DREAM OF ISOCHRONOUS RING, AGAIN

Hiroyuki Hama\textsuperscript{a}, Laboratory of Nuclear Science, Graduate School of Science, Tohoku University, 1-2-1 Mikamine, Taihaku-ku, Sendai 982-0826, Japan

\textbf{Abstract}

Experimental and theoretical studies of the isochronous synchrotron radiation ring, so far, suggest nonlinear effect resulted from higher order dispersion and chromaticity declines the “complete” isochronous system [1]. In addition, we have recently found that path-length deviation originated from the betatron motion is a much serious obstacle to preserve the bunch length [2].

However, it has been found out that a careful lattice design for the path-length cancellation seems to be very promising, at least for a wavelength region of THz. Tolerance of the path length along a turn of the ring may be within our reach. A concept to preserve bunch form-factor (or very short bunch) by using quasi-dispersion-free arcs having proper betatron phase advances in order to produce coherent THz radiation from every bending section. Furthermore, its extension toward prebunched-FEL and SASE-FEL will be also discussed.

\textbf{INTRODUCTION}

More than 20 years ago, D.A.G. Deacon proposed an isochronous storage ring for FEL to avoid bunch heating and decreasing a small signal gain [3]. Some of low momentum compaction operations have been carried out to research fundamental beam dynamics in the storage rings and reduction of equilibrium bunch lengths because the natural bunch length is proportional square root of momentum compaction factor ($\alpha$).

Recently coherent infrared radiation is observed on a 3rd generation light source, BESSY-II [4]. Because the 3rd generation rings are optimized to obtain very low emittance beam, the dispersion function in the arc sections are much reduced by introducing large bending radius, the momentum compaction factors are inevitably small and it might be relatively easy to quasi isochronous system. On the other hand, recently N.A. Vinokurov et al. proposed a ring type SASE-FEL based on a complete isochronous transport arc [5].

On the other hand, they have been discussed what the next generation (or 4th generation) light source is for a long period, and in consequence the most promising source of short wavelength radiation (below soft X-rays) shall be SASE FELs. Proof of principle of the SASE FEL was firstly demonstrated at TESLA-TTF by power saturation of FEL pulses at a wavelength of 80 nm [6], which was soon followed by a visible SASE FEL at the APS-LEUTL and BNL-VISA [7]. At present, a number of linac-based SASE-XFEL projects are being earnestly considered worldwide [8]. Those XFEL projects are going to cover higher photon energy around 10 keV with peak brilliance of $10^{24}$ photons/sec/mm$^2$/mrad$^2$/0.1%-Bandwidth. Time averaged brilliance of $10^{24}$ is also seems to be inside the target brightness frame, which will be brought by higher repetition rate and/or longer macropulse burst of the linac. Progress of superconducting accelerating structure is also another motive power for such an extremely high brilliant photon source.

Another candidate for the next generation light source is high brilliant radiations from adiabatic damped beam via high energy linacs connected with an extremely low emittance electron gun. Energy recovery linacs (ERLs) are the concept necessarily grown as high performance photoinjectors have been developed [9]. The normalized emittance of extracted beams from the photocathode RF guns presently reaches 1\,\pi mrad with ~ 1 nC charge, so that the beam accelerated up to 5 GeV performs 0.1 nmrad emittance in both the directions. Target brilliances of the ERLs at the X-ray region are mostly 2 orders above the 3rd generation light source, even in the time averaged domain.

In order to reach such higher brightness, the high intensity CW-like beam is of course necessary, which is being developed by using the advanced superconducting linacs [10]. For that reason, a consumed power for beam acceleration has to be required to collect from the used beam to save huge electricity, which is namely the energy recovery. The ERL seems to be a quite challenging concept because of many key technologies, i.e., pretty high brilliant electron gun in the 6-dimensional phase space, high field gradient superconducting accelerating structures, high efficient energy recovery and power recirculation. In addition, the round beam and few hundreds femtosecond pulse in an ERL offer new fascinating applications. Combined complex light sources of the ERL and the SASE FEL has been already proposed [11]. Vinokulov’s proposal is one of such next generations light source.

These rapid progresses, particularly in linac technologies, will be a fruitful competition for the next light source, we should notice there are many common technologies and a conceptual approach to be studied and considered, respectively.

