absorbed fraction is high enough to cause significant localized heating, effects of temperature dependence of refractive index, thermal expansion, stress-induced birefringence, and irreversible material damage can, in general, no longer be neglected. Although optical configurations and heat transport methods can be engineered to compensate for these effects, they add significant complexity and cost. Therefore, we seek materials in which these effects are minimized. These materials are the ones that have been extensively studied and developed for other high average power applications. The overall gain required to amplify the radiator's output from 0.1 mW average power to a desired level of 16 W for the modulator is large enough to eliminate almost all materials except those with the highest nonlinear coefficients. We used cadmium germanium arsenide (CdGeAs<sub>2</sub>), a positive uniaxial crystal with transparency ranging from 2.4–18  $\mu\text{m}$ . It can be phase matched over all wavelengths of interest, and has an extremely large nonlinear coefficient of 236 pm/V. The most advantageous configuration is type I phase matching at an angle of 33.8° to the optical axis, yielding both a high effective nonlinearity and large spectral bandwidth [15].

TABLE I. Summary of optical configurations.

Case	Pump energy (5.3 $\mu\text{m}$ [ $\mu\text{J}$ ])	Pump pulse length (FWHM [ns])	Pump beam size (FWHM [ $\mu\text{m}$ ])	Peak intensity ( $\text{MW}/\text{cm}^2$ )
OSC	80	2	400	20
Test	3500	150	300	20

Before optimizing the complete OSC system, two aspects of CdGeAs<sub>2</sub> must be investigated as there are trade-offs between crystal gain, wiggler length, bandwidth, and costs. The material of most samples of crystals absorbs relatively highly at the pump and signal wavelengths at the rate of 0.3–1.0 and 0.1–0.3 cm<sup>-1</sup>, respectively. The gain of the OPA exponentially depends on both the pump's intensity and the interaction length, which normally is the same as the crystal's total length. Neglecting pump absorption, the crystal length must be 3 cm for a gain of 160 000. However, the reduced intensity in the pump due to absorption in such a long crystal will severely lower the gain. Absorption is expected to fall as techniques for preparing crystals improve [16]. For example, it can be decreased dramatically by exposing the crystal to low energy electrons to introduce lattice defects that trap the free carriers primarily responsible for the high absorption [17]. In addition, by using several shorter crystals and introducing a new pump beam into every segment, we can compensate for the loss of pump energy from the remaining absorption.

The second closely related consideration is the damage threshold from thermal loading at high average power. Although CdGeAs has a thermal conductivity of 0.04 W/cm K, its thermal expansion coefficient of  $8.4 \times 10^{-6}$  is large, making it vulnerable to stress fracture. Cryogenic operation has been experimentally demonstrated to both increase thermal conductivity and reduce absorption [18] in samples of this and other nonlinear crystals, thereby raising the damage threshold. Because OSC of RHIC does not present fundamentally different conditions for the crystal, other than temporal structure, than those in the referenced work, cryogenic operation will allow significantly higher power to be sustained than at room temperature. We are investigating the thermal performance of this material, as well as the gain, bandwidth, and coherency of a CdGeAs<sub>2</sub>, based OPA using a test crystal.

Knowing the properties of the material, preliminary estimates were made of the behavior of a practical OPA based on the second harmonic CO<sub>2</sub> laser pumped CdGeAs<sub>2</sub>. We used standard analytical formulas for a single-pass OPA [19], and verified the results with the SNLO nonlinear optics code [20]. For this calculation we also assumed the material's nonlinearity and phase-matching condition described above, and the Sellmeier coefficients [15]. For the OSC of the RHIC, we assumed that a CO<sub>2</sub> pump laser produces a synchronous 2-ns pulse

train at the same repetition rate as the pickup radiation. The parameters are as follows: pump wavelength 5.3  $\mu\text{m}$ , pump peak intensity 20 MW/cm<sup>2</sup>, total crystal length 30 mm, and signal wavelength 12  $\mu\text{m}$ . In addition, we calculated the parameters of a test system we are now assembling from optics and lasers. These conditions are a single pump pulse of 150 ns at 5.3  $\mu\text{m}$ , signal wavelength 9.6  $\mu\text{m}$ , and the same pump intensity and crystal length as in the previous case. The latter value for the signal's wavelength is easily generated by CO<sub>2</sub> lasers, and is, therefore, easier to work with than the final desired wavelength of 12  $\mu\text{m}$ . Table I summarizes the initial parameters of our assessments. In both cases, we assume that the 10.6- $\mu\text{m}$  laser subsequently is frequency doubled to 5.3  $\mu\text{m}$  for OPA pumping, and the beam's size is optimized for a gain of  $2 \times 10^5$ .

Figure 2 shows the gain versus photon energy for the RHIC OSC case detailed above. The spectrum of the gain shows two characteristic peaks for the signal and idler wavelengths. The estimated bandwidth available for amplification is given by the FWHM of the 12  $\mu\text{m}$  gain peak, which is approximately 6% of the central energy of that peak.

#### IV. COOLING RATE CALCULATION FOR RHIC

We illustrated the efficiency of this new approach for OSC using a 12- $\mu\text{m}$  wavelength amplifier by investigating the cooling of a beam in BNL's RHIC, assuming the actual values of its lattice, machine functions, and various beam parameters: a beam of gold ions, with atomic mass  $A = 197$ , atomic number (and charge, for fully ionized

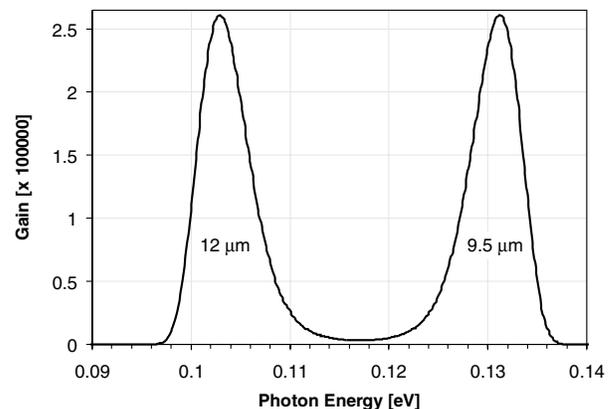


FIG. 2. Calculation of the gain of the OPA.

$^{79}\text{Au}$ )  $Z = 79$ , energy per nucleon of  $E_I = 100$  GeV,  $N_I = 1.2 \times 10^9$  ions per bunch, 120 bunches stored in each ring, an rms emittance of  $\varepsilon = 2.5 \times 10^{-8}$  m, and an energy spread of  $\sigma_\delta = 4 \times 10^{-4}$ . The ring's circumference is  $C = 3834$  m.

Figure 3 shows a possible setup for an OSC system in one of the RHIC's rings. We indicate the locations of the kicker and pickup undulators, the light path, and the optical amplifier. The regular part of the ring between the undulators serves as a bypass. An identical cooling system will be used in the other one of the RHIC's rings. This arrangement will reduce the ion beam's horizontal emittance and energy spread. Vertical emittance will not be directly cooled other than through coupling to the horizontal emittance, but this probably is acceptable since the excitation by IBS of vertical emittance is negligible [21], and can only be affected through the coupling to the horizontal plane.

The difference in the path lengths between the bypass and the optical path is only 2.4 cm even though both paths are approximately 200 m long. This is due to the large bending radius of RHIC's dipole magnets [22]. An additional time delay equivalent to approximately 2 cm path length accumulates due to the difference in the ion's velocity and the speed of light. Nevertheless, the total effective path-length differential of 4.4 cm is small and it suggests the need for a tight allotment of various time delays in the optical path and amplification. Certainly, a new layout for the bypass with a larger path-length differential can be designed, but in this study we elected not to change the RHIC's present configuration. The minimal modifications that we made (small change in the gradient in the eight quadrupoles around IP) were to adjust the time-of-flight matrix coefficients  $R_{51}$ ,  $R_{52}$ ,  $R_{56}$  of the bypass lattice (see the Appendix for a complete definition of these coefficients) and Twiss functions [23]  $\beta$ ,  $\alpha$ , dispersion function  $\eta$  and its derivative  $\eta'$  in the location of the undulators. For a modified bypass lattice we obtained

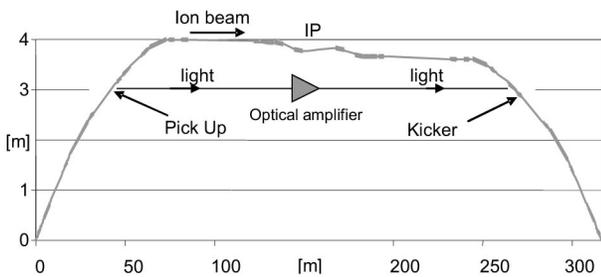


FIG. 3. Possible layout of a cooling system. The magnet-free area in the IP region is used for the pickup and kicker wigglers. (Focusing and bending magnets are shown on the plot with thickening of the lines. Vertical and horizontal axes show the scale of the transport line.)

$$\begin{pmatrix} R_{51} \\ R_{52} \\ R_{56} \end{pmatrix} = \begin{pmatrix} 0.001817 \\ 0.010511 \text{ m} \\ -0.004027 \text{ m} \end{pmatrix}, \quad \begin{pmatrix} \beta \\ \alpha \\ \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} 16.335 \text{ m} \\ -1.203 \\ 0.738 \text{ m} \\ 0.031 \end{pmatrix}. \quad (3)$$

These modifications gave us  $k\sigma_\Delta = 0.97$  and satisfied an important condition (A12) wherein particle delays in the bypass lattice yield the best cooling rate. We also tuned both damping partition numbers and obtained

$$\begin{aligned} f_x &= k\sigma_\delta(\eta R_{51} + \eta' R_{52}) = 0.35, \\ f_\delta &= -k\sigma_\delta(\eta R_{51} + \eta' R_{52} + R_{56}) = 0.50. \end{aligned} \quad (4)$$

Then, the optical amplifier with 16 W of average output power provides approximately 1 h of damping time.