However is the X-rays only the future light source in various field? To the contrary of approaching the X-ray targets, we have, however, thought such advanced linac beams is able to give many scientific opportunities by means of low energy coherent photons like THz radiations by using an isochronous ring configuration.

\textbf{SUB-PICOSECOND SHORT-BUNCH FROM A THERMIonic RF GUN}

These years, an RF gun assisted by advanced technologies of the laser and the cathode has reached to offer a charge of 1nC and nearly 1 $\pi$ nm mrad emittance.
In addition, the bunch compress skill is also developed to realize a sub-picosecond bunch length [12]. Such a frontier of the electron source is, however, not developed as common technologies and just for specific big projects.

The RF gun was invented many years ago, but not with a laser photo cathode [13]. Since a problem of extraordinary cathode heating due to an effect of beam back-bombardment was found soon, the thermionic RF gun has not been spread out worldwide. Today, the laser photo-cathode RF gun is being regularly employed for the SASE projects except SCSS at SPring-8 [14].

If the thermionic cathode is suitable for the other type of RF gun, it would be very convenient even for conventional use of small linacs. We have investigated the possible maximum performance of the thermionic RF gun [15]. A key issue for production the high brilliant beam might be proper geometry of the gun and fed powers for each cell including effect of the beam wakefield, which were found out through a 3-D simulation using an FDTD (Finite Difference Time Domain) method as a Maxwell’s equations solver [16].

**Velocity Bunching in ITC-RF Gun**

Detail of independently tunable cells (ITC) RF gun that has been under designing is reported elsewhere [17]. Since the electron is extracted at just the sign of the electric field changes from the thermionic cathode, so those extracted a bit after the first electron extracted may gain higher velocities. If we choose proper strengths of the electric fields and a length between cells, the same “velocity-bunching” like effect will happen at the entrance of the electric field of the 2nd cell.

![Figure 1: Calculated longitudinal electric fields excited in each cell. One can see the coupling between cells is almost negligible.](image)

Since the FDTD method includes fully space charge force and/or beam induced field, the simulation is expected to be close to actual one. At the moment, we have obtained the shortest microbunch peak of 100 fs (standard deviation) at the head of the continuum beam extracted from the ITC-RF gun. Since the correlation between the energy and the time is approximately still linear, we will select the bunch length and the energy spread by a slit in a dispersive arc.

![Figure 2: Example of simulated beam time spectrum from the ITC-Gun with 100 A/cm² cathode current. The spectrum is selected within a 1% energy width from a top energy electron.](image)

Because of a thermionic cathode is used, an electron charge bunched at the head of the beam train is not so large. However a couple of hundreds fs for the bunch length and the several ten pC charge may be obtained.

**Coherent THz Radiation**

It is well-known the wavelength region of synchrotron radiation (SR) longer than the bunch length is emitted coherently [18]. A very simple expression for the intensity of coherent part of SR can be written as

\[
I_{\text{int}}(\omega) = I_{\text{in-coh}}(\omega) \left( N_e + N_e^2 S(\omega) \right),
\]

where \( N_e \) is the number of electrons and \( S(\omega) \) is a form factor of the electron bunch. Ignoring transverse effect from the emittance, the bunch form factor is a Fourier transform of the bunch shape as

\[
S(\omega) = \int \rho(z) e^{i\omega z/dz} \left( \int \rho(z) dz = 1 \right).
\]

For the Gaussian shape of the bunch, the standard deviation of the angular frequency is just the inverse of the bunch length, so that the bunch length of 100 fs corresponds to 1.6 Terahertz (THz) (~188 μm).

Consequently, the electron beam of the bunch length of 100 fs is suitable to produce the coherent THz SR, less than mm-wavelength.
CONCEPT OF ISOCRONOUS RING TO PRODUCE COHERENT TERAHERTZ SR

Even for linac FELs, a THz FEL is a bit difficult because of qualities of lower energy beams. Of course development of high quality THz FEL sources should be one of the targets, because use of THz radiation is rapidly spreading worldwide.