## V. CONCLUSIONS

We explored the possibilities of optical-stochastic cooling of  $^{79}\text{Au}$  ions in the Relativistic Heavy Ion Collider. We showed that the cooling rate of heavy particles improves with increased wavelength as a square root of the wavelength. The optimal match for the RHIC is a  $12 \mu\text{m}$  operation wavelength. OSC can be implemented in RHIC using an optical parametric amplifier and relatively short undulators, all of which can be done with a modest investment. The projected cooling time is approximately 1 h. We also found that part of the RHIC regular lattice could be used as a bypass separating pickup and kicker undulators. It has open spaces that are long enough for the undulators, and only minor changes in the gradients of the magnetic field in few quadrupoles are need to tune the time-of-flight parameters of the bypass lattice to the desirable characteristics (A10)–(A14).

## APPENDIX

In this appendix we derive formulas for the cooling time calculation in a manner similar to Ref. [3], but for an arbitrary ion-beam transport channel. First, we assess the acceleration (deceleration) of a charged particle in the kicker undulator by the amplified signal of its own radiation emitted in the pickup undulator. It can be represented as the result of destructive (constructive) interference in the far-field region between the field of spontaneous radiation of the particle in the undulator  $E_r$  and the field of the amplified signal  $E_s$  [24]:

$$\Delta E = -2 \frac{c}{4\pi} \int E_s E_r dS \frac{d\omega}{2\pi}, \quad (A1)$$

where the integral is calculated over the area  $S$  and frequency. The amplified signal can be focused into the kicker undulator so that it has the same spatial configuration in the far-field region as spontaneous radiation. This is because the signal originates from the same spontaneous radiation produced in the pickup undulator, which

is similar to the kicker undulator. We also assume that the amplifier does not spatially distort the signal. Therefore,

$$\Delta E = -2\sqrt{A_s A_r} \frac{\Delta \omega_s}{\Delta \omega_r} \sin(\phi) = G E_b \sin(\phi), \quad (\text{A2})$$

where  $\phi$  is the phase difference between the radiation and signal fields that is due to the delay in the arrival of a particle in the kicker undulator and the arrival of its amplified signal,  $A_s$  is the energy of the pulse of spontaneous radiation, same as  $\delta E$  defined in Eq. (2),  $A_r$  is the energy of the pulse of the amplified signal, the ratio of the amplifier's bandwidth  $\Delta \omega_s$  to the bandwidth of the undulator's radiation  $\Delta \omega_r$  accounts for how much longer is the pulse of the amplified signal than that of spontaneous undulator radiation,  $\Delta \omega_s / \Delta \omega_r = N_u \Gamma$ , and  $G = -\Delta E / E_b$ , where  $E_b$  is the beam's energy.

Now we calculate the damping time. We define  $X = (x, x', s, \delta)^T$  as the particle 4D coordinate vector, where  $x, x'$  are transverse coordinates and angles,  $s$  is the longitudinal coordinate, and  $\delta$  is the particle's relative energy offset. Additionally, we use definition  $x = x_\beta + \eta \delta$  and  $x' = x_{\beta'} + h' \delta$ , where  $x_\beta$  and  $x_{\beta'}$  are the particle's betatron coordinate and angle, and  $\eta, \eta'$  are the dispersion function and its derivative. We identify the pickup undulator at a position  $A$  in the optics of the storage ring, and the kicker undulator at the position  $B$ . The beam transport from  $A$  to  $B$  can be written as  $X_B = R X_A$ . Consequently,  $X_A = R^{-1} X_B$  and we define

$$R^{-1} = \begin{bmatrix} R_{11} & R_{12} & 0 & R_{16} \\ R_{21} & R_{22} & 0 & R_{26} \\ R_{51} & R_{52} & 1 & R_{56} \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (\text{A3})$$

It is convenient to introduce new variables  $u = x_\beta$  and  $v = x_{\beta'} + \alpha / \beta u$ , where  $\alpha$  and  $\beta$  are Twiss functions in the location of the kicker undulator, and write the amplitude of the horizontal betatron oscillations

$$a^2 = \frac{1 + \alpha^2}{\beta} x_\beta^2 + 2\alpha x_\beta x_{\beta'} + \beta x_{\beta'}^2 = \frac{1}{\beta} u^2 + \beta v^2. \quad (\text{A4})$$