Usually, after passing through a bending magnet, the short bunch, which can emit coherent far infrared SR, is collapsed because of the path length deviation of each electron due to the dispersion function and the betatron motion. Except higher order effect of the dispersion function, the first order of momentum compaction factor is not difficult to be reduced to zero. At the moment, quasi-isochronous operation mode of the storage rings is preserve mm or several ps range [4, 19].

Path-Length Deviation Due to Betatron Motion

Path length deviation due to momentum deviations of each electron in a ring can be written as

$$\frac{\Delta C}{C} = \alpha_0 \frac{\Delta p}{p} + \alpha_1 \left( \frac{\Delta p}{p} \right)^2 + \alpha_2 \left( \frac{\Delta p}{p} \right)^3 + O(3) + ? \quad (3)$$

However Eq. 3 represents only the longitudinal dynamics in the ring. In general, the dispersion function increases as decreasing the bending radius, so that the 3rd generation light sources having large bending radiiuse have brought low dispersion function, thus low momentum compaction factor.

We have thought there would be only very long straight undulator line to extract the maximum performance of the advanced linac beam. This was a sincere motivation to study the beam dynamics in isochronous ring, again.

The equilibrium bunch length in the storage ring is proportional to the square root of $\alpha$ as

$$\sigma_{\text{Bunch}} \propto \sqrt{\alpha} \quad , \quad (4)$$

and $\alpha$ is calculated by a integral of the dispersion function. If we introduce some negative parts of the dispersion function,

$$\alpha = \frac{1}{C} \int_0^C \frac{\eta(s)}{\rho(s)} ds \sim 0 \quad (5)$$

is actually possible. This was a basic concept of isochronous storage ring like a New-SUBARU [19].

Here we assume a very small average dispersion function of $= 0.1$ m and a very sharp relative energy spread of $5 \times 10^{-3}$ and the bunch length of 100 fs (standard deviation) for an injected beam from an advanced linac. If we assume a bending radius of the ring dipoles of 3 m assumed, the estimated path length deviation is only ~ 50 fs, so that a convoluted bunch length is only 112 fs, which is not so significant to obtain the coherent THz radiation.

The path length is calculated from an equation,

$$L = \int_0^C \sqrt{1 + \frac{x}{\beta}} \frac{dx}{ds} + \frac{dy}{ds} \, ds$$

$$= \int_0^C \left[ 1 + \frac{1}{2} \left( \frac{2x}{\rho} + \left( \frac{x}{\beta} \right)^2 \right) + \left( \frac{dx}{ds} \right)^2 + \left( \frac{dy}{ds} \right)^2 - \frac{1}{2} \left( \frac{2x}{\rho} \right)^2 \right] ds + O(\Delta^3)$$

$$\quad (6)$$

The first order of the length deviation $\Delta L$ is caused in the bending magnets, so that

$$\Delta L = \int_0^C \frac{x}{\beta} ds + O(\Delta^2) \quad , \quad (7)$$

where the vertical betatron amplitude is also treated as a second order term. Eq. 7 is a general description of the path length deviation rather than $\alpha$, so that we estimate the path length deviation resulted from the betatron motion.

Assuming the horizontal emittance is 50 nmrad, the bending radius of 3 m and an averaged betatron function of 1.5 m, the path length deviation of 272 um (907 fs) is obtained for 180° bending arc, which is 20 times large than previous value resulted from the momentum deviation. Of course it depends various parameters of the beam and the ring (see Table 1), at least we can conclude that cancellation of path length difference due to the betatron motion is significant for the isochronous transport system.

<table>
<thead>
<tr>
<th>Table 1: Parameters to estimate the path length deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalized emittance $\varepsilon_n$</td>
</tr>
<tr>
<td>Energy spread $\Delta p/p$</td>
</tr>
<tr>
<td>Averaged dispersion $\eta_{ave}$</td>
</tr>
<tr>
<td>Averaged $\beta_i$-function $\beta_i_{ave}$</td>
</tr>
<tr>
<td>Bending radius $\rho$</td>
</tr>
<tr>
<td>Beam energy $E$</td>
</tr>
</tbody>
</table>

Cancellation Path-Length Deviation

The betatron motion is written as,

$$x_\beta = \sqrt{\mathcal{E}} \beta_i \left( s \right) \cos \left( \psi(s) + \theta_s \right) \quad , \quad (8)$$

then using Eq. 7, we obtain
\[ \Delta L = \int_0^C \frac{x}{\rho} ds = \int_0^C \frac{\sqrt{E_x(s) \cos \left( \psi(s) + \phi_0 \right)}}{\rho} ds \]  

\( (9) \)

Figure 3: Conceptual 180°-bend arc and the estimated path length deviations. Note the 80°-bend does not necessarily consist of one dipole only.

If we have identical 8 unit cells for 180° arc, \( \Delta L \) originated from the betatron motion is written as,

\[ \Delta L = F \left[ \sin \left( \psi_a + \phi_0 \right) - \sin \left( 0 + \phi_0 \right) \right] \]

\[ + F \left[ \sin \left( 2\psi_a + \psi_s + \phi_0 \right) - \sin \left( \psi_a + \psi_s + \phi_0 \right) \right] \]

\[ + F \left[ \sin \left( 2\psi_a + \psi_s + \phi_0 \right) - \sin \left( \psi_a + \psi_s + \phi_0 \right) \right] \]

\[ + F \left[ \sin \left( 3\psi_a + 2\psi_s + \phi_0 \right) - \sin \left( 2\psi_a + 2\psi_s + \phi_0 \right) \right] \]

\[ + F \left[ \sin \left( 4\psi_a + 3\psi_s + \phi_0 \right) - \sin \left( 3\psi_a + 3\psi_s + \phi_0 \right) \right] \]  

where \( \psi_a \) and \( \psi_s \) are phase advances in a bending magnet and between bends, respectively.

After some mathematical manipulation, Eq. 11 becomes to be

\[ \Delta L = F \left[ \sin \left( \phi_0 \right) \cdot \left[ \cos \left( \psi_a \right) + \cos \left( 2\psi_a + \psi_s \right) + \cos \left( 3\psi_a + 2\psi_s \right) \right] \]

\[ + \cos \left( 4\psi_a + 3\psi_s \right) + \cos \left( 5\psi_a + 4\psi_s \right) + \cos \left( 6\psi_a + 5\psi_s \right) + \cos \left( 7\psi_a + 6\psi_s \right) \]

\[ + \cos \left( 8\psi_a + 7\psi_s \right) - \cos \left( \psi_a + \psi_s \right) \]

\[ - \cos \left( 2\psi_a + 2\psi_s \right) - \cos \left( 3\psi_a + 3\psi_s \right) - \cos \left( 4\psi_a + 4\psi_s \right) \]

\[ - \cos \left( 5\psi_a + 5\psi_s \right) - \cos \left( 6\psi_a + 6\psi_s \right) - \cos \left( 7\psi_a + 7\psi_s \right) \]  

\( (12) \)

Here Eq. 12 is simply re-written as

\[ \Delta L = F \cdot \sin \left( \phi_0 \right) \cdot A_s + F \cdot \cos \left( \phi_0 \right) \cdot A_c \]  

\( (13) \)

Averaging over whole initial phase gives

\[ \left< \Delta L \right> = F \sqrt{\frac{A_s^2 + A_c^2}{2}} \]  

\( (14) \)

then

\[ \text{Coeff} = \sqrt{\frac{A_s^2 + A_c^2}{2}} \]  

\( (15) \)

is obtained.

After some numerical calculations, it was found that \( \text{Coeff} \) becomes 0 at the exit of 180° arc if following conditions are satisfied [20].

\[ \psi_a = 0.125 \times 2\pi \quad \text{or} \quad 0.250 \times 2\pi \]

\[ \psi_s = n \times 0.125 \times 2\pi \quad (n: \text{integer number}) \]  

\( (16) \)
Cancellation Path-Length Deviation with Dispersion Suppressor

When we assume a racetrack type ring, dispersion suppressors are required at both the ends of the 180° arc. Cancellation condition is now going to find for the 180° arc consisted with identical 6 bends and two dispersion suppressors.