We use similar substitutions for a dispersion function  $\eta_u = \eta$  and  $\eta_v = \eta' + \alpha / \beta \eta_u$ . With these new variables, the path-length difference on the trajectory from  $A$  to  $B$  written in terms of particle coordinates at a location  $B$  and taken relative to the equilibrium orbit is equal to

$$\begin{aligned} \Delta \ell &= -(R_{51} x + R_{52} x' + R_{56} \delta) \\ &= -\left\{ \left( R_{51} - \frac{\alpha}{\beta} R_{52} \right) u + R_{52} v \right. \\ &\quad \left. + \left[ \left( R_{51} - \frac{\alpha}{\beta} R_{52} \right) \eta_u + R_{52} \eta_v + R_{56} \right] \delta \right\}. \quad (\text{A5}) \end{aligned}$$

The signal must be delayed to let the particle enter the kicker undulator ahead of the signal. Moreover, the path length for a signal including the delay in the amplifier

must be chosen such that the equilibrium particle comes to the kicker undulator exactly at the crossover of the electric field with the electromagnetic wave of the signal. Then, the phase difference for a nonequilibrium particle is equal to

$$\varphi = k \Delta \ell. \quad (\text{A6})$$

Using (A2), we write the particle energy right after the energy kick as

$$\delta_c = \delta + G \sin(\varphi). \quad (\text{A7})$$

This energy kick also changes the particle's betatron amplitude because of dispersion in the kicker undulator:

$$\Delta a^2 = -2G \left( \frac{1}{\beta} u \eta_u + \beta v \eta_v \right) \sin(\varphi). \quad (\text{A8})$$

Now, assuming a Gaussian distribution of beam particles in  $u, v$ , and  $\delta$ , i.e.,  $\rho = (2\pi)^{-3/2} (\sigma_u \sigma_v \sigma_\delta)^{-1} \exp\{-u^2/2\sigma_u^2 - v^2/2\sigma_v^2 - \delta^2/2\sigma_\delta^2\}$ , where  $\sigma_u^2 = \beta \varepsilon$  and  $\sigma_v^2 = \varepsilon / \beta$ , and using (A6), we find

$$\begin{aligned} \frac{\overline{\Delta a^2}}{a^2} &= 2Gk \left[ \eta_u \left( R_{51} - \frac{\alpha}{\beta} R_{52} \right) + \eta_v R_{52} \right] \\ \exp\left\{-\frac{k^2 \sigma_\Delta^2}{2}\right\} &= 2Gk (\eta R_{51} + \eta' R_{52}) \exp\left\{-\frac{k^2 \sigma_\Delta^2}{2}\right\}, \quad (\text{A9}) \end{aligned}$$

where  $\sigma_\Delta$  is the rms of  $\Delta \ell$ :

$$\begin{aligned} \sigma_\Delta^2 &= \left( R_{51}^2 \beta - 2\alpha R_{51} R_{52} + \frac{R_{52}^2}{\beta} \right) \varepsilon \\ &\quad + (R_{51} \eta + R_{52} \eta' + R_{56})^2 \delta^2. \quad (\text{A10}) \end{aligned}$$

Similar to (A9), we obtain

$$\frac{\overline{\Delta \delta^2}}{\delta^2} = -2Gk (\eta R_{51} + \eta' R_{52} + R_{56}) \exp\left\{-\frac{k^2 \sigma_\Delta^2}{2}\right\}, \quad (\text{A11})$$

where  $\Delta \delta^2 = \delta_c^2 - \delta^2$ . For optimal cooling rate, we must maintain the condition

$$k \sigma_\Delta \approx 1. \quad (\text{A12})$$

Using (A12) we write for cooling decrements of horizontal and energy oscillations:

$$\alpha_x = -\frac{\overline{\Delta a^2}}{a^2} = -2G \frac{k}{\sqrt{e}} (\eta R_{51} + \eta' R_{52}) = -\frac{2G}{\sigma_\delta \sqrt{e}} f_x, \quad (\text{A13})$$

$$\begin{aligned} \alpha_\delta &= -\frac{\overline{\Delta \delta^2}}{\delta^2} = 2G \frac{k}{\sqrt{e}} (\eta R_{51} + \eta' R_{52} + R_{56}) \\ &= -\frac{2G}{\sigma_\delta \sqrt{e}} f_e, \quad (\text{A14}) \end{aligned}$$

where the dimensionless parameters  $f_x$  and  $f_e$  assume the meaning of damping partition numbers.

By defining  $P$  as  $P = A_s N_i f_L$  and  $I = q f_b N_i$ , where  $N_i$  is the number of particles per bunch, we obtain Eq. (1) using formulas (A1), (A13), and (A14).

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