From previous discussion and formulation, \( \text{Coeff} \) at the both sides of 6-bend is 0 with parameters
\[
\begin{align*}
\psi_D &= 0.250 \times 2\pi \\
\psi_S &= 5 / 24 \times 2\pi
\end{align*}
\]

For the dispersion suppressors, the path length deviation is calculated as
\[
\Delta L = F \cdot [\sin (\psi_D + \phi_0) - \sin (0 + \phi_0)] \\
+ F \cdot [\sin (2\psi_D + 2\psi_S + \psi_{\text{unit-cell}} + \phi_0) - \sin (\psi_D + 2\psi_S + \psi_{\text{unit-cell}} + \phi_0)]
\]
\[
= F \cdot [\sin (\psi_D) \cdot (\cos (\psi_D) - 1 + \cos (2\psi_D + 2\psi_S + \psi_{\text{unit-cell}})) \\
+ F \cdot \sin (2\psi_D + 2\psi_S + \psi_{\text{unit-cell}}) - \sin (\psi_D + 2\psi_S + \psi_{\text{unit-cell}})]
\]
\[
= F \cdot \sin (\phi_0) \cdot A_S + F \cdot \cos (\phi_0) \cdot A_C
\]

where \( \psi_D \) and \( \psi_S \) are phase advances in a bending magnet and other space in the dispersion suppressors, respectively. The phase advance of the 6-cell arc indicates \( \psi_{\text{unit-cell}} \).

Since \( \Delta L \) of the 6-unit-cell between the dispersion suppressors is already cancelled out, one can easily predict that the phase difference between the both sides of the 180° arc should be \( \pi \) to cancel \( \Delta L \) out.

**DEIGNED LATTICE AND PROOF OF THE PATH LENGTH COMPENSATION**

Figure 4 shows an example of 180° lattice and optical functions, which is still under designing. All 6-unit-cell bending magnets include focusing quadrupole moment and may be sextupole moment too. The bends at the dispersion suppressors include defocusing quad. We, of course, need certain length of the straight sections for beam injection and/or inserion devices, but we have not decided everything yet.

As one can see in Fig. 4, the dispersion function is very low. Because we introduce focus quad in the bend, the dispersion function is suppressed while keeping the horizontal beta function at lower. In order confirm the path length compensation, we have numerical beam tracking by using Hamiltonian equations in Curvilinear system to reduce numerical errors.

**SIMULATION OF THE COHERENT SYNCHROTRON RADIATION**

In general, the bunch form factor at a certain frequency is close to 1, the coherent radiation of the wavelength
corresponds to the frequency can be emitted. However transverse effect is not negligible, so that actual spectrum of the coherent radiation is not easy to be predicted. In order to estimate what spectrum can be obtained from the lattice indicated in Fig. 4, a calculation for radiation was performed by using the integral of Lienard-Wiechert potential [21],

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2}{4\pi^2c} \left| \int_{-\infty}^{\infty} e^{i\omega(t + R'(t)/c)} \frac{n \times (n - \beta \times \beta)}{(1 - \beta \cdot n)^2} dt' \right|^2.$$ \hspace{1cm} (18)

The double differential equation, which is a power spectrum of the radiation, should be evaluated at the time of the retarded time $t + R'(t)/c$, where $R$ is the distance between the electron and the observation point at a moment of the photon emission.

The simulation has done for a geometry indicated by Fig. 6.

Figure 6: Geometry for the simulation of radiation. Distance between an observation point and a radiation point on an ideal orbit is 5 m.

Since the coherent radiation is brought by many particle system, the in-coherent part and the coherent part of the synchrotron radiation is not normalized by an identical factor. However the interference with radiation emitted by a particle nearby can be seen if the bunch length is sufficiently short even small number of particles (in other words we do not have to employ actual huge number of electrons).

In this case the critical energy of SR is 5.9 eV (210 nm), which is clearly seen in the 2-dimesional spectrum. In addition, an intensity of the coherent THz part does not depends on the bending angle, which means the bunch length is kept at almost constant.

Figure 8: Calculated Spectra for different bending angles with one particle and 1000 particles. Intensity of coherent part is $10^6$ times higher than that of one-particle is clearly seen. Because of the coherent edge radiation, the intensity at long wavelength is extraordinarily higher at the large angle.

**SUMMARY AND PROSPECT**

The significant path difference due to the betatron motion is discussed for the isochronous system. The isochronous ring may have higher potential for the coherent SR and SASE-FEL or prebunched-FEL, if the complete system is realized. At least a ring for coherent THz source may be possible.

Further design of the ring and the RF gun are under way.

**REFERENCES**


